The Public Authority for Applied Education and Training

College of Technological Studies

Department of Civil Engineering Technology

CE 265 Reinforced Concrete (I)

Supplemental Class Notes

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CONTENTS

CHAPTER 1 REINFORCED CONCRETE STRUCTURES

1.1 INTRODUCTION

Many structures are built of reinforced concrete: bridges, viaducts, buildings, retaining walls, tunnels, tanks, conduits, and others.
Reinforced concrete is a logical union of two materials: plain concrete, which possesses high compressive strength but little tensile strength, and steel bars embedded in the concrete, which can provide the needed strength in tension.
First practical use of reinforced concrete was known in the mid-1800s. In the first decade of the 20th century, progress in reinforced concrete was rapid. Since the mid-1950s, reinforced concrete design practice has made the transition from that based on elastic methods to one based on strength.
Understanding of reinforced concrete behavior is still far from complete; building codes and specifications that give design procedures are continually changing to reflect latest knowledge.

1.2 REINFORCED CONCRETE MEMBERS

Every structure is proportioned as to both architecture and engineering to serve a particular function. Form and function go hand in hand, and the beat structural system is the one that fulfills most of the needs of the user while being serviceable, attractive, and economically cost efficient. Although most structures are designed for a life span of 50 years, the durability performance record indicates that properly proportioned concrete structures have generally had longer useful lives.
Reinforced concrete structures consist of a series of "members" (components) that interact to support the loads placed on the structures.
The components can be broadly classified into:

1. **Floor Slabs**

Floor slabs are the main horizontal elements that transmit the moving live loads as well as the stationary dead loads to the vertical framing supports of a structure. They can be:

- Slabs on beams,
- Waffle slabs,
- Slabs without beams (Flat Plates) resting directly on columns,
- Composite slabs on joists.

They can be proportioned such that they act in one direction (one-way slabs) or proportioned so that they act in two perpendicular directions (two-way slabs and flat plates).

2. **Beams**

Beams are the structural elements that transmit the tributary loads from floor slabs to vertical supporting columns. They are normally cast monolithically with the slabs and are

structurally reinforced on one face, the lower tension side, or both the top and bottom faces. As they are cast monolithically with the slab, they form a T-beam section for interior beams or an L beam at the building exterior.
The plan dimensions of a slab panel determine whether the floor slab behaves essentially as a one-way or two-way slab.

3. **Columns**

The vertical elements support the structural floor system. They are compression members subjected in most cases to both bending and axial load and are of major importance in the safety considerations of any structure. If a structural system is also composed of horizontal compression members, such members would be considered as beam-columns.

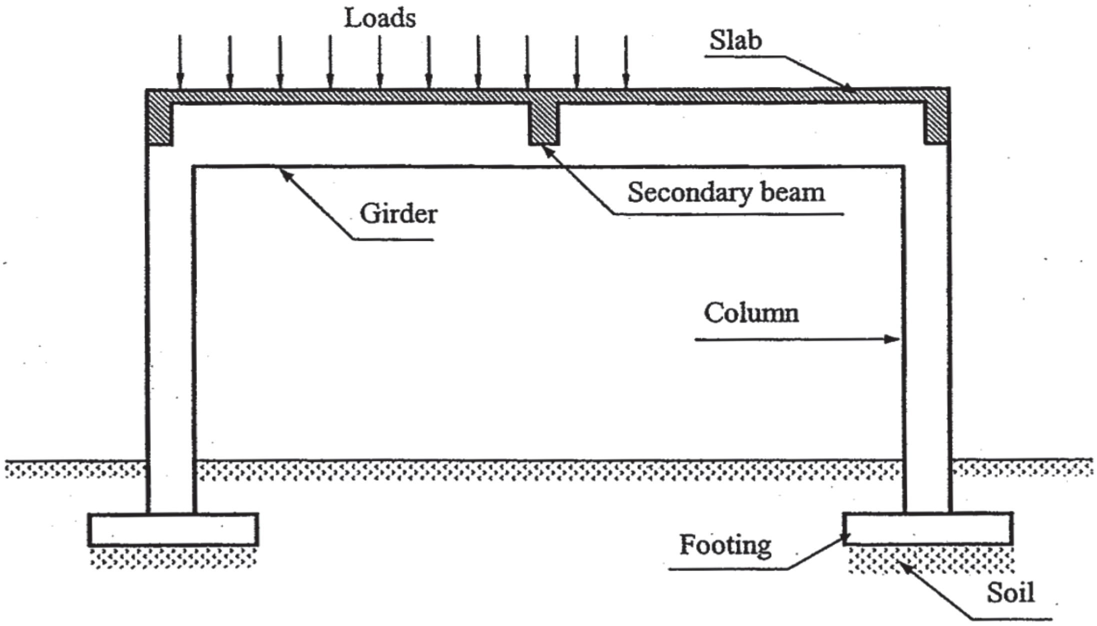
4. **Walls**

Walls are the vertical enclosures for building frames. They are not usually or necessarily made of concrete but of any material that esthetically fulfills the form and functional needs of the structural system. Additionally, structural concrete walls are often necessary as foundation walls, stairwell walls, and shear walls that resist horizontal wind loads and earthquake-induced loads.

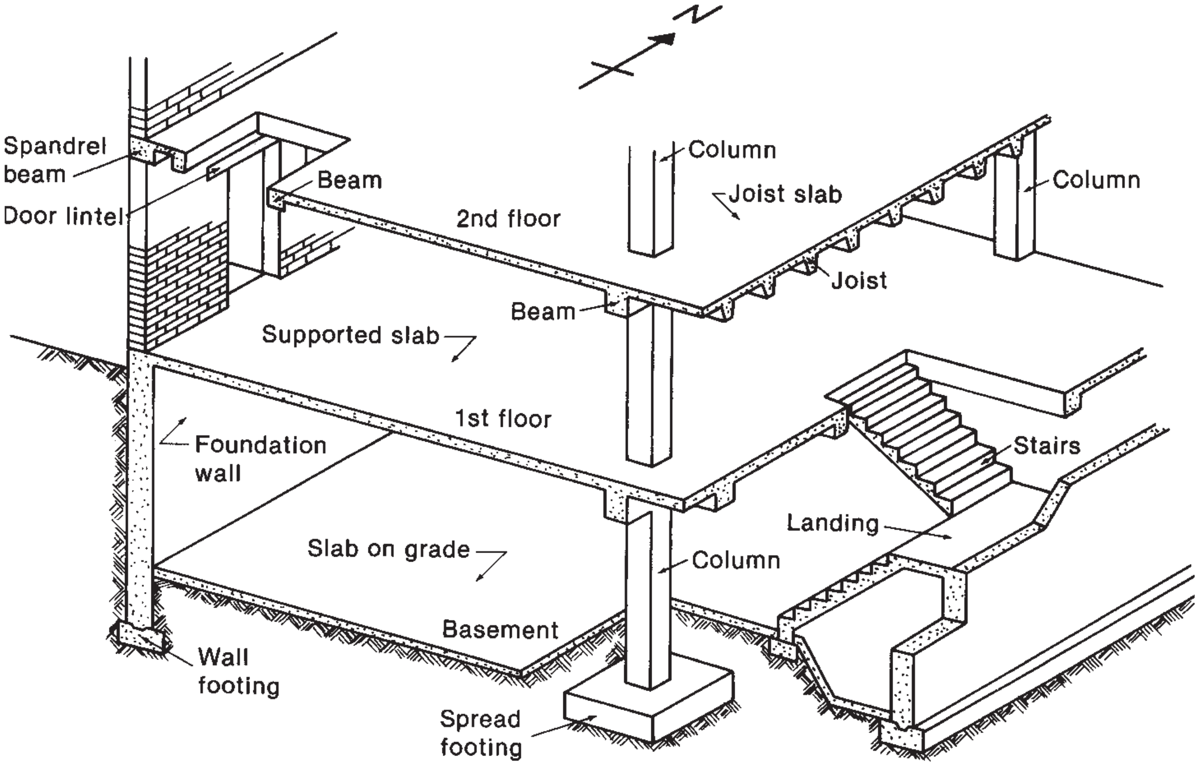
5. **Foundations**

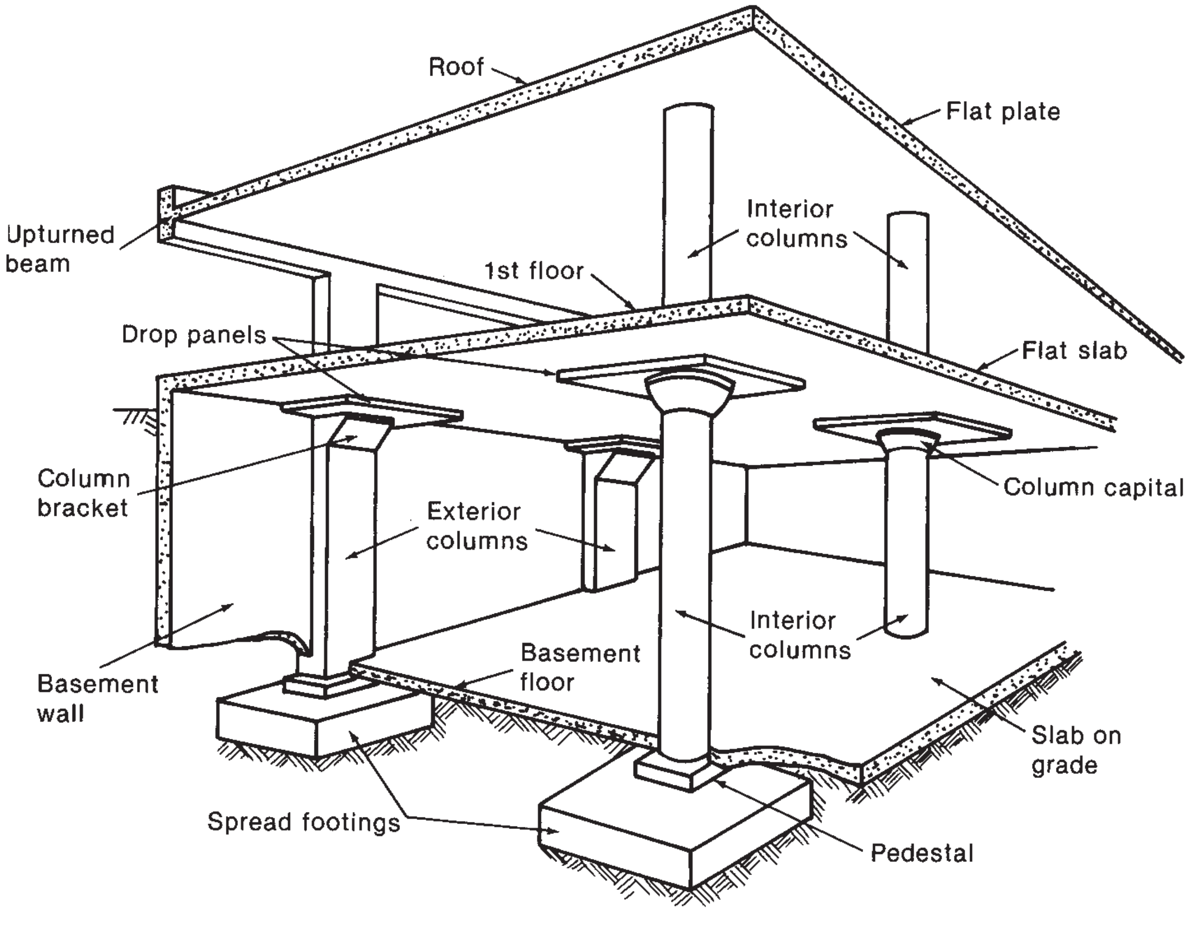
Foundations are the structural concrete elements that transmit the weight of the superstructure to the supporting soil. They could be in many forms:

- Isolated footing - the simplest one. It can be viewed as an inverted slab transmitting a distributed load from the soil to the column.
- Combined footings supporting more than one column.
- Mat foundations, and rafts which are basically inverted slab and beam construction.
- Strip footing or wall footing supporting walls.
- Piles driven to rock.



Typical reinforced concrete structural framing system

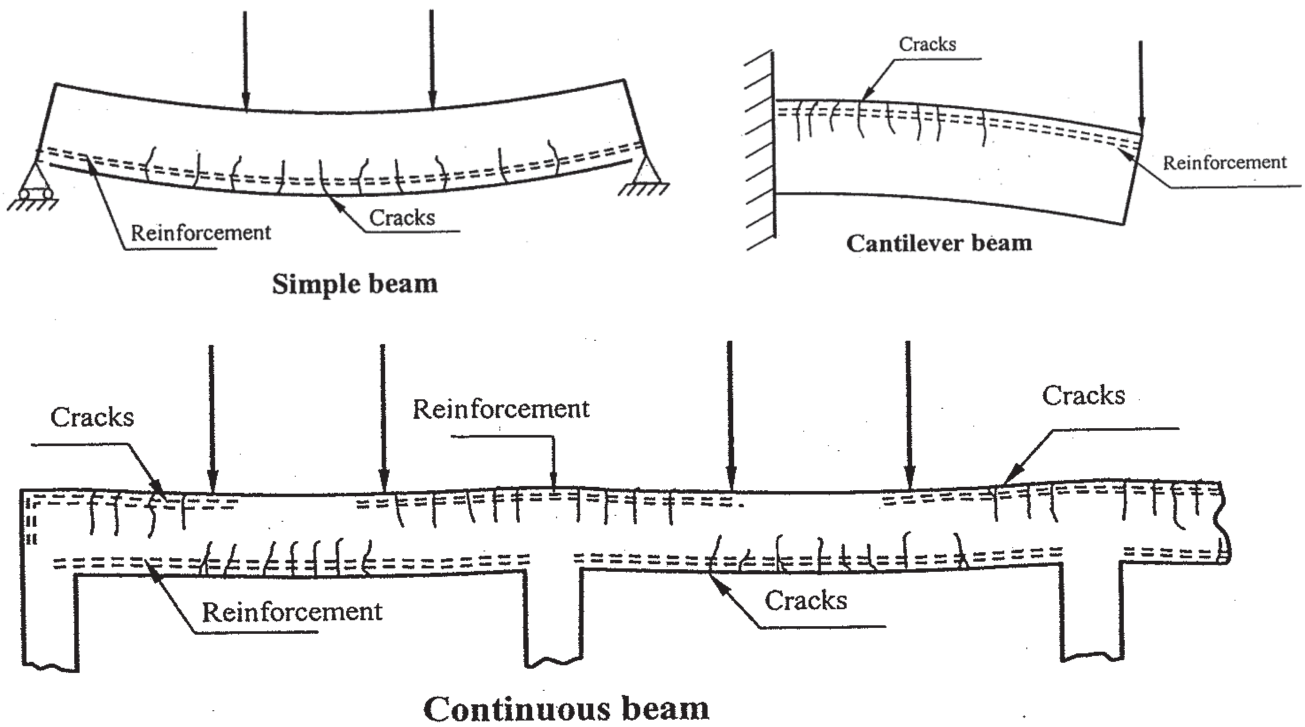




Reinforced concrete building elements

1.3 REINFORCED CONCRETE BEHAVIOR

The addition of steel reinforcement that bonds strongly to concrete produces a relatively ductile material capable of transmitting tension and suitable for any structural elements, e.g., slabs, beam, columns. Reinforcement should be placed in the locations of anticipated tensile stresses and cracking areas. For example, the main reinforcement in a simple beam is placed at the bottom fibers where the tensile stresses develop. However, for a cantilever, the main reinforcement is at the top of the beam at the location of the maximum negative moment. Finally for a continuous beam, a part of the main reinforcement should be placed near the bottom fibers where the positive moments exist and the other part is placed at the top fibers where the negative moments exist.



Reinforcement placement for different types of beams

CHAPTER 2 MATERIALS, AND PROPERTIES

2.1 CONCRETE

Plain concrete is made by mixing cement, fine aggregate, coarse aggregate, water, and frequently admixtures.
Structural concrete can be classified into:

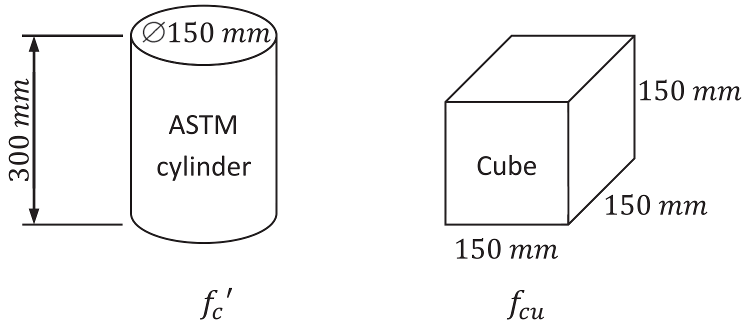
- Lightweight concrete with a unit weight from about $1350\ to\ 1850\ kg/m^3$ produced from aggregates of expanded shale, clay, slate, and slag.

Other lightweight materials such as pumice, scoria, perlite, vermiculite, and diatomite are used to produce insulating lightweight concretes ranging in density from about $250\ to\ 1450\ kg/m^3$.

- Normal-weight concrete with a unit weight from about $1800\ to\ 2400\ kg/m^3$ produced from the most commonly used aggregates— sand, gravel, crushed stone.
- Heavyweight concrete with a unit weight from about $3200\ to\ 5600\ kg/m^3$ produced from such materials such as barite, limonite, magnetite, ilmenite, hematite, iron, and steel punching or shot. It is used for shielding against radiations in nuclear reactor containers and other structures.

2.2 COMPRESSIVE STRENGTH

The strength of concrete is controlled by the proportioning of cement, coarse and fine aggregates, water, and various admixtures. The most important variable is (w/c) ratio.
Concrete strength (f_c') – uniaxial compressive strength measured by a compression test of a standard test cylinder ($150\ mm$ diameter by $300\ mm$ high) on the 28th day–ASTM C31, C39.
In many countries, the standard test unit is the cube ($200 \times 200 \times 200\ mm$).
The concrete strength depends on the size and shape of the test specimen and the manner of testing. For this reason the cylinder ($\varnothing 150\ mm$ by $300\ mm$ high) strength is 80% of the $150-mm$ cube strength and 83% of the $200-mm$ cube strength.



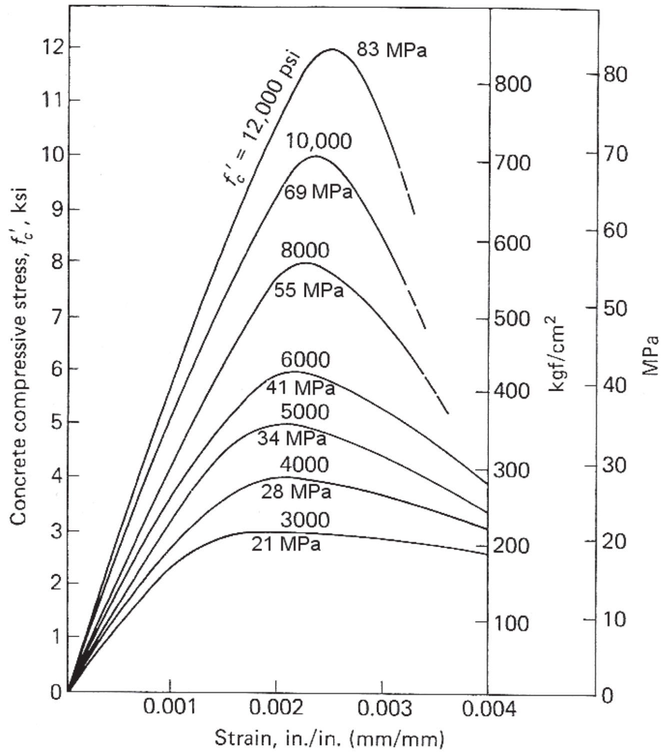
$$\boxed{f_c' \approx 0.80\ f_{cu}}$$

Stress-strain relationship: Typical curves for specimens ($150 \times 300\ mm$ cylinders) loaded in compression at 28 days.

Lower-strength concrete has greater deformability (ductility) than higher-strength concrete (length of the portion of the curve after the maximum stress is reached at a strain between 0.002 and 0.0025).

Ultimate strain at crushing of concrete varies from 0.003 to as high as 0.008.

- In usual reinforced concrete design f_c' of $(24\ to\ 35\ MPa)$ are used for nonprestressed structures.
- f_c' of $(35$ to $42\ MPa)$ are used for prestressed concrete.
- f_c' of $(42$ to $97\ MPa)$ are used particularly in columns of tall buildings.



2.3 TENSILE STRENGTH

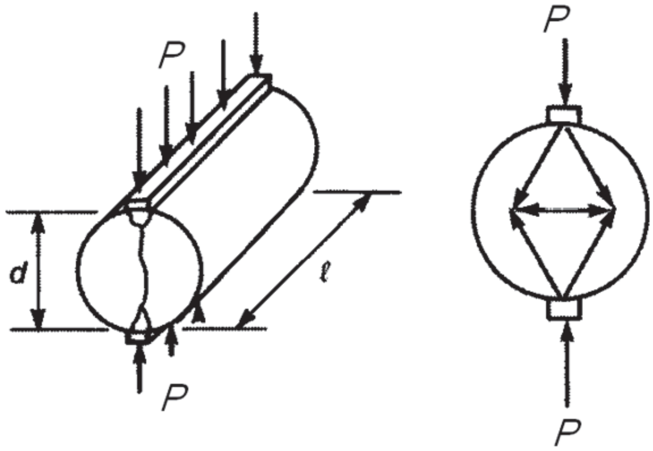
Concrete tensile strength is about $10\ to\ 15\%$ of its compressive strength.

The strength of concrete in tension is an important property that greatly affects that extent and size of cracking in structures.

Tensile strength is usually determined by using:

- Split-cylinder test (ASTM C496). A standard $150 \times 300\ mm$ compression test cylinder is placed on its side and loaded in compression along a diameter. The splitting tensile strength f_{ct} is computed as

$$f_{ct} = \frac{2P}{\pi l d}$$



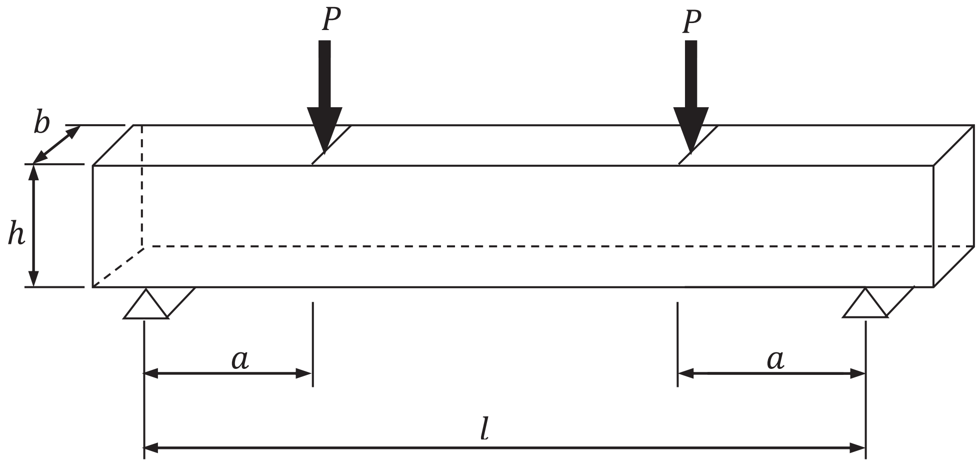
- Tensile strength in flexure (modulus of rupture) (ASTM C78 or C293). A plain concrete beam $150 \times 150\ mm\ \times 750\ mm\ long,$ is loaded in flexure at the third points of 600-mm span until it fails due to cracking on the tension face. Modulus of rupture f_r is computed as

$$f_r = \frac{M}{I}c = \frac{6M}{bh^2} = \frac{6Pa}{bh^2}$$

It is accepted (ACI 9.5.2.3) that an average value for f_r may be taken as

$$f_r = 0.62\,\lambda\sqrt{f_c'}\,, \qquad f_c' \quad in\ MPa$$

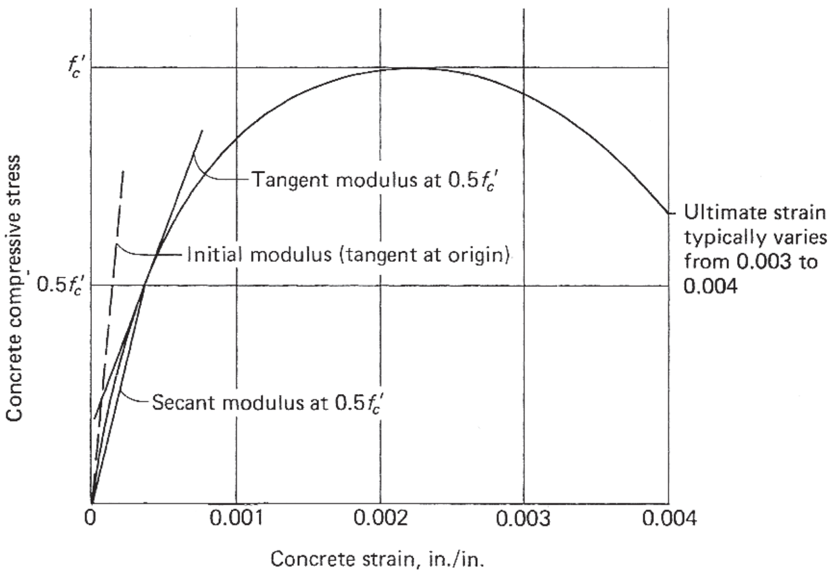
$$where \qquad \lambda = 1 \quad for\ normalweight\ concrete.$$



- Direct axial tension test. It is difficult to measure accurately and not in use today.

2.4 MODULUS OF ELASTICITY

The modulus of elasticity of concrete varies, unlike that of steel, with strength. A typical stress-strain curve for concrete in compression is shown. The initial modulus (tangent at origin), the tangent modulus (at $0.5\,f_c'$), and the secant modulus are noted. Usually the secant modulus at from $25\ to\ 50\%$ of the compressive strength f_c' is considered to be the modulus of elasticity. The empirical formula given by ACI-8.5.1



$$E_c = 0.043 w_c^{1.5}\sqrt{f_c'}$$

For normalweight concrete, E_c shall be permitted to be taken as $E_c = 4700\sqrt{f_c'}$,
where, $1440 \le w_c \le 2560\ \ kg/m^3 \quad and \quad f_c'\ in\ MPa$.

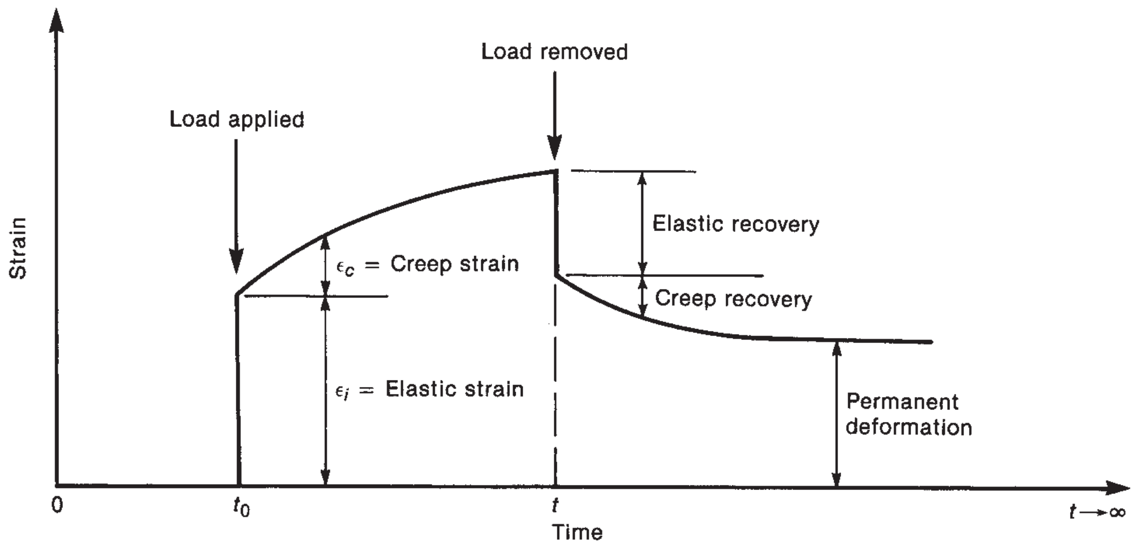
2.5 CREEP AND SHRINKAGE

Creep and shrinkage are time-dependent deformations that, along with cracking, provide the greatest concern for the designer because of the inaccuracies and unknowns that surround them. Concrete is elastic only under loads of short duration; and, because of additional deformation with time, the effective behavior is that of an inelastic material. Deflection after a long period of time is therefore difficult to predict, but its control is needed to assure serviceability during the life of the structure.

Creep (or plastic flow) is the property of concrete (and other materials) by which it continues to deform with time under sustained loads at unit stresses within the accepted elastic range (say, below $0.5\,f_c'$). This inelastic deformation increases at a decreasing rate during the time of loading, and its total magnitude may be several times as large as the short-time elastic deformation. Frequently creep is associated with shrinkage, since both are occurring simultaneously and often provide the same net effect: increased deformation with time.

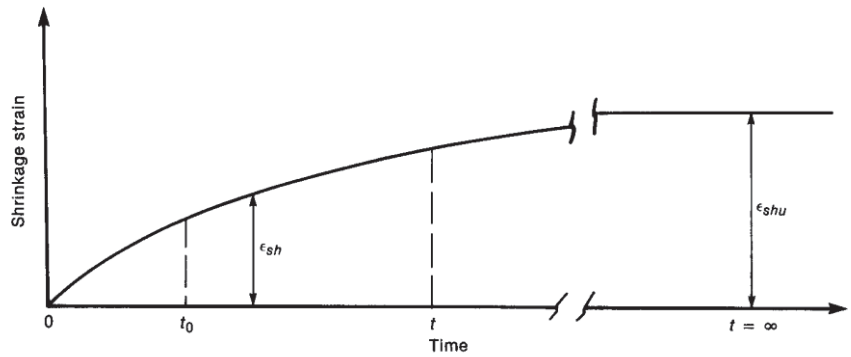
The internal mechanism of creep, or "plastic flow" as it is sometimes called, may be due to any one or a combination of the following: (1) crystalline flow in the aggregate and hardened cement paste; (2) plastic flow of the cement paste surrounding the aggregate; (3) closing of internal voids; and (4) the flow of water out of the cement gel due to external load and drying.

Factors affecting the magnitude of creep are (1) the constituents—such as the composition and fineness of the cement, the admixtures, and the size, grading, and mineral content of the aggregates: (2) proportions such as water content and water-cement ratio; (3) curing temperature and humidity; (4) relative humidity during period of use; (5) age at loading; (6) duration of loading; (7) magnitude of stress; (8) surface-volume ratio of the member; and (9) slump.



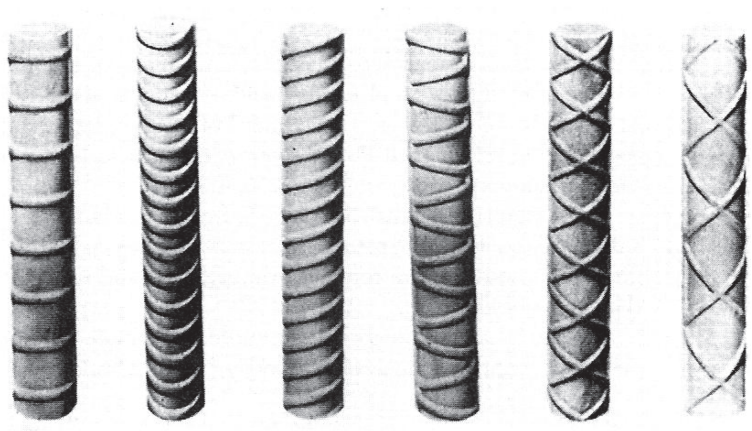
Creep of concrete will often cause an increase in the long-term deflection of members. Unlike concrete, steel is not susceptible to creep. For this reason, steel reinforcement is often provided in the compression zone of beams to reduce their long-term deflection.

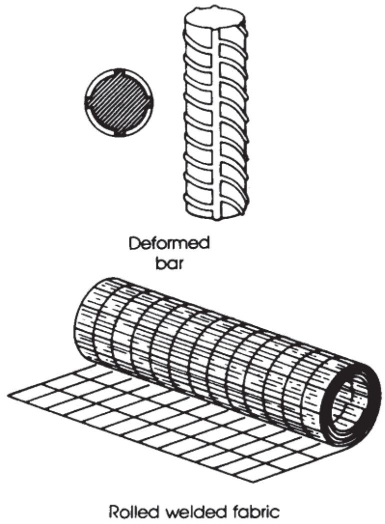
Shrinkage, broadly defined, is the volume change during hardening and curing of the concrete. It is unrelated to load application. The main cause of shrinkage is the loss of water as the concrete dries and hardens. It is possible for concrete cured continuously under water to increase in volume; however, the usual concern is with a decrease in volume. In general, the same factors have been found to influence shrinkage strain as those that influence creep—primarily those factors related to moisture loss.



2.6 STEEL REINFORCEMENT

The useful strength of ordinary reinforcing steels in tension as well as compression, the yield strength is about 15 times the compressive strength of common structural concrete and well over 100 times its tensile strength.



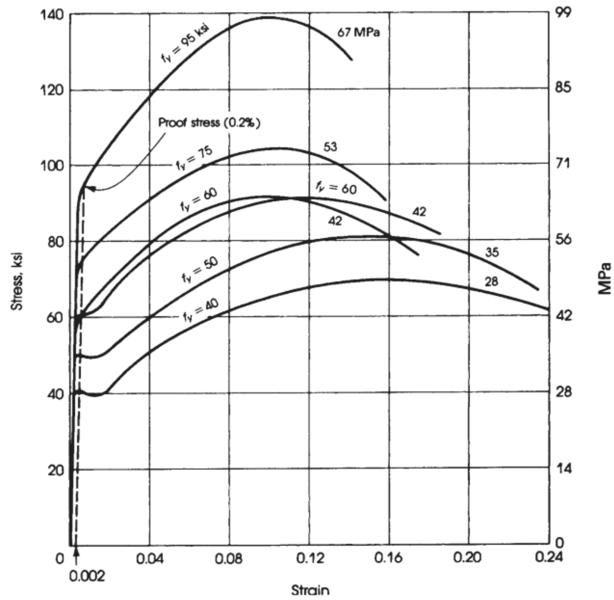


Steel reinforcement may consist of :

- Bars (deformed bars, as in picture below) – for usual construction.
- Welded wire fabric – is used in thins slabs, thin shells.
- Wires – are used for prestressed concrete.

The “Grade” of steel is the minimum specified yield stress (point) expressed in:

- MPa for SI reinforcing bar Grades 300, 350, 420, and 520.
- ksi for Inch-Pound reinforcing bar Grades 40, 50, 60, and 75.



The introduction of carbon and alloying additives in steel increases its strength but reduces its ductility. The proportion of carbon used in structural steels varies between 0.2% and 0.3%.

The steel modulus of elasticity (E_s) is constant for all types of steel. The ACI Code has adopted a value of $E_s = 2 \times 10^5\ MPa\ (29 \times 10^6\ psi\ )$.

Summary of minimum ASTM strength requirements

Product	ASTM Specification	Designation	Minimum Yield Strength, psi (MPa)	Minimum Tensile Strength, psi (MPa)
Reinforcing bars	A615	Grade 40 Grade 60 Grade 75	40,000 (280) 60,000 (420) 75,000 (520)	60,000 (420) 90,000 (620) 100,000 (690)
	A706	Grade 60	60,000 (420) [78,000 (540) maximum]	80,000 (550)[a]
	A996	Grade 40 Grade 50 Grade 60	40,000 (280) 50,000 (350) 60,000 (420)	60,000 (420) 80,000 (550) 90,000 (620)
	A1035	Grade 100	100,000 (690)	150,000 (1030)
Deformed bar mats	A184	Same as reinforcing bars		
Zinc-coated bars	A767	Same as reinforcing bars		
Epoxy-coated bars	A775, A934	Same as reinforcing bars		
Stainless-steel bars[b]	A955	Same as reinforcing bars		
Wire Plain	A82		70,000 (480)	80,000 (550)
Deformed	A496		75,000 (515)	85,000 (585)
Welded wire reinforcement Plain W1.2 and larger Smaller than W1.2	A185		 65,000 (450) 56,000 (385)	 75,000 (515) 70,000 (485)
Deformed	A497		70,000 (480)	80,000 (550)
Prestressing tendons Seven-wire strand	A416	Grade 250 (stress-relieved)	212,500 (1465)	250,000 (1725)
		Grade 250 (low-relaxation)	225,000 (1555)	250,000 (1725)
		Grade 270 (stress-relieved)	229,500 (1580)	270,000 (1860)
		Grade 270 (low-relaxation)	243,000 (1675)	270,000 (1860)
Wire	A421	Stress-relieved	199,750 (1375) to 212,500 (1465)[c]	235,000 (1620) to 250,000 (1725)[c]
		Low-relaxation	211,500 (1455) to 225,000 (1550)[c]	235,000 (1620) to 250,000 (1725)[c]
Bars	A722	Type I (plain) Type II (deformed)	127,500 (800) 120,000 (825)	150,000 (1035) 150,000 (1035)
Compacted strand[b]	A779	Type 245 Type 260 Type 270	241,900 (1480) 228,800 (1575) 234,900 (1620)	247,000 (1700) 263,000 (1810) 270,000 (1860)

[a] But not less than 1.25 times the actual yield strength.
[b] Not listed in ACI 318.
[c] Minimum strength depends on wire size.

Cross sectional Areas of standard steel bars for reinforced concrete structures

Diameter, mm	Area of bars for Number of bars , cm^2									Mass, Kg/ m
	1	2	3	4	5	6	7	8	9	
6	0.283	0.565	0.848	1.131	1.414	1.696	1.979	2.262	2.545	0.222
8	0.503	1.005	1.508	2.011	2.513	3.016	3.519	4.021	4.524	0.395
10	0.785	1.571	2.356	3.142	3.927	4.712	5.498	6.283	7.069	0.617
12	1.131	2.262	3.393	4.524	5.655	6.786	7.917	9.048	10.179	0.888
14	1.539	3.079	4.618	6.158	7.697	9.236	10.776	12.315	13.854	1.208
16	2.011	4.021	6.032	8.042	10.053	12.064	14.074	16.085	18.096	1.578
18	2.545	5.089	7.634	10.179	12.723	15.268	17.813	20.358	22.902	1.998
20	3.142	6.283	9.425	12.566	15.708	18.850	21.991	25.133	28.274	2.466
22	3.801	7.603	11.404	15.205	19.007	22.808	26.609	30.411	34.212	2.984
25	4.909	9.817	14.726	19.635	24.544	29.452	34.361	39.270	44.179	3.854
28	6.158	12.315	18.473	24.630	30.788	36.945	43.103	49.260	55.418	4.834
32	8.042	16.085	24.127	32.170	40.212	48.255	56.297	64.340	72.382	6.314
36	10.179	20.358	30.536	40.715	50.894	61.073	71.251	81.430	91.609	7.991
40	12.566	25.133	37.699	50.265	62.832	75.398	87.965	100.531	113.097	9.865
45	15.904	31.809	47.713	63.617	79.522	95.426	111.330	127.235	143.139	12.486

CHAPTER 3 DESIGN METHODS AND REQUIREMENTS

3.1 ACI BUILDING CODE

When two different materials, such as steel and concrete, act together, it is understandable that the analysis for strength of a reinforced concrete member has to be partly empirical. These principles and methods are being constantly revised and improved as results of theoretical and experimental research accumulate. The American Concrete Institute (ACI), serving as a clearinghouse for these changes, issues building code requirements, the most recent of which is the Building Code Requirements for Structural Concrete (ACI 318-08), hereafter referred to as the ACI Code.

The ACI Code is a Standard of the American Concrete Institute. In order to achieve legal status, it must be adopted by a governing body as a part of its general building code. The ACI Code is partly a specification-type code, which gives acceptable design and construction methods in detail, and partly a performance code, which states desired results rather than details of how such results are to be obtained. A building code, legally adopted, is intended to prevent people from being harmed; therefore, it specifies minimum requirements to provide adequate safety and serviceability. It is important to realize that a building code is not a recommended practice, nor is it a design handbook, nor is it intended to replace engineering knowledge, judgment, or experience. It does not relieve the designer of the responsibility for having a safe, economical structure.

ACI 318M-08 – Building Code Requirements for Structural Concrete and Commentary.

Two philosophies of design have long been prevalent:

- The working stress method (1900 – 1960).
- The strength design method (1960 till now, with few exceptions).

3.2 WORKING STRESS METHOD

In the working stress method, a structural element is so designed that the stresses resulting from the action of service loads (also called working loads) and computed by the mechanics of elastic members do not exceed some predesignated allowable values.

Service load is the load, such as dead, live, snow, wind, and earthquake, which is assumed actually to occur when the structure is in service.

The working stress method may be expressed by the following:

$$f \leq f_{allow}$$

where

$f -$ an elastic stress, such as by using the flexure formula $f = Mc/I$ for a beam, computed under service load.

$f_{allow} -$ a limiting or allowable stress prescribed by a building code as a percentage of the compressive strength f_c' for concrete, or of the yield stress for the steel reinforcing bars.

3.3 STRENGTH DESIGN METHOD

In the strength design method (formerly called ultimate strength method), the service loads are increased by factors to obtain the load at which failure is considered to be "imminent". This load is called the factored load or factored service load. The structure or structural element is then proportioned such that the strength is reached when the factored load is acting. The computation of this strength takes into account the nonlinear stress-strain behavior of concrete.
The strength design method may be expressed by the following,

strength provided ≥ [strength required to carry factored loads]

where the "strength provided" (such as moment strength) is computed in accordance with the provisions of a building code, and the "strength required" is that obtained by performing a structural analysis using factored loads.

3.4 SAFETY PROVISIONS

Structures and structural members must always be designed to carry some reserve load above what is expected under normal use. Such reserve capacity is provided to account for a variety of factors, which may be grouped in two general categories:

- factors relating to overload
- factors relating to understrength (that is, less strength than computed by acceptable calculating procedures).

Overloads may arise from changing the use for which the structure was designed, from underestimation of the effects of loads by oversimplification in calculation procedures, and from effects of construction sequence and methods. Understrength may result from adverse variations in material strength, workmanship, dimensions, control, and degree of supervision, even though individually these items are within required tolerances.
In the strength design method, the member is designed to resist factored loads, which are obtained by multiplying the service loads by load factors. Different factors are used for different loadings. Because dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty. Several load combinations must be considered in the design to compute the maximum and minimum design forces. Reduction factors are used for some combinations of loads to reflect the low probability of their simultaneous occurrences. The ACI Code presents specific values of load factors to be used in the design of concrete structures.
In addition to load factors, the ACI Code specifies another factor to allow an additional reserve in the capacity of the structural member. The nominal strength is generally calculated using accepted analytical procedure based on statistics and equilibrium; however, in order to account for the degree of accuracy within which the nominal strength can be calculated, and for adverse variations in materials and dimensions, a strength reduction factor, ϕ, should be used in the strength design method.

To summarize the above discussion, the ACI Code has separated the safety provision into an overload or load factor and to an undercapacity (or strength reduction) factor, ϕ. A safe design is achieved when the structure's strength, obtained by multiplying the nominal strength by the reduction factor, ϕ, exceeds or equals the strength needed to withstand the factored loadings (service loads times their load factors).
The requirement for strength design may be expressed:

$$\text{Design strength } \geq \text{Factored load (i.e., required strength)}$$
$$\phi P_n \geq P_u$$
$$\phi M_n \geq M_u$$
$$\phi V_n \geq V_u$$

where P_n, M_n, and V_n are "nominal" strengths in axial compression, bending moment, and shear, respectively, using the subscript n.
P_u, M_u, and V_u are the factored load effects in axial compression, bending moment, and shear, respectively, using the subscript u.
Given a load factor of 1.2 for dead load and a load factor of 1.6 for live load, the overall safety factor for a structure loaded be a dead load, D, and a live load, L, is

$$\text{Factor of Safety} = \frac{1.2D + 1.6\,L}{D + L}\left(\frac{1}{\phi}\right)$$

3.5 LOAD FACTORS AND STRENGTH REDUCTION FACTORS

Overload Factors *U*
The factors U for overload as given by ACI-9.2 are:

$$U = 1.4(D + F)$$
$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$$
$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$$
$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$$
$$U = 1.2D + 1.0E + 1.0L + 0.2S$$
$$U = 0.9D + 1.6W + 1.6H$$
$$U = 0.9D + 1.0E + 1.6H$$

where
D - dead load; L - live load; L_r- roof live load; S - snow load; R - rain load; W - wind load; E - earthquake load; F - load due to weights and pressures of fluids with well-defined densities and controllable maximum heights; H - load due to weight and pressure of soil, water in soil or other materials; T - the cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage compensating concrete.

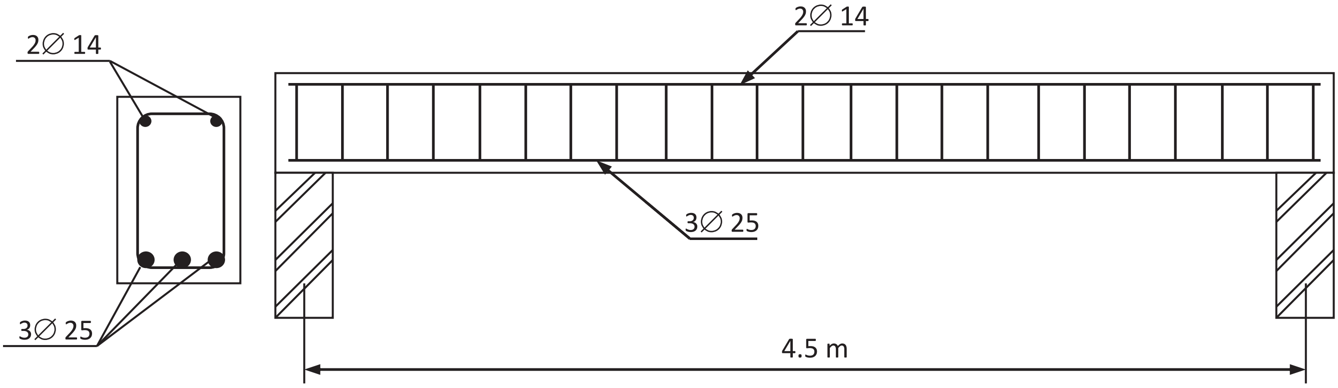
Strength Reduction Factors ϕ

The factors ϕ for understrength are called strength reduction factors according to ACI-9.3. and are as follows:

Strength Condition	ϕ **Factors**
1. Flexure (with or without axial force)	
Tension-controlled sections	0.90
Compression-controlled sections	
Spirally reinforced	0.75
Others	0.65
2. Shear and torsion	0.75
3. Bearing on concrete	0.65
4. Post-tensioned anchorage zones	0.85
5. Struts, ties, nodal zones, and bearing areas in strut-and-tie models	0.75

Example:

A simple beam is loaded with a dead load of 40 KN/m and a live load of 30 KN/m. Check the strength requirement according to ACI code if the nominal bending moment $M_n = 275\ KN.m$



<u>Solution</u>:

$$M_n = 275\ KN.m \qquad and \qquad \phi = 0.9$$

$$w_u = 1.2D + 1.6L = 1.2 \cdot 40 + 1.6 \cdot 30 = 96\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{96 \cdot 4.5^2}{8} = 243\ KN \cdot m$$

$$\phi M_n \geq M_u$$

$0.9 \cdot 275 = 247.5\ KN \cdot m > 243\ KN \cdot m$ OK Strength requirement is satisfied

$$\text{Factor of Safety} = \frac{1.2D + 1.6\ L}{D + L}\left(\frac{1}{\phi}\right) = \frac{96}{40 + 30}\left(\frac{1}{0.9}\right) = 1.52$$

CHAPTER 4 FLEXURE IN BEAMS

4.1 INTRODUCTION

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of steel, wood, or any other structural material.

Two different types of problems arise in the study of reinforced concrete:

1. Analysis. Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.
2. Design. Given a factored design moment, normally designated as M_u. select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

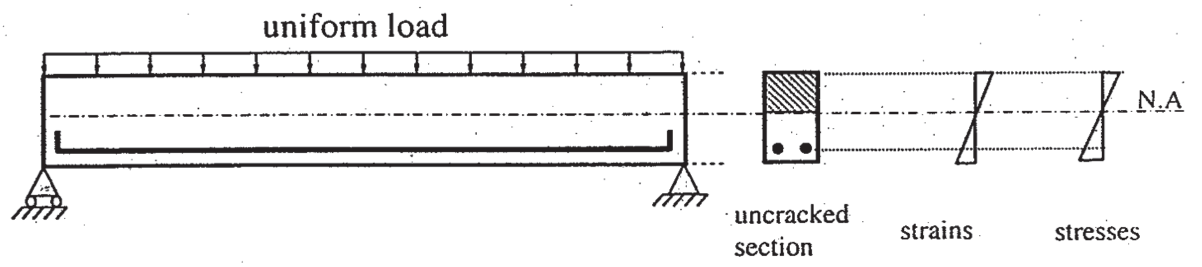
The Strength Design Method requires the conditions of static equilibrium and strain compatibility across the depth of the section to be satisfied.

The following are the assumptions for Strength Design Method:

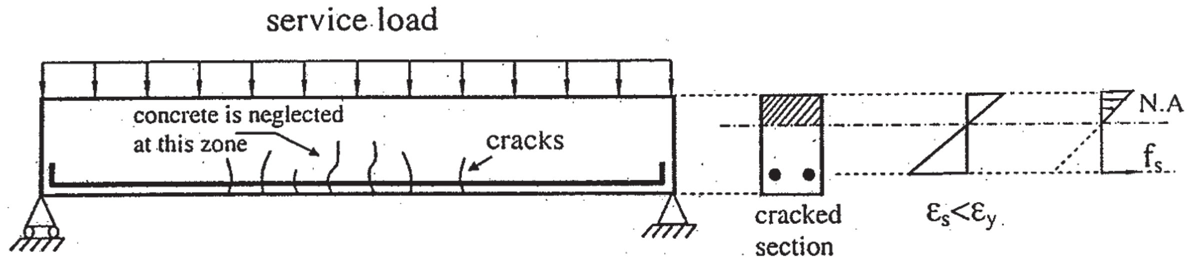
1. Strains in reinforcement and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
2. Concrete sections are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
3. Stress in reinforcement varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
4. Tensile strength of concrete is neglected.
5. Compressive stress distribution of concrete can be represented by the corresponding stress-strain relationship of concrete. This stress distribution may be simplified by a rectangular stress distribution as described later.

4.2 REINFORCED CONCRETE BEAM BEHAVIOR

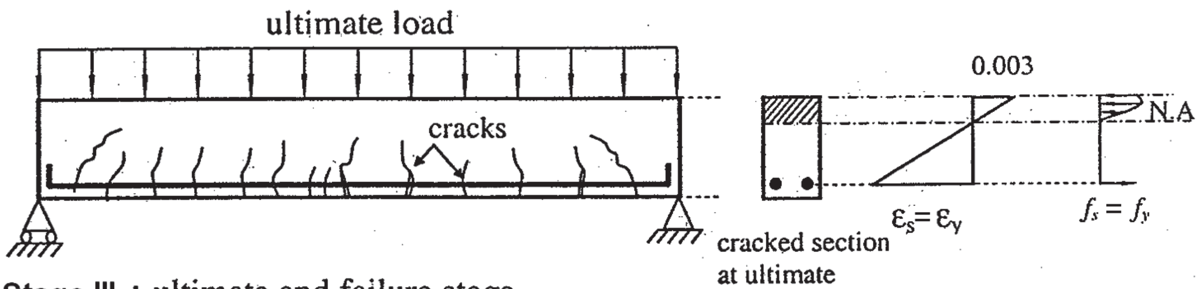
Consider a simply supported and reinforced concrete beam with uniformly distributed load on top. Under such loading and support conditions, flexure-induced stresses will cause compression at the top and tension at the bottom of the beam. Concrete, which is strong in compression, but weak in tension, resists the force in the compression zone, while steel reinforcing bars are placed in the bottom of the beam to resist the tension force. As the applied load is gradually increased from zero to failure of the beam (ultimate condition), the beam may be expected to behave in the following manner:



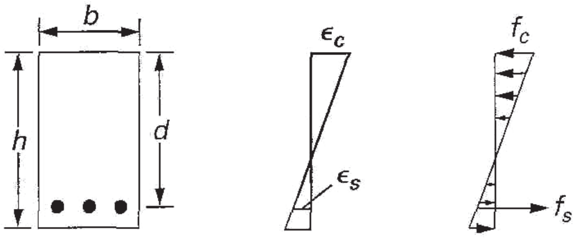
Stage I : before cracking



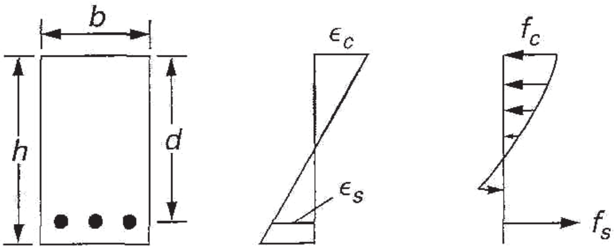
Stage II : cracking stage, before yield, working load



Stage III : ultimate and failure stage

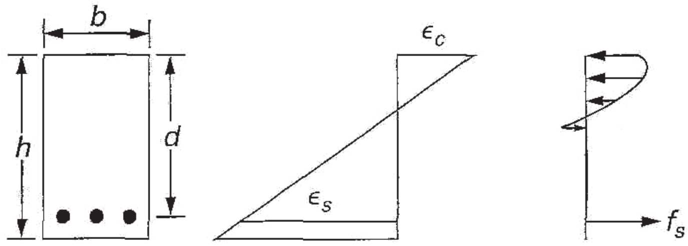


Stage I: when the applied load is low, the stress distribution is essentially linear over the depth of the section. The tensile stresses in the concrete are low enough so that the entire cross-section remains uncracked and the stress distribution is as shown in (a). In the compression zone, the concrete stresses are low enough (less than about $0.5\,f_c'$) so that their distribution is approximately linear.



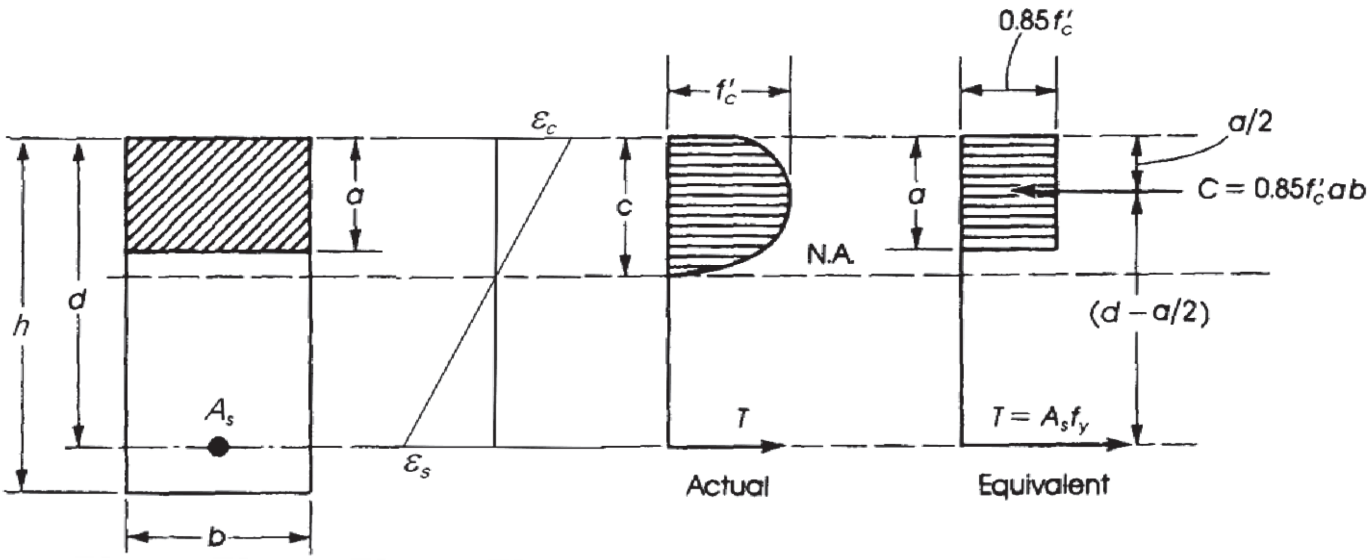
Stage II: On increasing the applied load, the tensile stresses at the bottom of the beam become high enough to exceed the tensile strength at which the concrete cracks. After cracking, the tensile force is resisted mainly by the steel reinforcement. Immediately below the neutral axis, a small portion of the beam remains uncracked. These tensile stresses in the concrete offer, however, only a small contribution to the flexural strength. The concrete stress distribution in the compression zone becomes nonlinear.

Stage III: at nominal (so,-called ultimate) strength, the neutral axis has moved farther upward as flexural cracks penetrate more and more toward the compression face. The steel reinforcement has yielded and the concrete stress distribution in the compression zone becomes more nonlinear. Below the neutral axis, the concrete is cracked except for a very small zone.



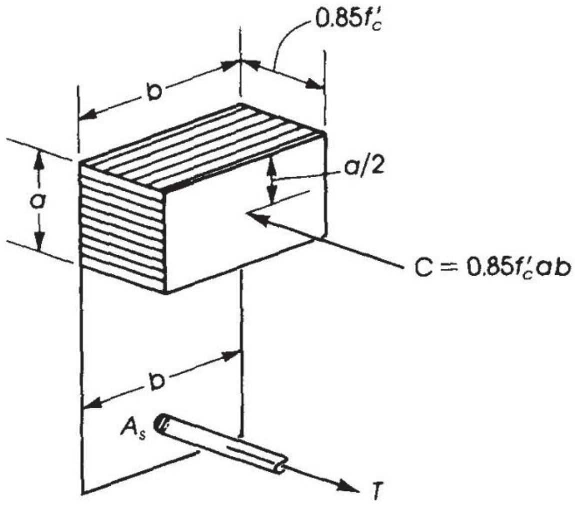
At the ultimate stage, two types of failure can be noticed. If the beam is reinforced with a small amount of steel, ductile failure will occur. In this type of failure, the steel yields and the concrete crushes after experiencing large deflections and lots of cracks. On the other hand, if the beam is reinforced with a large amount of steel, brittle failure will occur. The failure in this case is sudden and occurs due to the crushing of concrete in the compression zone without yielding of the steel and under relatively small deflections and cracks. This is not a preferred mode of failure because it does not give enough warning before final collapse.

4.3 THE EQUIVALENT RECTANGULAR COMPRESSIVE STRESS DISTRIBUTION (COMPRESSIVE STRESS BLOCK)



The actual distribution of the compressive stress in a section has the form of rising parabola. It is time consuming to evaluate the volume of compressive stress block. An equivalent rectangular stress block can be used without loss of accuracy.

The flexural strength M_n, using the equivalent rectangular, is obtained as follows:



$$C = 0.85\, f_c' ab$$
$$T = A_s f_y$$

$\sum F_x = 0$ gives $T = C$

$$A_s f_y = 0.85\, f_c' ab$$

or

$$a = \frac{A_s f_y}{0.85\, f_c' b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' ab\left(d - \frac{a}{2}\right)$$

Notation:

a − depth of rectangular compressive stress block,
b − width of the beam at the compression side,
c − depth of the neutral axis measured from the extreme compression fibers,
d − effective depth of the beam, measured from the extreme compression fibers to the centroid of the steel area,
h − total depth of the beam,
ε_c − strain in extreme compression fibers,
ε_s − strain at tension steel,
f_c' − compressive strength of concrete,
f_y − yield stress of steel,
A_s − area of the tension steel,
C − resultant compression force in concrete,
T − resultant tension force in steel,
M_n − nominal moment strength of the section.

Example:

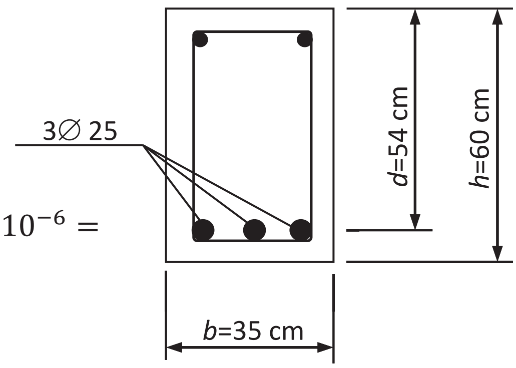
Determine the nominal moment strength of the beam section. Take $f_c' = 20\, MPa$, $f_y = 400\, MPa$.

Solution:

$A_s(3\varnothing\, 25) = 14.72\, cm^2$

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{14.72 \cdot 100 \cdot 400}{0.85 \cdot 20 \cdot 350} = 98.96\, mm$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 14.72 \cdot 100 \cdot 400\left(540 - \frac{98.96}{2}\right) \cdot 10^{-6} =$$
$$= 288.82\, KN \cdot m$$

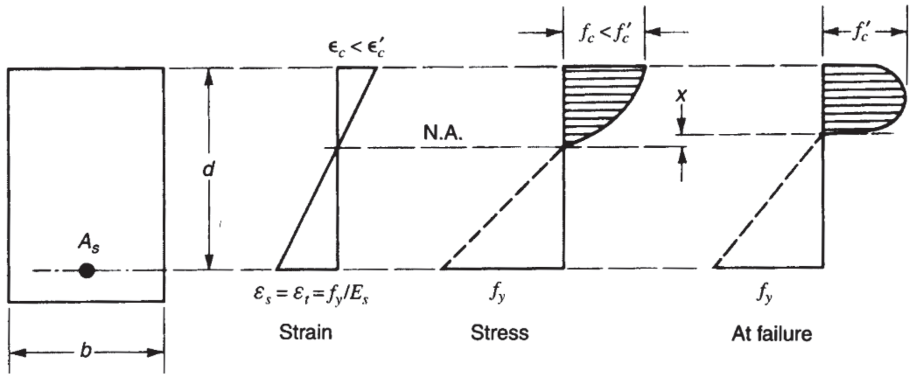


4.4 TYPES OF FAILURE AND STRAIN LIMITS

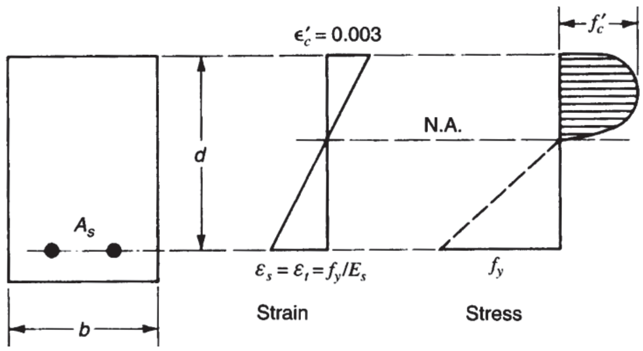
Types of failure

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

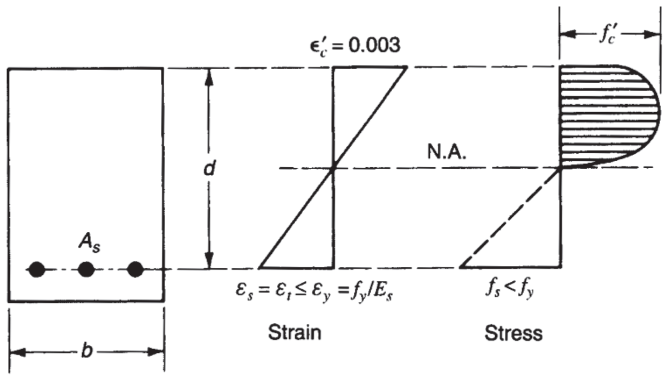
1. Steel may reach its yield strength before the concrete reaches its maximum strength, In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and is called a **tension-controlled section**.



2. Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a **balanced section**.



3. Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is, f_s is less than f_y. The strain in the steel is equal to or less than 0.002. This section is called a **compression-controlled section**.



The ACI Code assumes that concrete fails in compression when the concrete strain reaches 0.003.

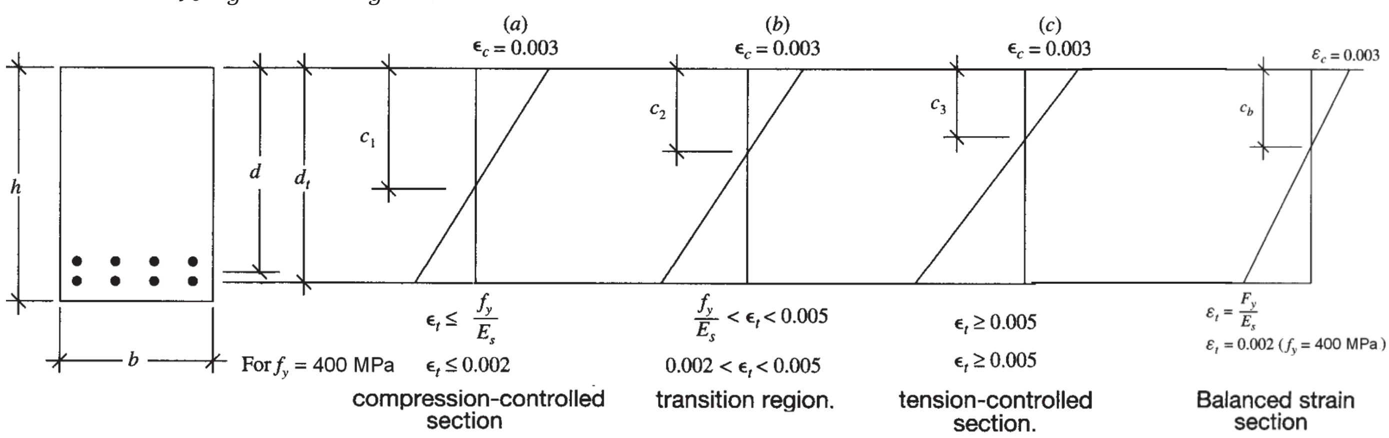
In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code **does not allow** this type of design.

Strain Limits for Tension and Tension-Controlled Sections

The ACI Code, Section 10.3. defines the concept of tension or compression-controlled sections in terms of net tensile strain ε_t (net tensile strain in the reinforcement closest to the tension face). Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition.
These four conditions are defined as follows:

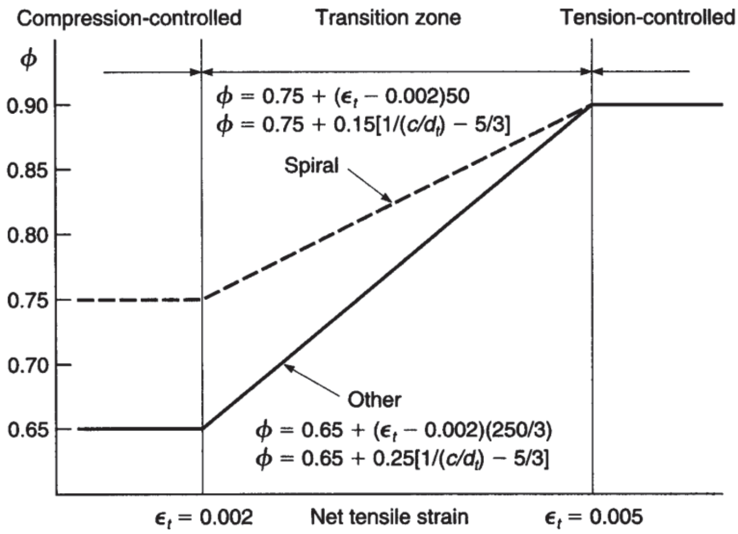
1. Compression-controlled sections are those sections in which ε_t at nominal strength is equal to or less than the compression-controlled strain limit (the compression-controlled strain limit may be taken as a net strain of $\varepsilon_y = 0.002 -$ for $f_y = 400\ MPa$) at the time when concrete in compression reaches its assumed strain limit of 0.003, ($\varepsilon_c = 0.003$). This case occurs mainly in columns subjected to axial forces and moments.
2. Tension-controlled sections are those sections in which the ε_t is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003
3. Sections in which the ε_t lies between the compression-controlled strain limit of 0.002 (for $f_y = 400\ MPa$) and the tension-controlled strain limit of 0.005 constitute the transition region.
4. The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, f_y or $\varepsilon_s = \frac{f_y}{E_s}$, just as the maximum strain in concrete at the extreme compression fibers reaches 0.003.

In addition to the above four conditions, Section 10.3.5 of the ACI Code indicates that the net tensile strain, ε_t, at nominal strength, within the transition region, shall not be less than 0.004 for reinforced concrete flexural members without or with an axial load less than $0.10\ f_c' A_g$, where $A_g =$ gross area of the concrete section.



Strain limit distribution

Note that in cases where strain is less than 0.005 namely, the section is in the transition zone, a value of the reduction ϕ lower than 0.9 for flexural has to be used for final design moment, with a strain not less than 0.004 as a limit.



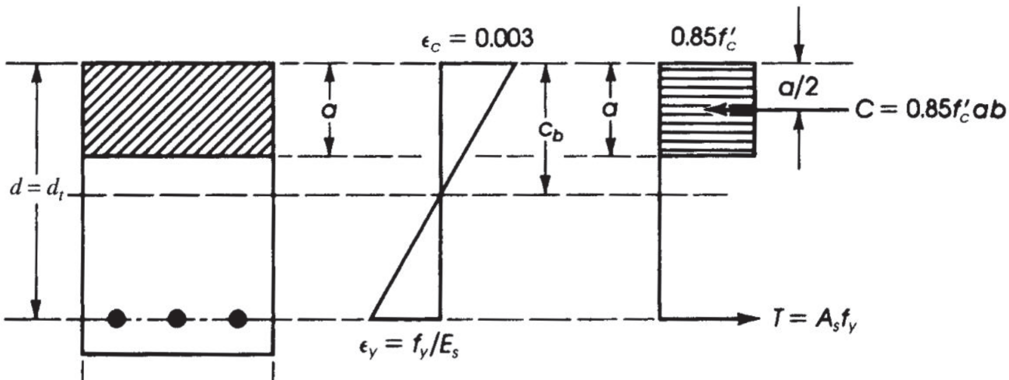
For transition region ϕ may be determined by linear interpolation:

$$\phi = 0.75 + (\varepsilon_t - 0.002)50 - \ for\ spiral\ members$$

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) - \ for\ other\ members$$

4.5 THE BALANCED CONDITION

Let us consider the case of balanced section, which implies that at ultimate load the strain in concrete equals 0.003 and that of steel equals $\varepsilon_t = \frac{f_y}{E_s}$ (at distance d_t).



$$\frac{c_b}{0.003} = \frac{d}{0.003 + \frac{f_y}{E_s}}, \qquad or \qquad c_b = \frac{d}{0.003 + \frac{f_y}{E_s}} 0.003$$

Substituting $E_s = 200\,000\ MPa$

$$c_b = \frac{600}{600 + f_y} d$$

From equation of equilibrium $\sum F_x = 0$

$$T = C \qquad \Longrightarrow \qquad A_s f_y = 0.85\, f_c' ab$$

$a -$ the depth of compressive block and equal $a = \beta_1 c$.

For balanced condition, $a_b = \beta_1 c_b$.

where β_1 as defined in ACI 10.2.7.3 equal:

$$\beta_1 = 0.85 - 0.007(f_c' - 28) \qquad 0.65 \le \beta_1 \le 0.85$$

The reinforcement ratio for tension steel

$$\rho = \frac{A_s}{bd} \qquad \textit{and balanced reinforcement ratio } \rho_b = \frac{(A_s)_b}{bd}$$

$$\frac{(A_s)_b}{bd} = 0.85\,\frac{f_c'}{f_y}\beta_1 \frac{600}{600 + f_y}$$

$$\rho_b = 0.85\,\frac{f_c'}{f_y}\beta_1 \left(\frac{600}{600 + f_y}\right)$$

4.6 UPPER AND LOWER (MINIMUM) STEEL PERCENTAGES.

The maximum reinforcement ratio ρ_{max} that ensures a minimum net tensile steel strain of 0.004.

$$\rho\ (\varepsilon_t = 0.004) = \frac{0.003 + \varepsilon_y}{0.003 + 0.004}\rho_b = \frac{0.003 + \varepsilon_y}{0.007}\rho_b = \rho_{max}$$

For Grade 420 reinforcing bars $\varepsilon_y = 0.002$, then

$$\rho_{max} = \frac{0.003 + 0.002}{0.007}\rho_b = \frac{0.005}{0.007}\rho_b = 0.724\rho_b$$

If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment, the calculation may show that very small or no steel reinforcement is required. The ACI Code, 10.5, specifies a minimum steel area, $A_{s,min}$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y} b_w d$$

and not less than

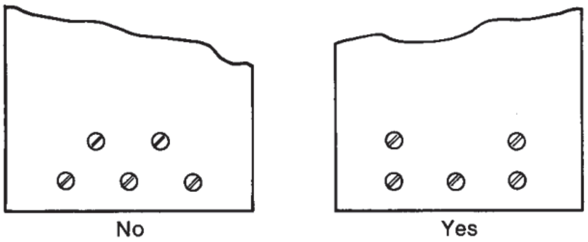
$$A_{s,min} = \frac{1.4}{f_y} b_w d$$

The above requirements of $A_{s,min}$ need not be applied if, at every section, A_s provided is at least one-third greater than that required by analysis ($A_{s,provided} \ge 1.33 A_{s,required}$). This exception provides sufficient additional reinforcement in large members where the amount required by the above equations would be excessive.

$b_w -$ width of section, width of web for T-section, mm.

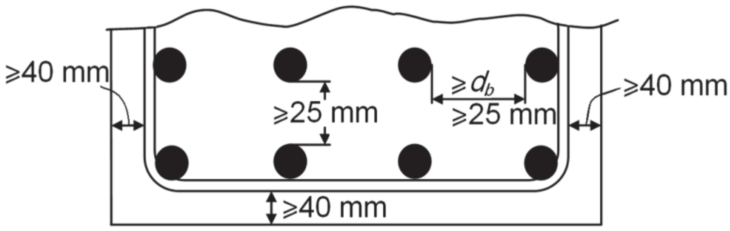
4.7 SPACING LIMITS AND CONCRETE PROTECTION FOR REINFORCEMENT.

The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking.



Arrangement of bars in two layers (ACI Section 7.6.2).

According to ACI 7.6. The minimum clear spacing between parallel bars in a layer shall be d_b, but not less than $25\,mm$. Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than $25\,mm$.



In addition, the nominal maximum size of coarse aggregate shall be not larger than:

(a) $1/5$ the narrowest dimension between sides of forms, nor

(b) $1/3$ the depth of slabs, nor

(c) $3/4$ the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, individual tendons, bundled tendons, or ducts.

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where concrete cover is prescribed for a class of structural members, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars. According to ACI, 7.7, minimum clear cover in cast-in-place concrete beams and columns should not be less than $40\,mm$.

To limit the widths of flexural cracks in beams and slabs, ACI Code Section 10.6.4 defines upper limit on the center-to-center spacing between bars in the layer of reinforcement closest to the tension face of a member. In some cases, this requirement could force a designer to select a larger number of smaller bars in the extreme layer of tension reinforcement. The spacing limit is:

$$s = 380\left(\frac{280}{f_s}\right) - 2.5C_c \qquad \text{but} \qquad s \le 300\left(\frac{280}{f_s}\right)$$

where C_c is the least distance from surface of reinforcement to the tension face. It shall be permitted to take f_s as $\frac{2}{3}f_y$.

4.8 ANALYSIS OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

Given: section dimensions b, h; reinforcement A_s; material strength f_c', f_y.

Required: M_n − Nominal moment strength.

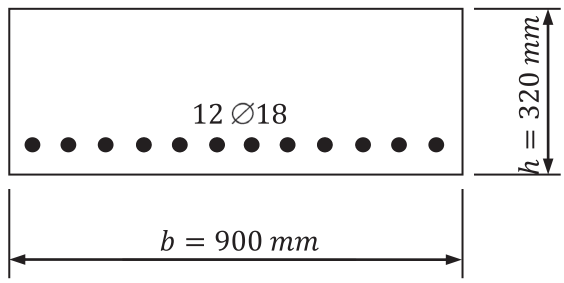
$$T = C \qquad \Longrightarrow \qquad A_s f_y = 0.85\, f_c' ab \qquad \Longrightarrow \qquad a = \frac{A_s f_y}{0.85\, f_c' b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' ab \left(d - \frac{a}{2}\right)$$

Example:

Determine the nominal moment strength of the beam section. Take $f_c' = 30\ MPa,\ f_y = 420\ MPa.$



Solution:

$A_s(12\varnothing\ 18) = 30.536\ cm^2 = 3053.6\ mm^2$

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{3053.6\ \cdot 420}{0.85 \cdot 30 \cdot 900} = 55.88\ mm$$

$$d = 320 - 40 - 10 - \frac{18}{2} = 261\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3053.6\ \cdot 420\left(261 - \frac{55.88}{2}\right) \cdot 10^{-6} = 298.9\ KN \cdot m$$

Check for strain:

$$\varepsilon_s = 0.003\left(\frac{d - c}{c}\right)$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{55.88}{0.836} = 66.84\ mm$$

$$\varepsilon_s = 0.003\left(\frac{261 - 66.84}{66.84}\right) = 0.00871 > 0.005$$

Take $\phi = 0.9\ for\ flexure$

$$\phi M_n = 0.9 \cdot 298.9 = 269.01\ KN \cdot m$$

4.9 DESIGN OF SINGLY REINFORCED CONCRETE RECTANGULAR SECTIONS FOR FLEXURE.

Given: M_u − factored moment ($M_u \leq \phi M_n$); material strength f_c', f_y.

Required: section dimensions b, h; reinforcement A_s.

The two conditions of equilibrium are

$$T = C \tag{1}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right) \tag{2}$$

Reinforcement ratio

$$\rho = \frac{A_s}{bd} \qquad or \qquad A_s = \rho bd$$

Substituting into (1)

$$\rho bdf_y = 0.85\, f_c' ab$$

$$a = \rho\left(\frac{f_y}{0.85 f_c'}\right) d \tag{3}$$

Substituting (3) into (2)

$$M_n = \rho bdf_y\left[d - \frac{\rho}{2}\left(\frac{f_y}{0.85 f_c'}\right) d\right] \tag{4}$$

A strength coefficient of resistance R_n is obtained by dividing (4) by (bd^2) and letting

$$m = \left(\frac{f_y}{0.85 f_c'}\right)$$

Thus

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{\rho m}{2}\right) \tag{5}$$

From which ρ may be determined

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \tag{6}$$

Design Procedure:

1. Set $M_u = \phi M_n = \phi R_n\, bd^2$
2. For ductile behavior such that beam is well into the tension controlled zone, a reinforcement percentage ρ should be chosen in the range of $(40 - 60)\%$ of ρ_b. Assume $\rho = (0.4 - 0.6)\rho_b$.

$$\rho_b = 0.85\,\frac{f_c'}{f_y}\beta_1\left(\frac{600}{600 + f_y}\right).$$

3. Find the flexural resistance factor R_n

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right), \qquad\qquad m = \left(\frac{f_y}{0.85 f_c'}\right)$$

4. Determine the required dimensions b, d

$$bd^2 = \frac{M_n}{R_n} = \frac{M_u}{\phi R_n}$$

5. Determine the required steel area for the chosen b, d

$$A_s = \rho bd$$

Where

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right), \qquad R_n = \frac{M_n}{bd^2}, \qquad m = \left(\frac{f_y}{0.85f_c'}\right)$$

6. Check for minimum steel reinforcement area

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

Or

$$\rho_{min} = 0.25\frac{\sqrt{f_c'}}{f_y} \geq \frac{1.4}{f_y}$$

$$If \quad A_{s,provided} \geq \frac{4}{3}A_{s,required} \quad - \quad NO\ need\ to\ use\ A_{s,min}$$

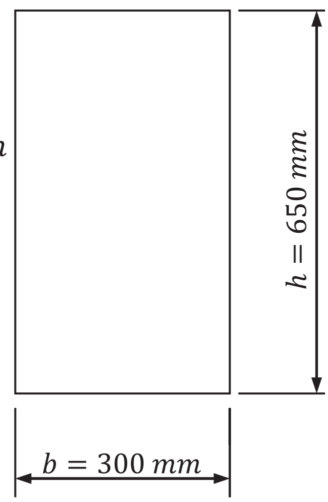
7. Check for strain ($\varepsilon_s \geq 0.005$) – tension-controlled section.
8. Check for steel bars arrangement in section.

Example:

Calculate the area of steel reinforcement required for the beam. $M_u = 360\ KN \cdot m$

Take $f_c' = 30\ MPa,\ f_y = 400\ MPa.$

Assume ∅ 25 with one layer arrangement.



<u>Solution:</u>

$$d = h - cover - \varnothing\text{stirrups} - \frac{\varnothing\text{bar}}{2} = 650 - 40 - 10 - \frac{25}{2} = 587.5\ mm$$

Take $\phi = 0.9\ for\ flexure$

$$R_n = \frac{M_n}{bd^2} = \frac{M_u}{\phi bd^2} = \frac{360 \cdot 10^6}{0.9 \cdot 300 \cdot 587.5^2} = 3.86\ MPa$$

$$m = \left(\frac{f_y}{0.85f_c'}\right) = \frac{400}{0.85 \cdot 30} = 15.69$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{15.69}\left(1 - \sqrt{1 - \frac{2 \cdot 15.69 \cdot 3.86}{400}}\right) = 0.0105$$

$$A_s = \rho bd = 0.0105 \cdot 300 \cdot 587.5 = 1850.625\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{30}}{400}300 \cdot 587.5 = 603.35\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 300 \cdot 587.5 = \ 617\ mm^2 \quad -control$$

$$A_s = 1850.625\ mm^2 > A_{s,min} = 617\ mm^2 \ \ -OK$$

Use $4 \varnothing 25$ with $A_s(4 \varnothing 25) = 19.634\ cm^2 > \ A_{s,req} = 18.5\ cm^2 \quad -OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{1963.4\ \cdot 400}{0.85 \cdot 30 \cdot 300} = 102.66\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$$c = \frac{102.66}{0.836} = 122.8\ mm$$

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{587.5 - 122.8}{122.8}\right) = 0.01135 > 0.005 \qquad OK$$

Check for bar placement:

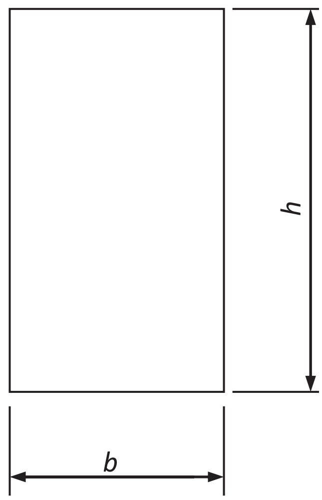
$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 4 \times 25}{3} = 33.33\ mm > d_b = 25\ mm, \quad > 25\ mm \qquad OK$$

Example:

Select an economical rectangular beam sizes and select bars using ACI strength method. The beam is a simply supported span of a 12 m and it is to carry a live load of 20 KN/m and a dead load of 25 KN/m including beam weight.

Take $f_c' = 28\ MPa$, $f_y = 400\ MPa$.

Assume $d \approx 2b$



__Solution:__

$$w_u = 1.2DL + 1.6LL = 1.2 \cdot 25 + 1.6 \cdot 20 = 62\ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{62 \cdot 12^2}{8} = 1116\ KN \cdot m$$

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85$ $(f_c' = 28\ MPa)$

$$\rho_b = 0.85\ \frac{f_c'}{f_y} \beta_1 \left(\frac{600}{600 + f_y}\right) = 0.85\ \frac{28}{400} 0.85 \left(\frac{600}{600 + 400}\right) = 0.030345$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.030345 = 0.012138$$

$$m = \left(\frac{f_y}{0.85 f_c'}\right) = \left(\frac{400}{0.85 \cdot 28}\right) = 16.807$$

$$R_n = \rho f_y \left(1 - \frac{\rho m}{2}\right) = 0.012138 \cdot 400 \left(1 - \frac{0.012138 \cdot 16.807}{2}\right) = 4.36\ MPa$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{1116 \cdot 10^6}{0.9 \cdot 4.36} = 4b^3 \qquad \rightarrow \quad b = \sqrt[3]{\frac{1116 \cdot 10^6}{4 \cdot 0.9 \cdot 4.36}} = 414.28\, mm$$

Take $b = 400\, mm$ and $d = 2b = 2 \cdot 400 = 800\, mm$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{1116 \cdot 10^6}{0.9 \cdot 400 \cdot 800^2} = 4.84\, MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{16.807}\left(1 - \sqrt{1 - \frac{2 \cdot 16.807 \cdot 4.84}{400}}\right) = 0.01367$$

$$A_s = \rho b d = 0.01367 \cdot 400 \cdot 800 = 4374.54\, mm^2$$

$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

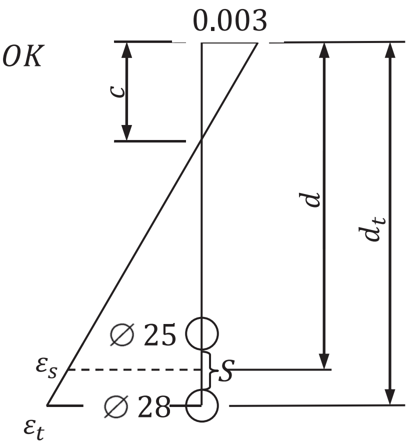
$$A_{s,min} = 0.25 \frac{\sqrt{28}}{400} 400 \cdot 800 = 1058.3\, mm^2$$

$$A_{s,min} = \frac{1.4}{400} 400 \cdot 800 = \ 1120\, mm^2 \quad - control$$

$$A_s = 4374.54\, mm^2 > A_{s,min} = 1120\, mm^2 \quad - OK$$

Take $4 \varnothing 28 + 4 \varnothing 25$ in two layers with

$$A_s = 24.63 + 19.635 = 44.265\, cm^2 > \ A_{s,req} = 43.74\, cm^2 \quad - OK$$



Check for strain:

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{4426.5 \quad \cdot 400}{0.85 \cdot 28 \cdot 400} = 185.99\, mm$$

$$c = \frac{a}{\beta_1} = \frac{185.99}{0.85} = 218.81\, mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 800 + \frac{25}{2} + \frac{28}{2} = 826.5\, mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{826.5 - 218.81}{218.81}\right) = 0.00833 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{400 - 40 \times 2 - 10 \times 2 - 4 \times 28}{3} = 62.67\, mm > d_b = 28\, mm, \quad > 25\, mm \qquad OK$$

$$h = d_t + \frac{d_b}{2} + \varnothing\text{stirrups} + cover = 826.5 + \frac{28}{2} + 10 + 40 = 890.5\, mm$$

Take $b = 400\, mm$ and $h = 900\, mm.$

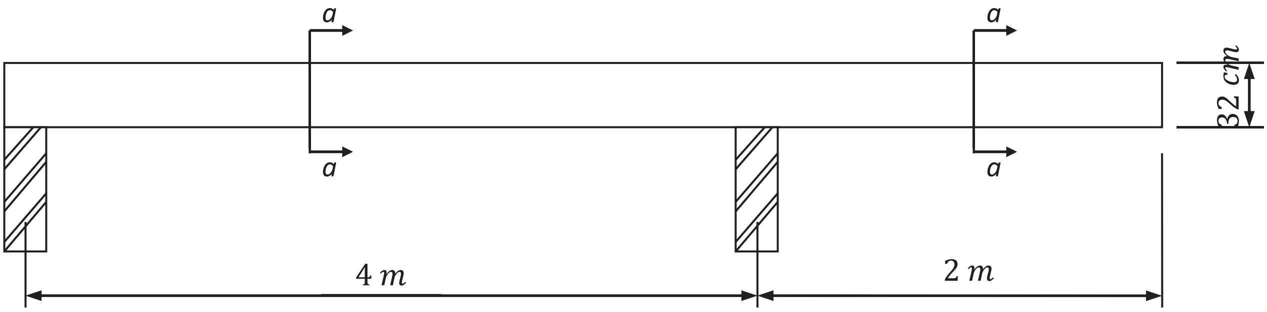
Example:

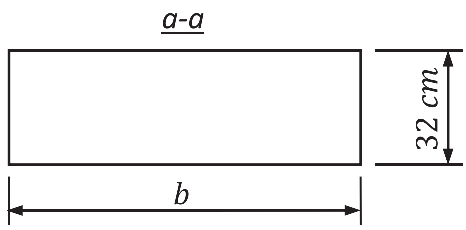
The beam shown below is loaded by service (unfactored) dead load of $45\, KN/m$ and service live load of $25\, KN/m$. Design the beam for flexure given the following information:

$f_c' = 24\ MPa, \qquad f_y = 420\ MPa.$

Assume the depth of the beam $h = 32\ cm$

Use bars $\varnothing\, 16$





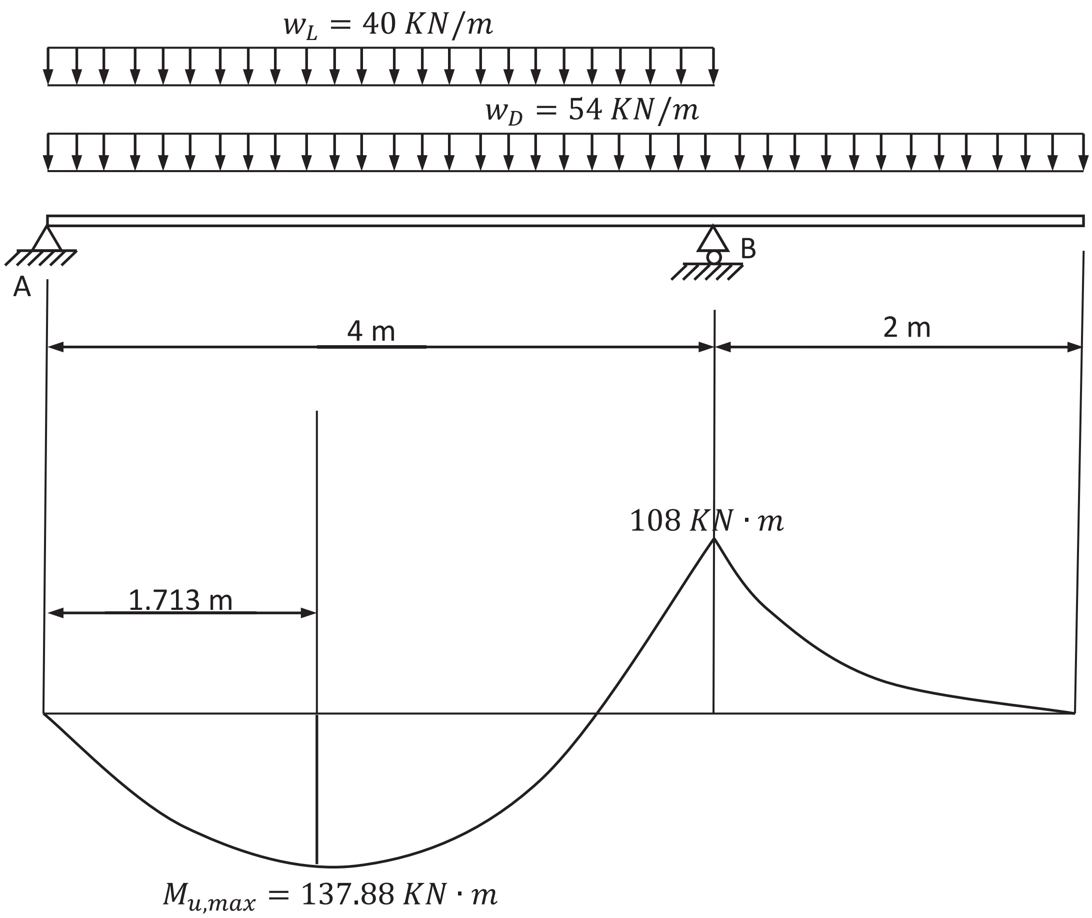
Solution:

$w_D = 1.2 \cdot 45 = 54\ KN/m$

$w_L = 1.6 \cdot 25 = 40\ KN/m$

Determination the maximum positive and negative bending moments for the beam:

- Maximum positive bending moment.



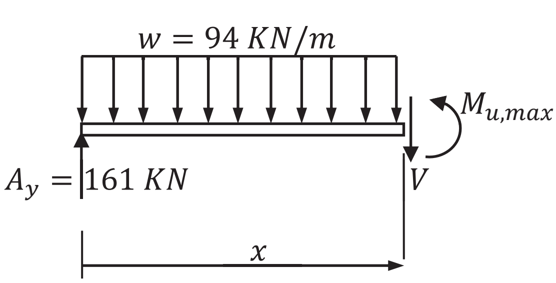
$\curvearrowright + \sum M_B = 0, \qquad A_y \cdot 4 - 94 \cdot 4 \cdot 2 + 54 \cdot 2 \cdot 1 = 0 \qquad\qquad A_y = 161\ KN$

Location of Maximum positive moment at distance x from support A from condition of zero shear force.

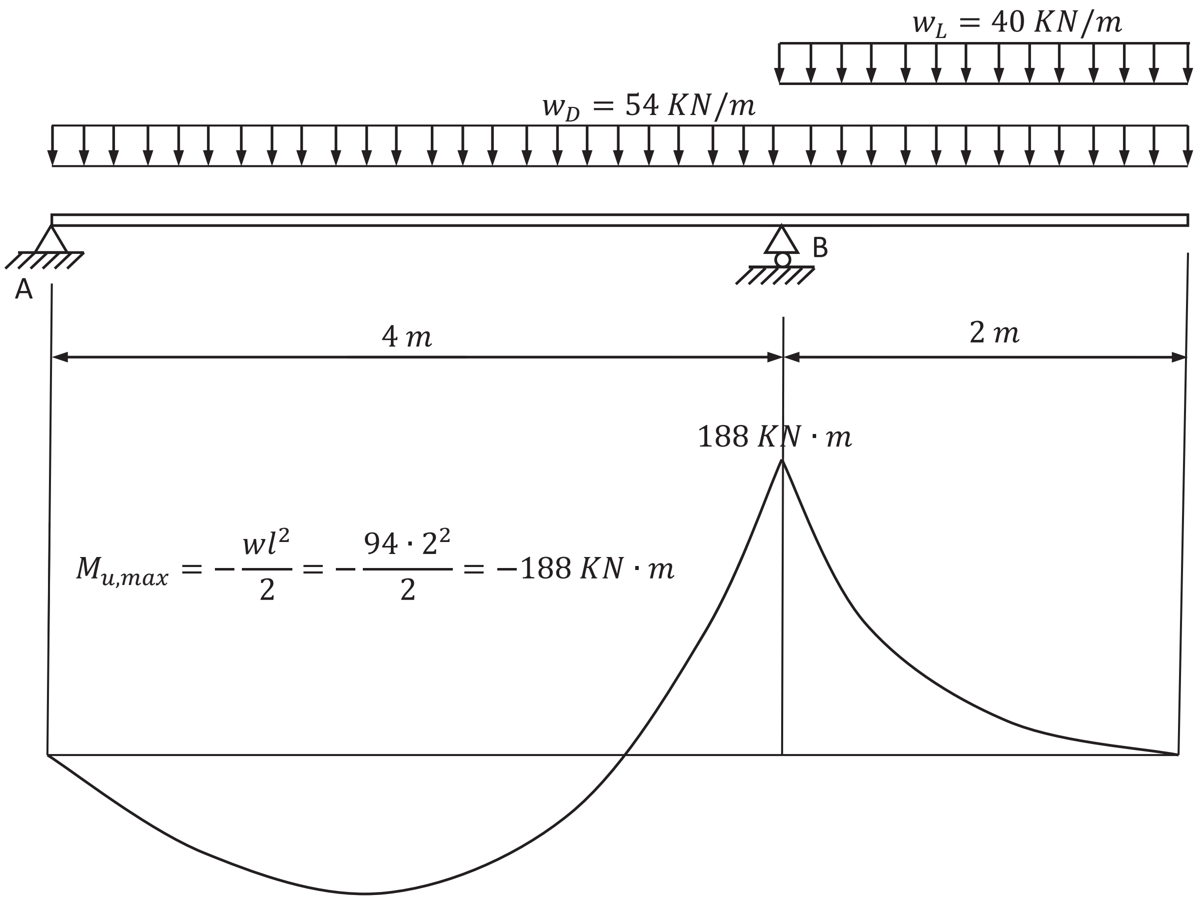
$V(x) = 0, \qquad 161 - 94 \cdot x = 0 \qquad x = 1.713\ m$

$$M_{u,max} = 161 \cdot 1.713 - 94 \cdot \frac{1.713^2}{2} = 137.88\ KN \cdot m$$

$$M_B = -54 \cdot \frac{2^2}{2} = -108\ KN \cdot m$$



- Maximum negative bending moment.

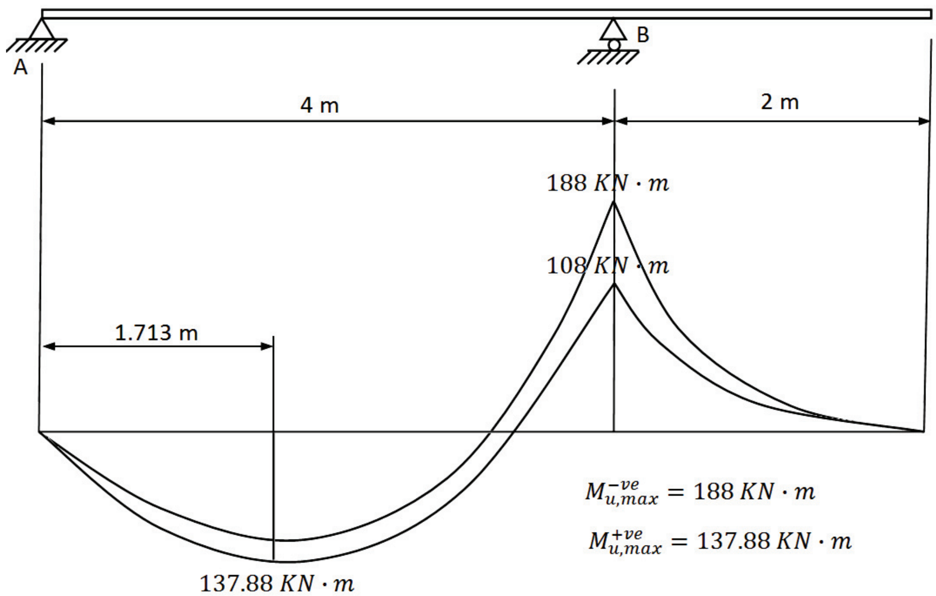


$$M_{u,max} = -\frac{wl^2}{2} = -\frac{94 \cdot 2^2}{2} = -188\ KN \cdot m$$

The maximum moments from all cases (envelope):

$M^{+ve}_{u,max} = 137.88\ KN \cdot m$

$M^{-ve}_{u,max} = 188\ KN \cdot m$



Determination of the beam width b and Design for negative moment $M_u = 188\ KN \cdot m$

Take $\phi = 0.9$ for flexure as tension-controlled section

Assume $\rho = 0.4\rho_b$.

Take $\beta_1 = 0.85$ ($f_c' = 24\ MPa$)

$$\rho_b = 0.85\ \frac{f_c'}{f_y}\beta_1\left(\frac{600}{600+f_y}\right) = 0.85\ \frac{24}{420}0.85\left(\frac{600}{600+420}\right) = 0.02429$$

$$\rho = 0.4\rho_b = 0.4 \cdot 0.02429 = 0.01$$

$$m = \left(\frac{f_y}{0.85f_c'}\right) = \left(\frac{420}{0.85 \cdot 24}\right) = 20.6$$

$$R_n = \rho f_y\left(1 - \frac{\rho m}{2}\right) = 0.01 \cdot 420\left(1 - \frac{0.01 \cdot 20.6}{2}\right) = 3.767\ MPa$$

$$d = h - cover - \varnothing\text{stirrups} - \frac{\varnothing\text{bar}}{2} = 320 - 40 - 10 - \frac{16}{2} = 262\ mm$$

$$bd^2 = \frac{M_u}{\phi R_n} = \frac{188 \cdot 10^6}{0.9 \cdot 3.767} = b \cdot 262^2 \qquad \rightarrow \qquad b = \frac{188 \cdot 10^6}{0.9 \cdot 3.767 \cdot 262^2} = 807.8\ mm$$

Take $b = 900\ mm$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{188 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 3.38\ MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1 - \sqrt{1 - \frac{2 \cdot 20.6 \cdot 3.38}{420}}\right) = 0.0089$$

$$A_s = \rho b d = 0.0089 \cdot 900 \cdot 262 = 2099\ mm^2$$

$$A_{s,min} = 0.25\frac{\sqrt{f_c'}}{f_y}b_w d \geq \frac{1.4}{f_y}b_w d$$

$$A_{s,min} = 0.25\frac{\sqrt{24}}{420}900 \cdot 262 = 688\ mm^2$$

$$A_{s,min} = \frac{1.4}{420}900 \cdot 262 = 786\ mm^2 \quad - control$$

$A_s = 2099\ mm^2 > A_{s,min} = 786\ mm^2 \quad - OK$

Take 11 $\varnothing$ 16 in one layer with $A_s = 22.11\ cm^2 > A_{s,req} = 20.99\ cm^2 \quad - OK$

Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{2211 \cdot 420}{0.85 \cdot 24 \cdot 900} = 50.6\ mm$$

$$c = \frac{a}{\beta_1} = \frac{50.6}{0.85} = 59.5\ mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{262 - 59.5}{59.5}\right) = 0.01 > 0.005 \qquad OK$$

Check for bar placement:

$$S_b = \frac{900 - 40 \times 2 - 10 \times 2 - 11 \times 16}{10} = 62.4\,mm > 25\,mm \qquad OK$$

Design for positive moment $M_u = 137.88\ KN \cdot m$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{137.88 \cdot 10^6}{0.9 \cdot 900 \cdot 262^2} = 2.48\,MPa$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) = \frac{1}{20.6}\left(1 - \sqrt{1 - \frac{2 \cdot 20.6 \cdot 2.48}{420}}\right) = 0.0063$$

$$A_s = \rho b d = 0.0063 \cdot 900 \cdot 262 = 1486\,mm^2$$

$$A_{s,min} = 786\,mm^2$$

$$A_s = 1486\,mm^2 > A_{s,min} = 786\,mm^2 \quad - OK$$

Take 8 ∅ 16 in one layer with $A_s = 16.08\,cm^2 > A_{s,req} = 14.86\,cm^2 \quad - OK$

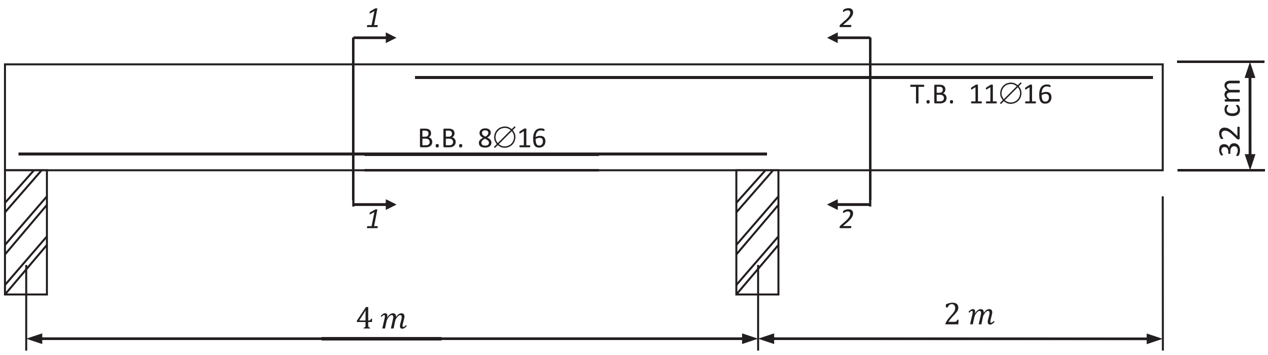
Check for strain:

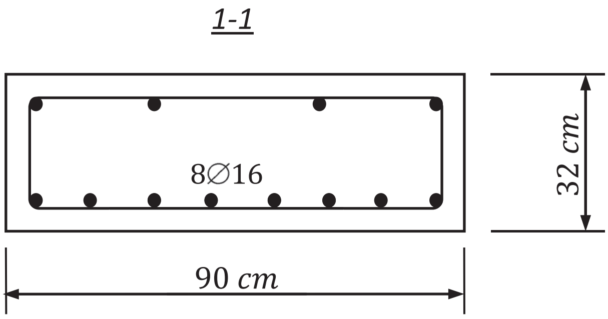
$$a = \frac{A_s f_y}{0.85\,f_c' b} = \frac{1608 \cdot 420}{0.85 \cdot 24 \cdot 900} = 36.78\,mm$$

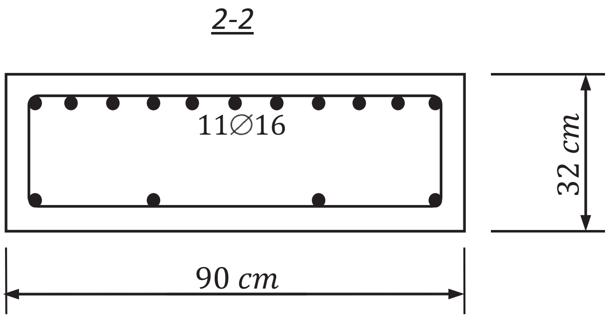
$$c = \frac{a}{\beta_1} = \frac{37}{0.85} = 43.28\,mm$$

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{262 - 43.28}{43.28}\right) = 0.0152 > 0.005 \qquad OK$$

Check for bar placement: $S_b > 25\,mm \qquad OK$



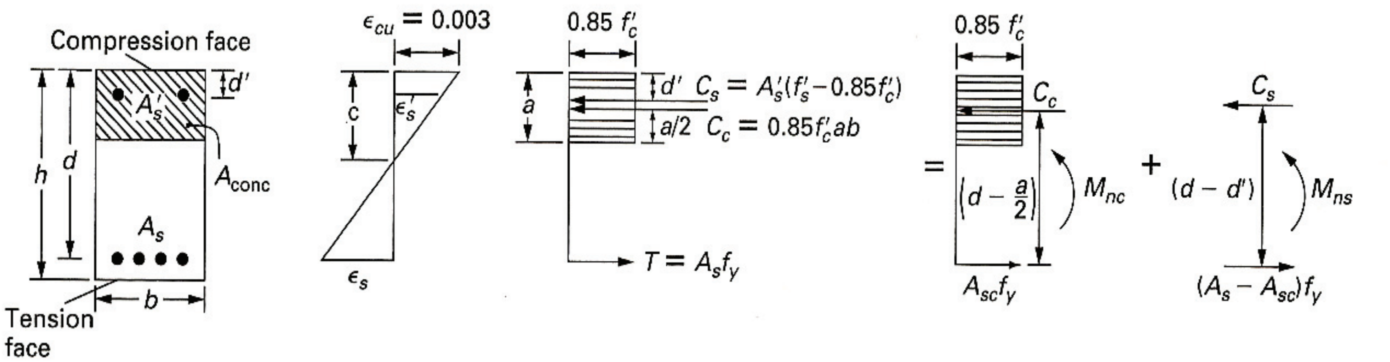




4.10 DOUBLY REINFORCED CONCRETE SECTIONS (SECTIONS WITH COMPRESSION REINFORCEMENT).

Flexural members are designed for tension reinforcement. Any additional moment capacity required in the section is usually provided by increasing the section size or the amount of tension reinforcement.
However, the cross-sectional dimensions in some applications may be limited by architectural or functional requirements (architectural limitations restrict the beam web depth at midspan, or the midspan section dimensions are not adequate to carry the support negative moment even when tensile steel at the support is sufficiently increased), and the extra moment capacity may have to be provided by additional tension and compression reinforcement. The extra steel generates an internal force couple, adding to the sectional moment capacity without changing the ductility of the section. In such cases, the total moment capacity consists of two components:

1. moment due to the tension reinforcement that balances the compression concrete, M_{nc}, and
2. moment generated by the internal steel force couple consisting of compression reinforcement and equal amount of additional tension reinforcement, M_{ns} as illustrated in figure below.



(a) Cross section (b) Strain diagram (c) Stress diagram (d) Moment carried as a singly reinforced beam, M_{nc} (e) Moment carried by the compression steel, M_{ns}

Notation:

ε_s' – strain in compression steel.

$f_s' = E_s\varepsilon_s' \leq f_y$ – compression steel stress

A_s' – area fo compression steel

d' – distance from extreme compression fiber to centroid of compression steel

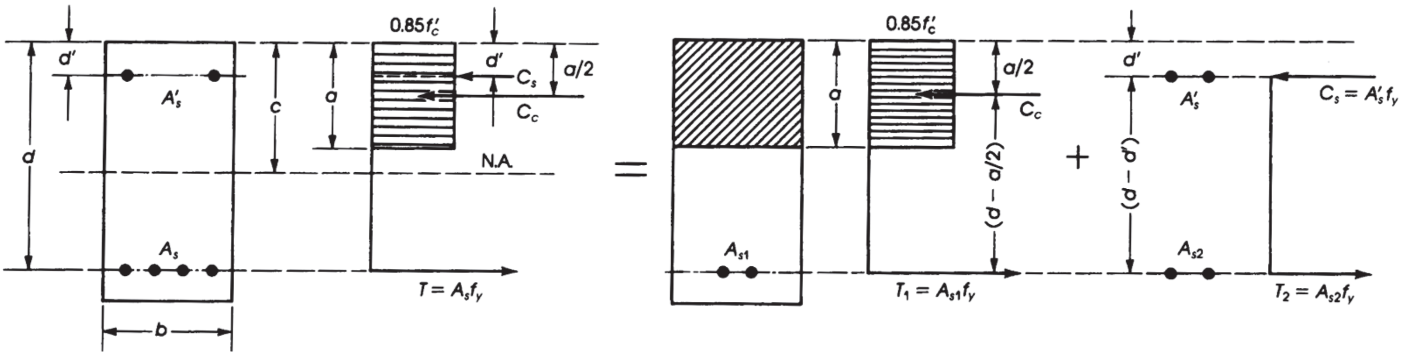
$\rho' = \dfrac{A_s'}{bd}$ – compression steel reinforcement ratio.

A_{sc} – part of the tension steel that match C_c.

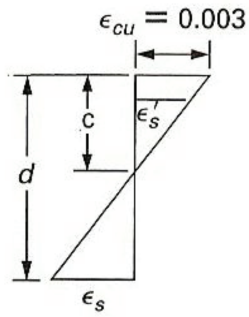
C_c – concrete compression resultant for a beam without compression reinforcement.

C_s – compression steel resultant as if A_s' were stressed at $(f_s' - 0.85f_c')$.

4.10.1 Analysis of doubly reinforced concrete sections



- **<u>Compression steel is yielded</u>**



Compression steel is yielded when $\varepsilon'_s \geq \varepsilon_y = \dfrac{f_y}{E_s}$

$$\varepsilon'_s = 0.003\left(\frac{c-d'}{c}\right), \qquad c = \frac{a}{\beta_1}$$

$$A_{s1} = A_s - A_{s2}, \qquad A_{s2} = A'_s$$

$$T_1 = A_{s1} f_y = C_c \quad and \quad M_{nc} = A_{s1} f_y\left(d - \frac{a}{2}\right) \;\; or \;\; M_{nc} = (A_s - A'_s) f_y\left(d - \frac{a}{2}\right)$$

where $a = \dfrac{A_{s1} f_y}{0.85\, f'_c b} = \dfrac{(A_s - A'_s) f_y}{0.85\, f'_c b}$ $\qquad \rho = \dfrac{A_s}{bd}$ $\qquad \rho' = \dfrac{A'_s}{bd}$

substituting "a" into $\mathrm{c} = \dfrac{a}{\beta_1} = \dfrac{(A_s - A'_s) f_y}{\beta_1 \cdot 0.85\, f'_c b} = \dfrac{(\rho - \rho') f_y d}{\beta_1 \cdot 0.85\, f'_c}$

substituting "c" into $\varepsilon'_s = 0.003\left(1 - \dfrac{d'}{c}\right) = 0.003\left[1 - \dfrac{0.85\beta_1 f'_c d'}{(\rho - \rho') d f_y}\right]$

Compression steel is yielded when

$$\varepsilon'_s \geq \varepsilon_y = \frac{f_y}{E_s}$$

$$0.003\left[1 - \frac{0.85\beta_1 f'_c d'}{(\rho - \rho') d f_y}\right] \geq \frac{f_y}{E_s = 200000\, MPa}$$

or in the form

$$\rho - \rho' \geq \frac{0.85 f'_c d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right)$$

$$\rho \geq \bar{\rho}_{cy}$$

where
$$\bar{\rho}_{cy} = \frac{0.85 f'_c d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right) + \rho' \qquad (*)$$

$\bar{\rho}_{cy}$ − minimum tensile reinforcement ratio that will ensure yielding of compression steel at failure.

In the previous equation of $\bar{\rho}_{cy}$ was ignored that part of the compression zone is occupied by the compression reinforcement, the value of ignored compressive force is $A_s'(0.85f_c{}')$. So the depth of stress block can be expressed

$$a = \frac{A_s f_y - A_s'(f_y - 0.85f_c{}')}{0.85f_c{}'b},$$

and

$$\bar{\rho}_{cy} = \frac{0.85f_c' d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right) + \rho'(1 - \frac{0.85f_c'}{f_y})$$

In all calculations, the equation $(*)$ for $\bar{\rho}_{cy}$ will be used.

$T = A_s f_y = C_c + C_s = T_1 + T_2$

$C_c = 0.85f_c' ab, \qquad C_s = A_s'(f_y - 0.85f_c')$

$A_s f_y = 0.85f_c' ab + A_s'(f_y - 0.85f_c') \qquad from\ where \qquad a = \dfrac{A_s f_y - A_s'(f_y - 0.85f_c{}')}{0.85f_c{}'b}$

The nominal moment strength for rectangular section with tension and **compression steel is yielded**

$$M_n = \left(A_s f_y - A_s'(f_y - 0.85f_c{}')\right)\left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85f_c{}')(d - d'),$$

$$or \qquad M_n = 0.85f_c' ab\left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85f_c{}')(d - d').$$

For simplicity, $A_s'(0.85f_c{}')$ can be ignored and then:

$$T = A_s f_y = C_c + C_s = T_1 + T_2, \qquad C_c = 0.85f_c' ab, \qquad C_s = A_s' f_y \qquad a = \frac{(A_s - A_s')f_y}{0.85f_c{}'b}$$

$$M_n = (A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s' f_y(d - d') = 0.85f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_y(d - d')$$

- **<u>Compression steel is NOT yielded</u>**

Compression steel is NOT yielded when

$$\varepsilon_s' < \varepsilon_y = \frac{f_y}{E_s} \qquad \text{or} \qquad f_s' = \varepsilon_s' E_s < f_y \qquad \text{or} \qquad \rho < \bar{\rho}_{cy}$$

$$f_s' = \varepsilon_s' E_s = 0.003\left(\frac{c - d'}{c}\right)200\,000 = 600\left(\frac{c - d'}{c}\right)$$

$T = A_s f_y = C_c + C_s = T_1 + T_2$

$C_c = 0.85f_c' ab, \qquad C_s = A_s'(f_s' - 0.85f_c')$

$$A_s f_y = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c') \quad from\ where \quad a = \frac{A_s f_y - A_s'(f_s' - 0.85 f_c')}{0.85 f_c' b} = \beta_1 c.$$

Note that in the above equation two unknowns "c" and "f_s'". Substitiuting $f_s' = 600\left(\frac{c-d'}{c}\right)$ in "a" we get an quadratic equation in "c", the only unknown, which is easily solved for "c".

The nominal moment strength for rectangular section with tension and compression **steel is NOT yielded**

$$M_n = \left(A_s f_y - A_s'(f_s' - 0.85 f_c')\right)\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d'),$$

$$or \qquad M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c')(d - d').$$

For simplicity, $A_s'(0.85 f_c')$ can be ignored and then:

$$T = A_s f_y = C_c + C_s = T_1 + T_2, \qquad C_c = 0.85 f_c' ab, \qquad C_s = A_s' f_s' \qquad a = \frac{A_s f_y - A_s' f_s'}{0.85 f_c' b}$$

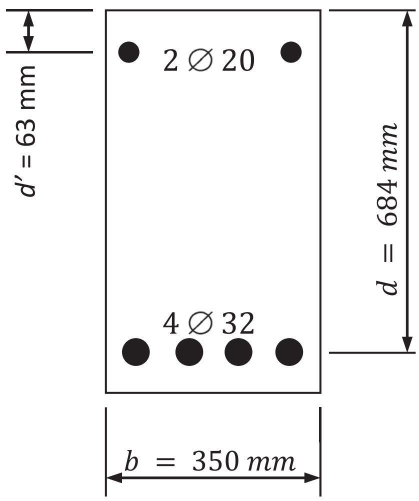
$$M_n = \left(A_s f_y - A_s' f_s'\right)\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d') = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s' f_s'(d - d')$$

For both cases (compression steel is yielded and is NOT yielded) $\varepsilon_s \geq 0.005$ (tension-controlled section).

Example:

Determine the nominal positive moment strength of the section of rectangular cross sectional beam. The beam is reinforced with $4 \varnothing 32$ in the tension zone and $2 \varnothing 20$ in the compression zone.

Take $f_c' = 20\ MPa, \quad f_y = 400\ MPa.$



<u>Solution:</u>

$A_s(4 \varnothing 32) = 32.17\ cm^2$

$A_s'(2 \varnothing 20) = 6.28\ cm^2$

$$\rho = \frac{A_s}{bd} = \frac{3217}{350 \cdot 684} = 0.0134$$

$$\rho' = \frac{A_s'}{bd} = \frac{628}{350 \cdot 684} = 0.0026, \qquad \beta_1 = 0.85,$$

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y}\beta_1\left(\frac{600}{600 - f_y}\right) + \rho' = \frac{0.85 \cdot 20 \cdot 63}{684 \cdot 400} 0.85\left(\frac{600}{600 - 400}\right) + 0.0026 = 0.01258$$

$\rho = 0.0134 > \bar{\rho}_{cy} = 0.01258$ compression steel is yielded ($\varepsilon_s' \geq \varepsilon_y$)

$T = A_s f_y = C_c + C_s$

$A_s f_y = 0.85 f_c' ab + A_s'(f_y - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c{}')}{0.85 f_c{}' b} = \frac{3217 \cdot 400 - 628 \cdot (400 - 0.85 \cdot 20)}{0.85 \cdot 20 \cdot 350} = 175.84\, mm,$$

$$c = \frac{a}{\beta_1} = \frac{175.84}{0.85} = 206.88\, mm,$$

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2}\right) + A_s'(f_y - 0.85 f_c{}')(d - d') =$$

$$= \left[0.85 \cdot 20 \cdot 175.84 \cdot 350 \left(684 - \frac{175.84}{2}\right) + 628(400 - 0.85 \cdot 20)(684 - 63)\right] \times 10^{-6} =$$

$$= 773.01\, KN \cdot m$$

Check for $\varepsilon_s \geq 0.005$:

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{684 - 206.88}{206.88}\right) = 0.00691 > 0.005 \qquad OK$$

Take $\phi = 0.9$ for flexure as tension-controlled section.

$$\phi M_n = 0.9 \cdot 773.01 = 695.71\, KN \cdot m$$

Example:

Repeat the previous example using $f_c' = 30\, MPa.$

Solution:

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y} \beta_1 \left(\frac{600}{600 - f_y}\right) + \rho' = \frac{0.85 \cdot 30 \cdot 63}{684 \cdot 400} 0.836 \left(\frac{600}{600 - 400}\right) + 0.0026 = 0.0173$$

$\rho = 0.0134 < \bar{\rho}_{cy} = 0.0173$ compression steel is NOT yielded ($\varepsilon_s' < \varepsilon_y$)

$$T = A_s f_y = C_c + C_s$$

$$f_s' = 600 \left(\frac{c - d'}{c}\right), \qquad \beta_1 = 0.85 - 0.007(f_c' - 28) = 0.85 - 0.007(30 - 28) = 0.836$$

$A_s f_y = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_s' - 0.85 f_c{}')}{0.85 f_c{}' b} = \beta_1 c$$

$$\frac{3217 \cdot 400 - 628 \cdot \left(600 \left(\frac{c - 63}{c}\right) - 0.85 \cdot 30\right)}{0.85 \cdot 30 \cdot 350} = 0.836\, c$$

$$103.755 + \frac{2659.764}{c} = 0.836\, c, \qquad \Longrightarrow \quad 0.836\, c^2 - 103.755\, c - 2659.764 = 0,$$

solution of quadratic equation $\qquad x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$c^2 - 124.109\, c - 3181.536 = 0,$$

$$c_{1,2} = \frac{124.109 \pm \sqrt{124.109^2 - 4 \cdot 1 \cdot (-3181.536)}}{2} = \frac{124.109 \pm 167.718}{2}$$

Choose only $c > 0$, $c = 145.91\ mm$

$a = \beta_1 c = 0.836 \cdot 145.91 = 121.98\, mm,$

$$f_s' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{145.91-63}{145.91}\right) = 340.94\, MPa \quad < f_y = 400\, MPa,$$

$$M_n = 0.85 f_c' ab\left(d - \frac{a}{2}\right) + A_s'(f_s' - 0.85 f_c{}')(d-d') =$$

$$= \left[0.85 \cdot 30 \cdot 121.98 \cdot 350\left(684 - \frac{121.98}{2}\right) + 628(340.94 - 0.85 \cdot 30)(684-63)\right] \times 10^{-6} =$$

$$= 801.27\, KN \cdot m$$

Check for $\varepsilon_s \geq 0.005$:

$$\varepsilon_s = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{684-145.91}{145.91}\right) = 0.011 > 0.005 \qquad OK$$

Take $\phi = 0.9$ for flexure as tension-controlled section.

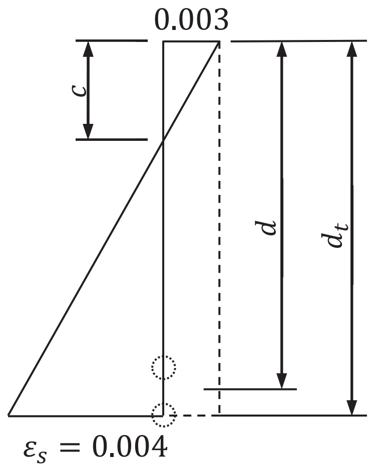
$\phi M_n = 0.9 \cdot 801.27 = 721.14\, KN \cdot m$

4.10.2 Design of doubly reinforced concrete sections.

When the factored moment M_u is greater than the design strength ϕM_n of the beam when it is reinforced with the maximum permissible amount of tension reinforcement, compression reinforcement becomes necessary.

The logical procedure for designning a doubly reinforced sections is to determine first whether compression steel is needed for strength. This may be done by comparing the required moment strength with the moment strength of a singly reinforced section with the maximum permissible amount of tension steel ρ_{max}.

For example, for steel Grade 420 $\rho_{max} = 0.724\rho_b$ which defined from strain conditon $\varepsilon_t = 0.004$ for beams.



$$\frac{c}{0.003} = \frac{d_t}{0.003 + 0.004} \quad \Longrightarrow \quad c = \frac{3}{7} d_t, \qquad a = \beta_1 c$$

The maximum moment strength as a singly reinforced section

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right),$$

If $M_u > \phi M_{n,max}$ Design the section as doubly reinforced section,

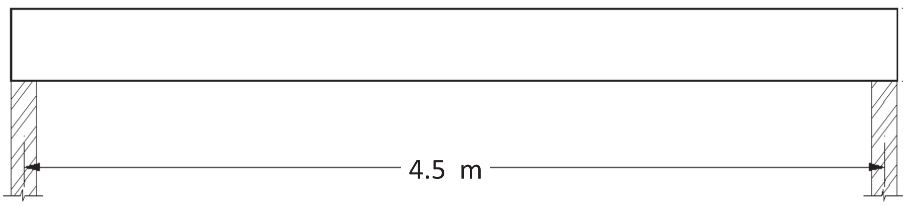
where $\phi = 0.65 + (0.004 - 0.002)\dfrac{250}{3} = 0.817 \approx 0.82$

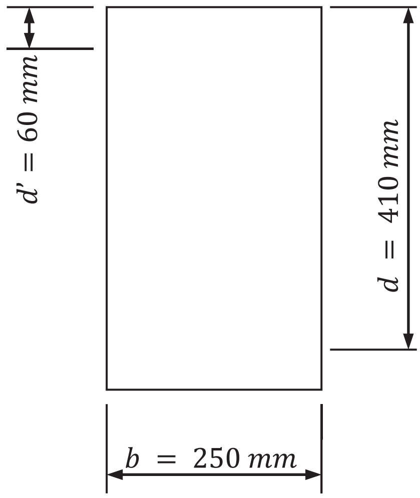
Example:

The beam is loaded by a uniform service $DL = 25\ KN/m$ and a uniform service $LL = 35\ KN/m$. Compute the area of steel reinforcement for the section.

Take $f_c' = 20\ MPa, \quad f_y = 400\ MPa.$

Assume $d' = 60\ mm$, and one layer arrangement of tension steel.





<u>Solution:</u>

$$w_u = 1.2D + 1.6L = 1.2 \cdot 25 + 1.6 \cdot 35 = 86\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{86 \cdot 4.5^2}{8} = 217.7\ KN \cdot m$$

Maximum nominal moment strength from strain condition $\varepsilon_s = 0.004$

$$c = \frac{3}{7}d = \frac{3}{7}410 = 175.7\ mm, \qquad \beta_1 = 0.85$$

$$a = \beta_1 c = 0.85 \cdot\ 175.7 = 149.4\ mm$$

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) = 0.85 \cdot 20 \cdot 149.4 \cdot 250\left(410 - \frac{149.4}{2}\right) \times 10^{-6} = 212.9\ KN \cdot m$$

$$\phi = 0.82$$

$$M_u = 217.7\ KN \cdot m > \phi M_n = 0.82 \cdot 212.9 = 174.6\ \ KN \cdot m$$

Design the section as doubly reinforced concrete section.

$$M_{ns} = \frac{M_u}{\phi} - M_{nc} = \frac{217.7}{0.82} -\ 212.9 = 52.59\ KN \cdot m$$

$$M_{ns} = C_s(d - d') = A_s'(f_s{}' - 0.85 f_c{}')(d - d') \quad \Longrightarrow \quad A_s' = \frac{M_{ns}}{(f_s{}' - 0.85 f_c{}')(d - d')}$$

$$f_s' = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{175.7 - 60}{175.7}\right) = 395.1\ MPa\ < f_y = 400\ MPa,$$

Compression steel does NOT yield

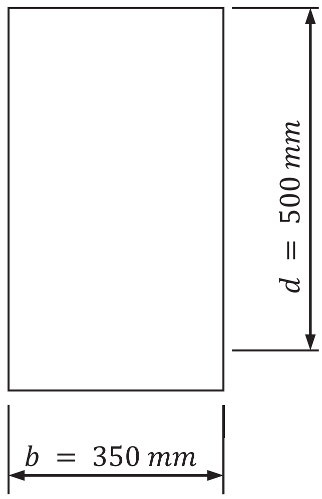
$$A_s' = \frac{M_{ns}}{(f_s{}' - 0.85 f_c{}')(d - d')} = \frac{52.59 \cdot 10^6}{(395.1\ - 0.85 \cdot 20)(410 - 60)} = 397.4\ mm^2$$

$$T = C_c + C_s = 0.85 f_c' ab + A_s'(f_s' - 0.85 f_c{}') =$$

$$= [0.85 \cdot 20 \cdot 149.4 \cdot 250 + 397.4(395.1 - 0.85 \cdot 20)] \times 10^{-3} = 785.21\ KN$$

$$A_s = \frac{T}{f_y} = \frac{785.21 \cdot 10^3}{400} = 1963.02\ mm^2$$

Example:



The beam section shown below is loaded by a factored bending moment $M_u = 520\ KN \cdot m$. Design the beam for flexure given the following information:

$f_c' = 24\ MPa, \qquad f_y = 400\ MPa.$

Use bars $\varnothing\, 25\ mm$, and assume one layer arrangement of tension steel.

<u>Solution:</u>

Check whether the section will be designed as singly or doubly:

Maximum nominal moment strength from strain condition $\varepsilon_s = 0.004$

$$c = \frac{3}{7}d = \frac{3}{7}500 = 214.29\ mm, \qquad \beta_1 = 0.85$$

$$a = \beta_1 c = 0.85 \cdot\ 214.29 = 182.14\ mm$$

$$M_{n,max} = 0.85 f_c' ab\left(d - \frac{a}{2}\right) = 0.85 \cdot 24 \cdot 182.14 \cdot 350\left(500 - \frac{182.14}{2}\right) \times 10^{-6} = 531.81\ KN \cdot m$$

$$\phi = 0.82$$

$$M_u = 520\ KN \cdot m > \phi M_n = 0.82 \cdot 531.81 = 436.1\ \ KN \cdot m$$

Design the section as doubly reinforced concrete section.

$$M_{ns} = \frac{M_u}{\phi} - M_{nc} = \frac{520}{0.82} -\ 531.81 = 102.34\ KN \cdot m$$

$$M_{ns} = C_s(d - d') = A_s'(f_s{}' - 0.85 f_c{}')(d - d') \quad \Longrightarrow \qquad A_s' = \frac{M_{ns}}{(f_s{}' - 0.85 f_c{}')(d - d')}$$

$$d' = cover + \varnothing stirrups + \frac{\varnothing bar}{2} = 40 + 10 + \frac{25}{2} = 62.5\ mm$$

$$f_s' = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{214.29 - 62.5}{214.29}\right) = 425\ MPa > f_y = 400\ MPa,$$

Compression steel is yielded. Take $\quad f_s' = f_y = 400\ MPa$

$$A_s' = \frac{M_{ns}}{(f_y - 0.85 f_c{}')(d - d')} = \frac{102.34 \cdot 10^6}{(400\ - 0.85 \cdot 24)(500 - 62.5)} = 616.23\ mm^2$$

$$T = C_c + C_s = 0.85 f_c' ab + A_s'(f_y - 0.85 f_c{}') =$$

$$= [0.85 \cdot 24 \cdot 182.14 \cdot 350 + 616.23(400 - 0.85 \cdot 24)] \times 10^{-3} = 1534.4\ KN$$

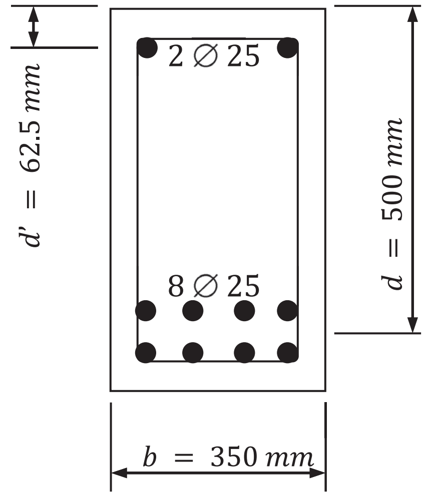
$$A_s = \frac{T}{f_y} = \frac{1534.4\ \cdot 10^3}{400} = 3836\ mm^2$$

Take 8 $\varnothing$ 25 in two layers with $A_s = 39.27\ cm^2 >\ A_{s,req} = 38.36\ cm^2 \quad - OK$

Take 2 $\varnothing$ 25 in one layer with $A_s' = 9.817\ cm^2 >\ A_{s,req}' = 6.16\ cm^2 \quad - OK$

Now it's an analysis problem of doubly reinforced concrete section.

Check whether compression steel has yielded:



$$\rho = \frac{A_s}{bd} = \frac{3927}{350 \cdot 500} = 0.02244$$

$$\rho' = \frac{A_s'}{bd} = \frac{981.7}{350 \cdot 500} = 0.00561, \qquad \beta_1 = 0.85,$$

$$\bar{\rho}_{cy} = \frac{0.85 f_c' d'}{d f_y} \beta_1 \left(\frac{600}{600 - f_y} \right) + \rho' =$$

$$= \frac{0.85 \cdot 24 \cdot 62.5}{500 \cdot 400} 0.85 \left(\frac{600}{600 - 400} \right) + 0.00561 = 0.02187$$

$$\rho = 0.02244 > \bar{\rho}_{cy} = 0.02187$$

compression steel is yielded ($\varepsilon_s' \geq \varepsilon_y$)

Check for $\varepsilon_t \geq 0.005$:

$T = A_s f_y = C_c + C_s \quad \Rightarrow \quad A_s f_y = 0.85 f_c' ab + A_s' (f_y - 0.85 f_c')$ from where

$$a = \frac{A_s f_y - A_s'(f_y - 0.85 f_c')}{0.85 f_c' b} = \frac{3927 \cdot 400 - 981.7 \cdot (400 - 0.85 \cdot 24)}{0.85 \cdot 24 \cdot 350} = 167.81\ mm,$$

$$c = \frac{a}{\beta_1} = \frac{167.81}{0.85} = 197.42\ mm, \qquad d_t = d + \frac{S}{2} + \frac{d_b}{2} = 500 + \frac{25}{2} + \frac{25}{2} = 525\ mm$$

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c} \right) = 0.003 \left(\frac{525 - 197.42}{197.42} \right) = 0.00498 < 0.005$$

When $0.004 < \varepsilon_t < 0.005$ (in transition zone between compression-controlled section and tension-controlled section), it is obvious here that the nominal moment strength of the section will satisfy the strength condition $\phi M_n \geq M_u,\ where \quad 0.82 < \phi < 0.9$.

This step is a proof of the above statement.

$$\phi = 0.65 + (0.00498 - 0.002) \frac{250}{3} = 0.8983 > 0.82 - as\ was\ used$$

$$M_n = 0.85 f_c' ab \left(d - \frac{a}{2} \right) + A_s' (f_s' - 0.85 f_c')(d - d') =$$

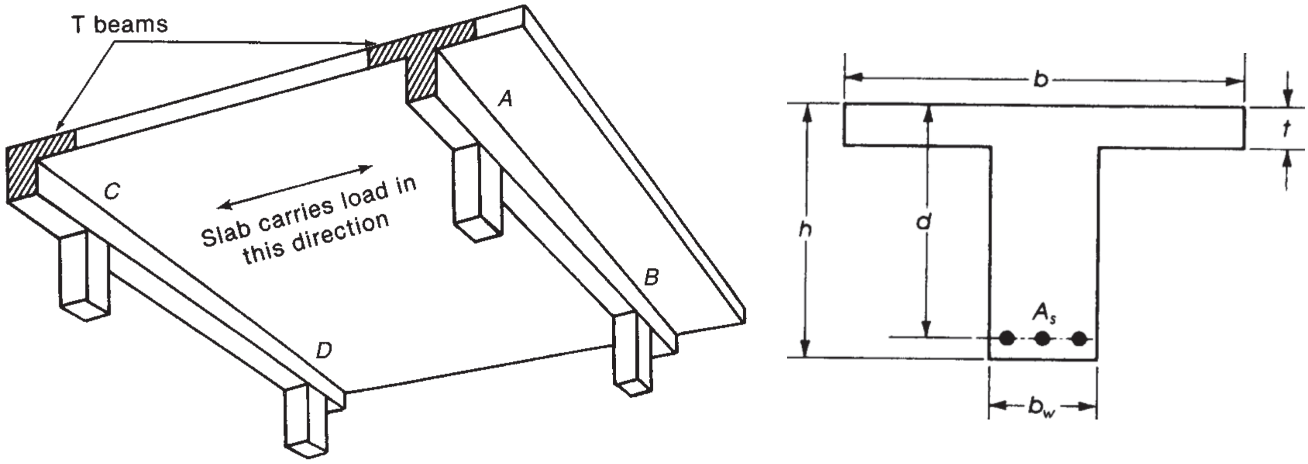
$$= \left[0.85 \cdot 24 \cdot 167.81 \cdot 350 \left(500 - \frac{167.81}{2} \right) + 981.7(400 - 0.85 \cdot 24)(500 - 62.5) \right] \times 10^{-6} =$$

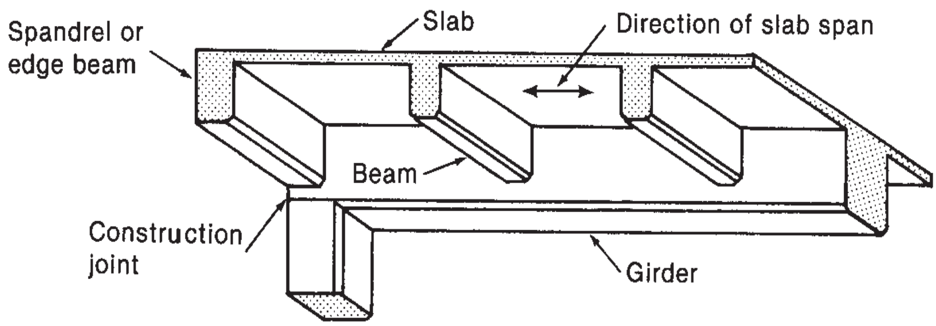
$$= 661.59\ KN \cdot m$$

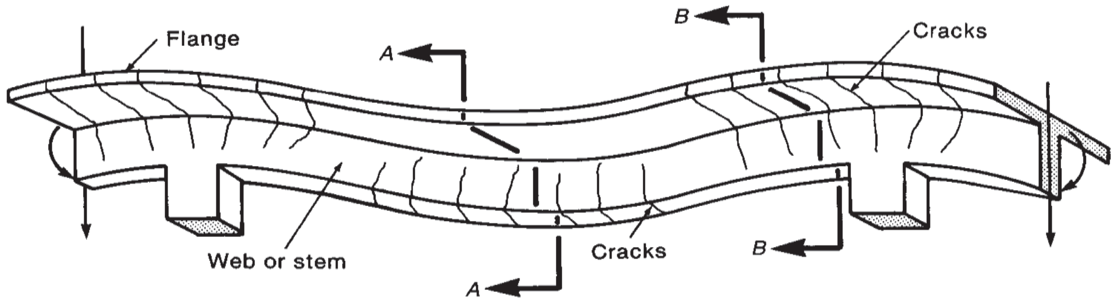
$$\phi M_n = 0.8983 \cdot 661.59 = 594.31\ KN \cdot m > M_u = 520 \quad KN \cdot m$$

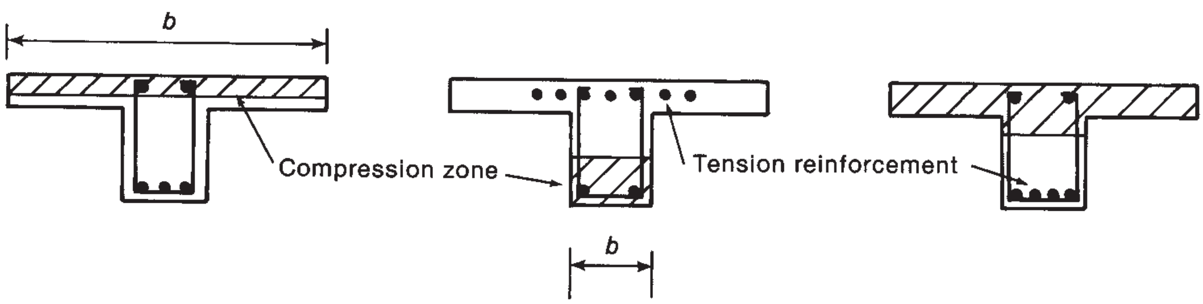
4.1 REINFORCED CONCRETE FLANGED SECTIONS (T- AND L- SECTIONS).

It is normal to cast concrete slabs and beams together, producing a monolithic structure. Slabs have smaller thicknesses than beams. Under bending stresses, those parts of the slab on either side of the beam will be subjected to compressive stresses, depending on the position of these parts relative to the top fibers and relative to their distances from the beam. The part of the slab acting with the beam is called the flange. The rest of the section is called the stem, or web.









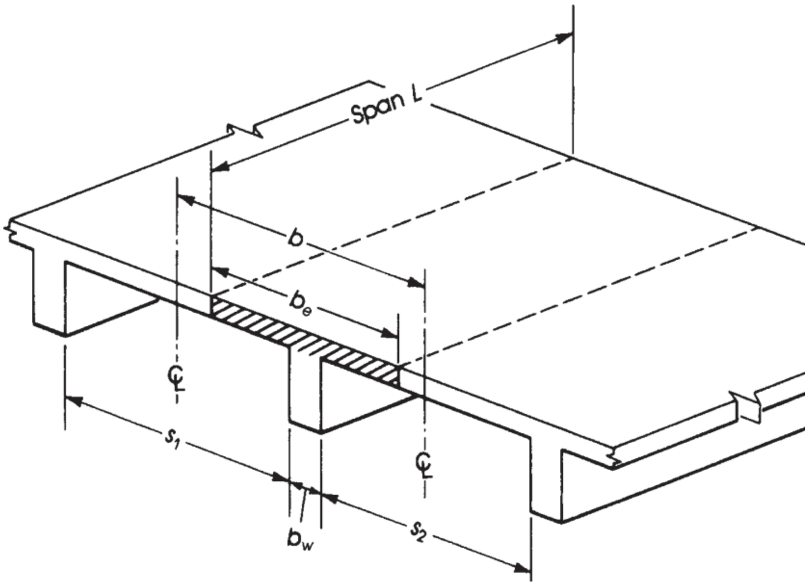
(b) Section *A-A* (rectangular compression zone).

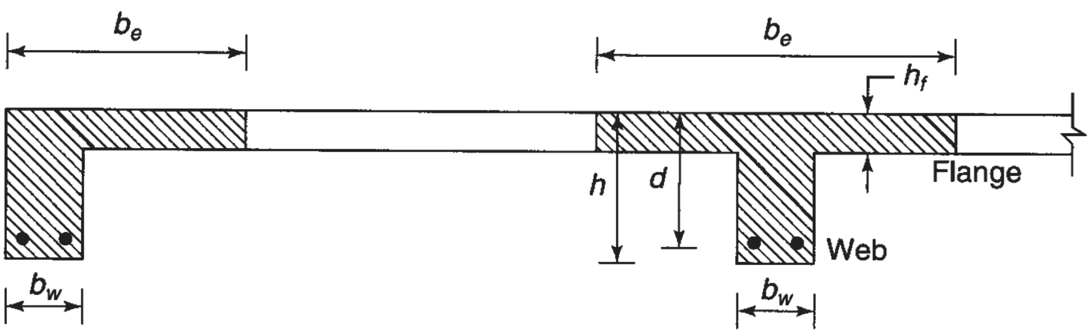
(c) Section *B-B* (negative moment).

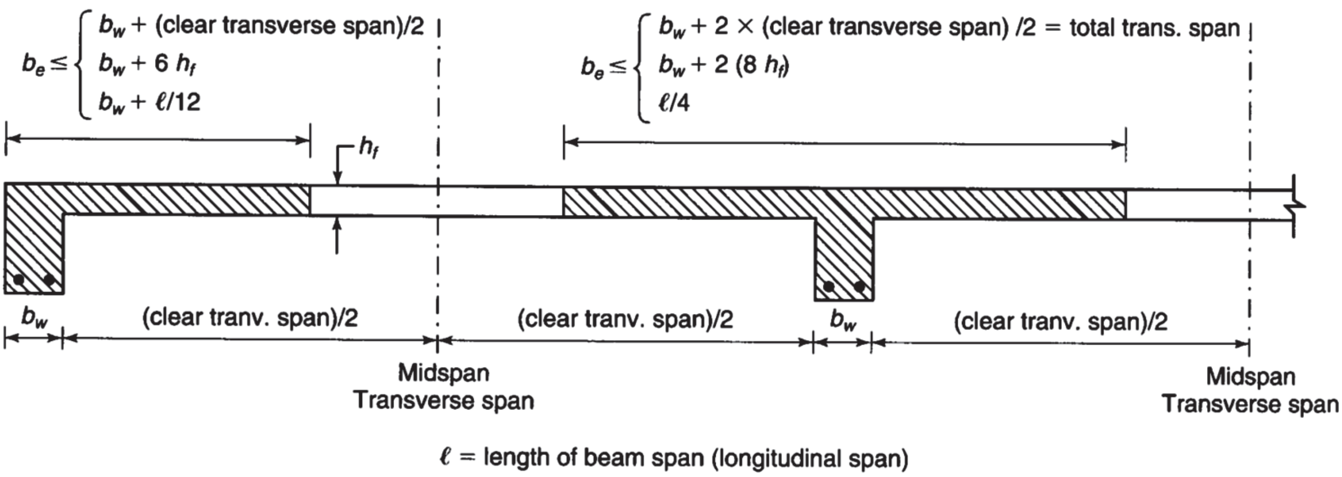
(d) Section *A-A* (T-shaped compression zone).

4.11.1 Effective width.

The ACI Code definitions for the effective compression flange width for T- and inverted L-shapes in continuous floor systems are illustrated in figure below.







For T-shapes, the total effective compression flange width, b_e, is limited to one-quarter of the span length of the beam (L), and the effective overhanging portions of the compression flange on each side of the web are limited to

(a) eight times the thickness of the flange (slab), and

(b) one-half the clear distance to the next beam web.

The ACI Code, 8.12.2, prescribes a limit on the effective flange width, b_e , of interior T-section to the smallest of the following:

$$(a)\quad b_e \le \frac{L}{4}$$

$$(b)\quad b_e \le b_w + 16h_f$$

$$(c)\quad b_e \le b_w + \frac{1}{2}\text{ the clear distance to the next beam web from both sides}$$

For symmetrical T-section (the clear distance to the next beam web from both sides is the same) the previous (c) will be

$$(c)\quad b_e \le \text{Center to Center spacing between adjacent beams}$$

For inverted L-shapes, the following three limits are given for the effective width of the overhanging portion of the compression flange:

(a) one-twelfth of the span length of the beam,

(b) six times the thickness of the flange (slab), and

(c) one-half the clear transverse distance to the next beam web.

The ACI Code, 8.12.3, prescribes a limit on the effective flange width, b_e , of exterior T-section (L-shape) to the smallest of the following:

$$(a)\quad b_e \le b_w + \frac{L}{12}$$

$$(b)\quad b_e \le b_w + 6h_f$$

$$(c)\quad b_e \le b_w + \frac{1}{2}\text{ the clear distance to the next beam web.}$$

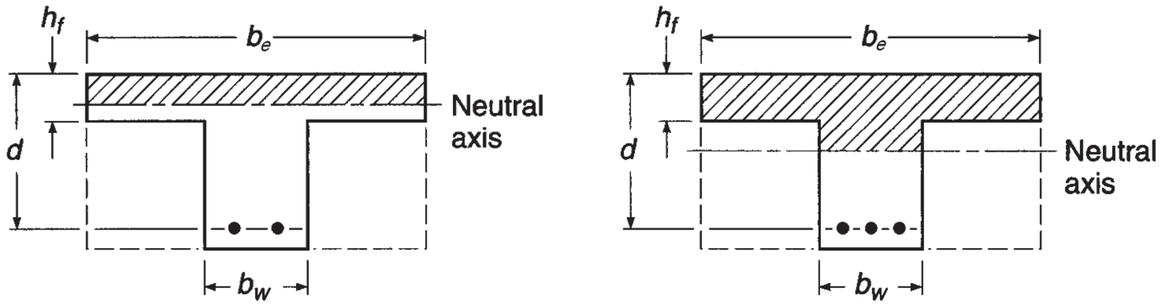
Isolated beams, in which the T-shape is used to provide a flange for additional compression area, shall have a flange thickness (ACI 8.12.4)

$$(a)\quad b_e \le 4b_w$$

$$(b)\quad t \ge \frac{1}{2}b_w$$

4.11.2 Analysis of T-sections.

The neutral axis of a T-section beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strength of the materials.



Procedure of analysis:

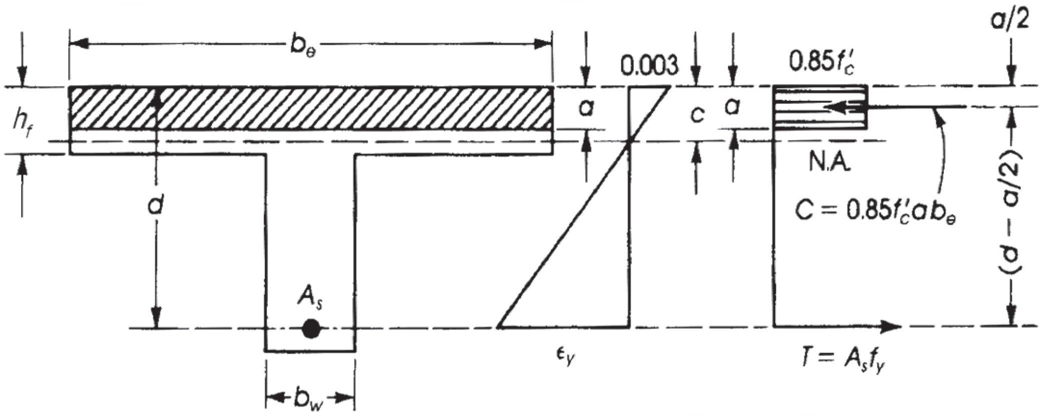
1. Assume that T-section is a rectangular section with total b_e width.

$$T = C \qquad \Rightarrow \qquad A_s f_y = 0.85\, f_c' a b_e \qquad \Rightarrow \qquad a = \frac{A_s f_y}{0.85\, f_c' b_e}$$

2. Compare a with $h_f -$ the thickness of flange.

Here may be TWO CASES:

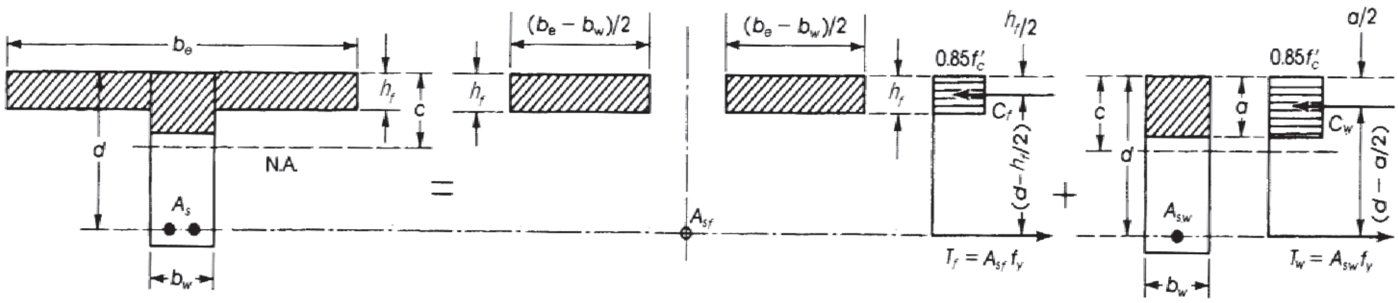
Case I: $a \leq h_f$ analyze as rectangular section.



$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85\, f_c' a b_e \left(d - \frac{a}{2}\right)$$

$$A_s = \frac{0.85\, f_c' a b_e}{f_y}$$

Case II: $a > h_f$ analyze as T-section.



1. $M_n = M_{nf} + M_{nw}$

where $M_n -$ Moment capacity of the T-section,

M_{nf} − Moment capacity of the flange,
M_{nw} − Moment capacity of the web.

2. $M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 0.85\, f_c'(b_e - b_w) h_f \left(d - \frac{h_f}{2}\right)$

$$T_f = C_f \quad \Longrightarrow \quad A_{sf} f_y = 0.85\, f_c'(b_e - b_w) h_f \quad \Longrightarrow \quad A_{sf} = \frac{0.85\, f_c'(b_e - b_w) h_f}{f_y}$$

3. $M_{nw} = A_{sw} f_y \left(d - \frac{a}{2}\right) = 0.85\, f_c' b_w a \left(d - \frac{a}{2}\right), \qquad A_{sw} = A_s - A_{sf}$

$$T_w = C_w \quad \Longrightarrow \quad A_{sw} f_y = 0.85\, f_c' b_w a \quad \Longrightarrow \quad a = \frac{A_{sw} f_y}{0.85\, f_c' b_w}$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right)$$

or

$$M_n = 0.85\, f_c'(b_e - b_w) h_f \left(d - \frac{h_f}{2}\right) + 0.85\, f_c' b_w a \left(d - \frac{a}{2}\right)$$

4. Check for strain $\varepsilon_s \geq 0.005$.

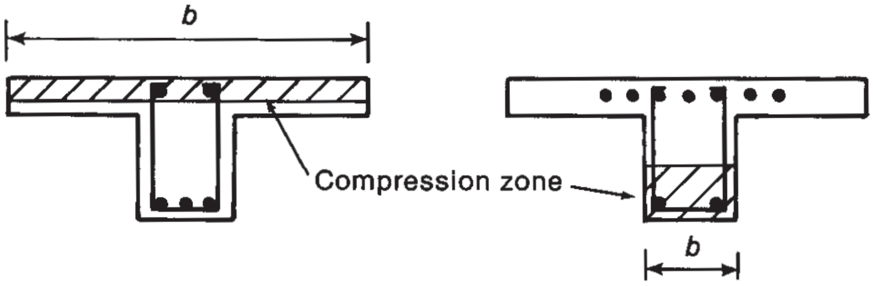
4.11.3 Minimum reinforcement of flexural T-section members.

$A_{s,min}$ for T-sections is as in 4.6 (page 23).
For statically determinate members with a flange in tension, ACI Code, 10.5.2., as in the case of cantilever beams, $A_{s,min}$ shall not be less than the value given by equations in section 4.6 (see page 23), except that b_w is replaced by either $2b_w$ or the width of the flange, whichever is smaller.

$$A_{s,min} = \frac{0.5\sqrt{f_c'}}{f_y}\, b_w d, \qquad A_{s,min} = \frac{0.25\sqrt{f_c'}}{f_y}\, bd.$$

According to ACI code, 10.6.6, where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in 8.12, or a width equal to one-tenth the span, whichever is smaller. If the effective flange width exceeds one-tenth the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

4.11.4 Analysis of the positive-moment capacity of a T-section.



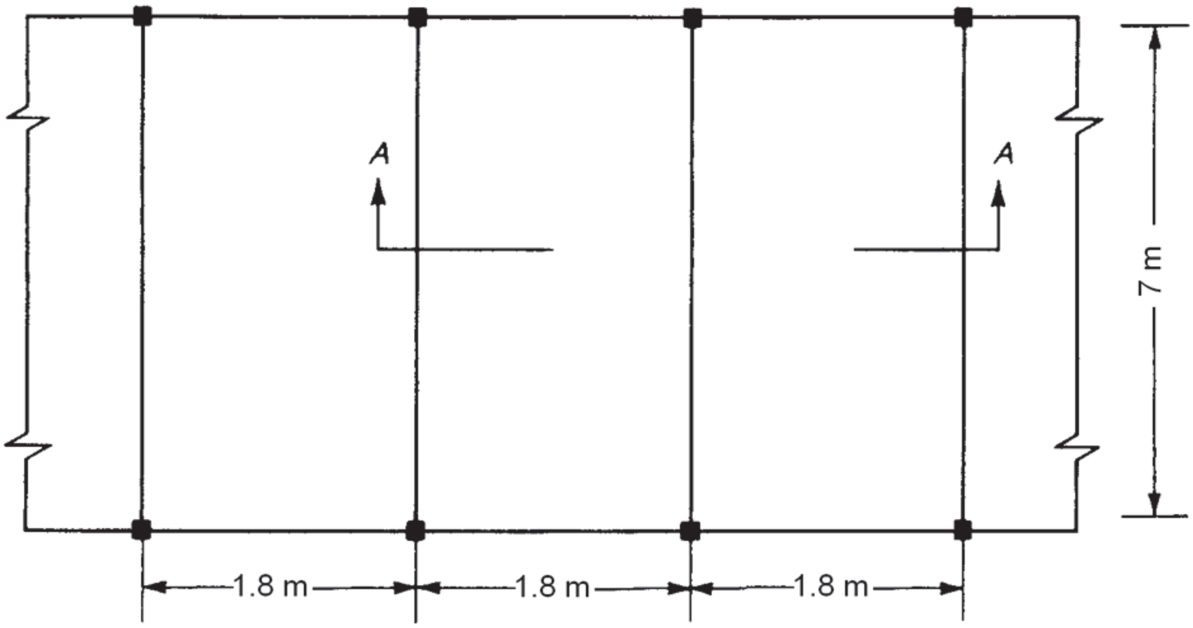
Midspan section
Positive moment
(Compression in flange)

Support section
Negative moment
(Compression in web)

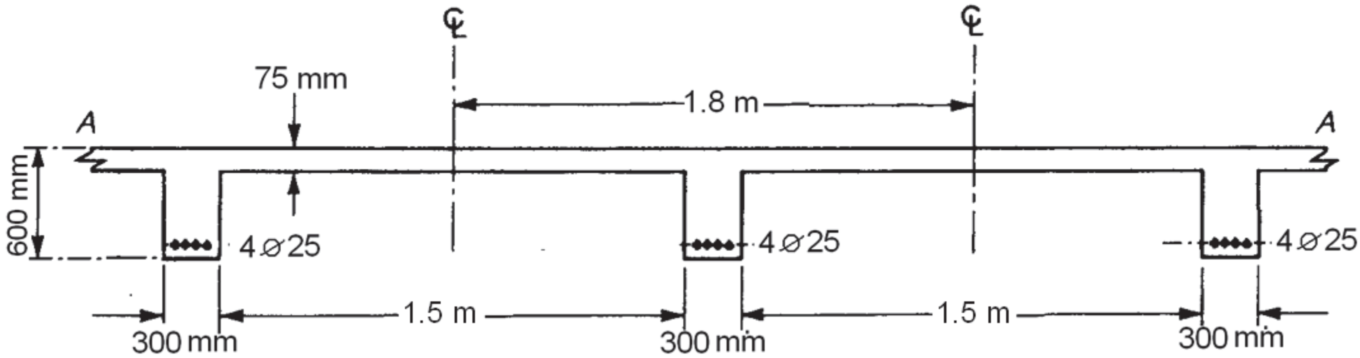
Example:

Calculate the design strength ϕM_n for one of the T beams in the positive moment region. The beam has a clear span of 7 m (face to face).

$f_c' = 28\ MPa, \qquad f_y = 420\ MPa.$



(a)



(b)

<u>Solution:</u>

From the Geometry of T-section:

$b_w = 300\ mm,$ $\qquad h = 600\ mm,$ $\qquad t = h_f = 75\ mm$

$A_s(4\varnothing\, 25) = 1963.5\ mm^2$

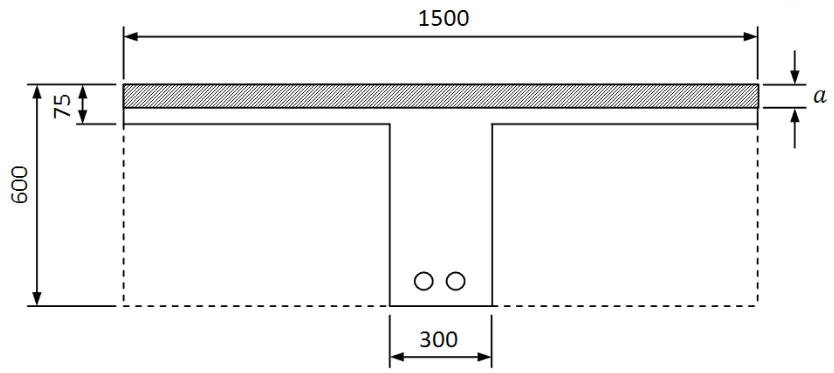
b_e is the smallest of:

$$(a)\ \ b_e \leq \frac{L}{4} = \frac{7000}{4} = 1750\ mm,$$

$$(b)\ \ b_e \leq b_w + 16h_f = 300 + 16 \cdot 75 = 1500\ mm, \qquad -\ control$$

(c) $b_e \leq$ Center to Center spacing between adjacent beams $= 1800\ mm.$

Take $b_e = 1500\ mm$.



$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{1963.5\ \cdot 420}{0.85\ \cdot 28 \cdot 1500} = 23.1\ mm\ \ < h_f = 75\ mm$$

The beam section will be considered as rectangular with $b = b_e = 1500\ mm$.

$$d = 600 - 40 - 10 - \frac{25}{2} = 537.5\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1963.5 \cdot 420 \left(537.5 - \frac{23.1}{2}\right) \times 10^{-6} = 433.74\ KN \cdot m$$

Check for strain $\varepsilon_s \geq 0.005$

$$c = \frac{a}{\beta_1} = \frac{23.1}{0.85} = 27.18\ mm, \qquad\qquad \beta_1 = 0.85$$

$$\varepsilon_s = 0.003 \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{537.5 - 27.18}{27.18}\right) = 0.0565 > 0.005 \qquad OK$$

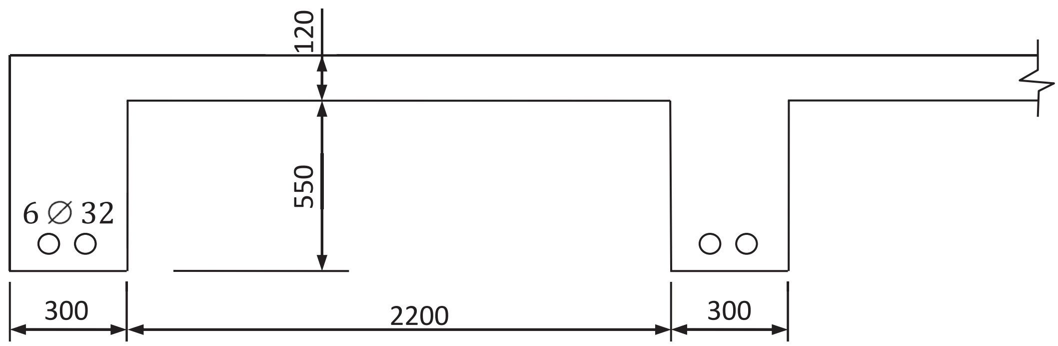
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 433.74 = 390.37\ KN \cdot m$$

Example:

Determine the positive moment capacity of the edge L-section beam. The beam has a clear span of 6 m (face to face).

$f_c' = 20\ MPa, \qquad f_y = 400\ MPa.$



<u>Solution:</u>

From the Geometry of T-section:

$b_w = 300\ mm,$ $h = 670\ mm,$ $t = h_f = 120\ mm$

$A_s(6\varnothing\ 32) = 4825.5\ mm^2$

b_e is the smallest of:

$$(a)\quad b_e \le b_w + \frac{L}{12} = 300 + \frac{6000}{12} = 800\ mm, \qquad -control$$
$$(b)\quad b_e \le b_w + 6h_f = 300 + 6\cdot 120 = 1020\ mm,$$
$$(c)\quad b_e \le b_w + \frac{1}{2}\text{ the clear distance to the next beam web} = 300 + \frac{2200}{2} = 1400\ mm.$$

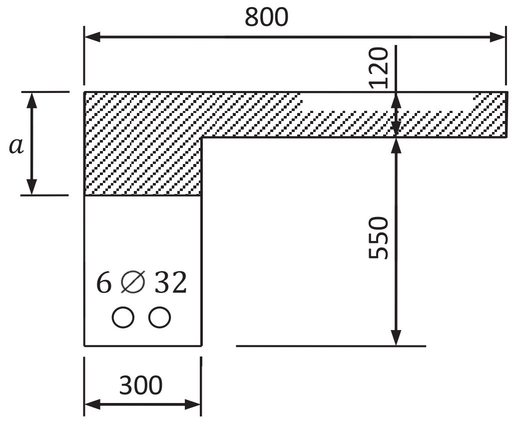
Take $b_e = 800\ mm.$

Check if $a > h_f$

$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{4825.5\ \cdot 400}{0.85\ \cdot 20\cdot 800} = 141.93\ mm\ > h_f = 120\ mm$$

The beam section will be considered as L-section with

$b_e = 800\ mm.$



$$A_{sf} = \frac{0.85\ f_c'(b_e - b_w)h_f}{f_y} =$$
$$= \frac{0.85\ \cdot 20(800-300)120}{400} = 2550\ mm^2$$
$$A_{sw} = A_s - A_{sf} = 4825.5 - 2550 = 2275.5\ mm^2$$
$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{2275.5\cdot 400}{0.85\ \cdot 20\cdot 300} = 178.47\ mm$$

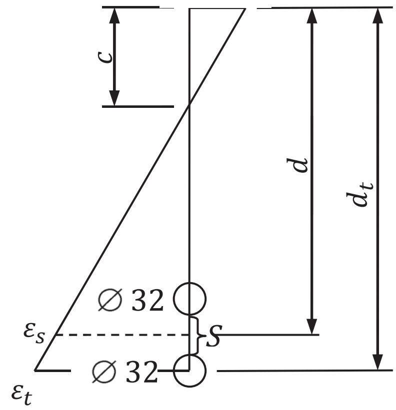
$A_s\ (6\varnothing\ 32)$ are arranged in two layers

$$d = 670 - 40 - 10 - 32 - \frac{25}{2} = 575.5\ mm$$

$$M_n = A_{sf} f_y\left(d - \frac{h_f}{2}\right) + A_{sw} f_y\left(d - \frac{a}{2}\right) =$$
$$= \left[2550\cdot 400\left(575.5 - \frac{120}{2}\right) + 2275.5\cdot 400\left(575.5 - \frac{178.47}{2}\right)\right]\times 10^{-6} = 968.4\ KN\cdot m$$

Check for strain $\varepsilon_s \ge 0.005$

$$c = \frac{a}{\beta_1} = \frac{178.47}{0.85} = 209.96\ mm, \qquad \beta_1 = 0.85$$
$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 575.5 + \frac{25}{2} + \frac{32}{2} = 604\ mm$$
$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{604 - 209.96}{209.96}\right) =$$
$$= 0.00563 > 0.005 \qquad OK$$



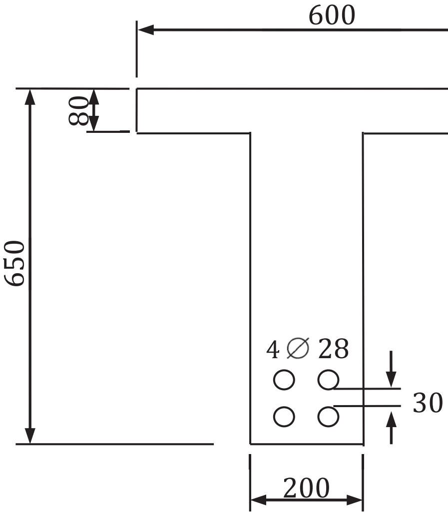
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 961.65 = 865.49\ KN \cdot m$$

Example:

Compute the positive design moment capacity of the T-section beam.

$f_c' = 20\ MPa, \qquad f_y = 420\ MPa.$



<u>Solution:</u>

From the Geometry of T-section:

$b_w = 200\ mm, \qquad h = 650\ mm, \qquad t = h_f = 80\ mm$

$A_s(4\varnothing\ 28) = 2463\ mm^2$

Check if $a > h_f$

$$a = \frac{A_s f_y}{0.85\ f_c' b_e} = \frac{2463\ \cdot 420}{0.85\ \cdot 20 \cdot 600} = 101.42\ mm$$

$a = 101.42\ mm > h_f = 80\ mm$. The beam section will be considered as T-section.

$$A_{sf} = \frac{0.85\ f_c'(b_e - b_w)h_f}{f_y} = \frac{0.85\ \cdot 20(600 - 200)80}{420} = 1295.2\ mm^2$$

$$A_{sw} = A_s - A_{sf} = 2463 - 1295.2 = 1167.76\ mm^2$$

$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{1167.76\ \cdot 420}{0.85\ \cdot 20 \cdot 200} = 144.25\ mm$$

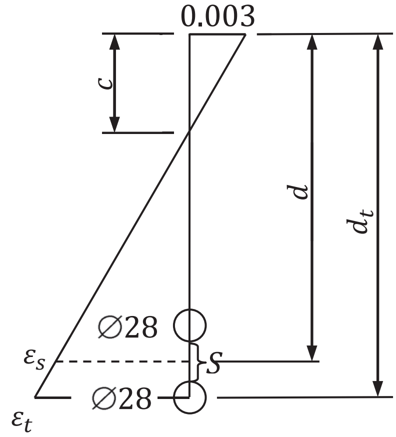
$A_s\ (4\varnothing\ 28)$ are arranged in two layers

$$d = 650 - 40 - 10 - 28 - \frac{30}{2} = 557\ mm$$

$$M_n = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) =$$

$$= \left[1295.2\ \cdot 420\left(557 - \frac{80}{2}\right) + 1167.76 \cdot 420\left(557 - \frac{144.25}{2}\right)\right] \times 10^{-6} = 519.05\ KN \cdot m$$

Check for strain $\varepsilon_s \geq 0.005$



$$c = \frac{a}{\beta_1} = \frac{144.25}{0.85} = 169.7\ mm, \qquad \beta_1 = 0.85$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 557 + \frac{30}{2} + \frac{28}{2} = 586\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{586 - 169.7}{169.7}\right) =$$

$$= 0.00736 > 0.005 \qquad OK$$

Take $\phi = 0.9$ for flexure as tension-controlled section.

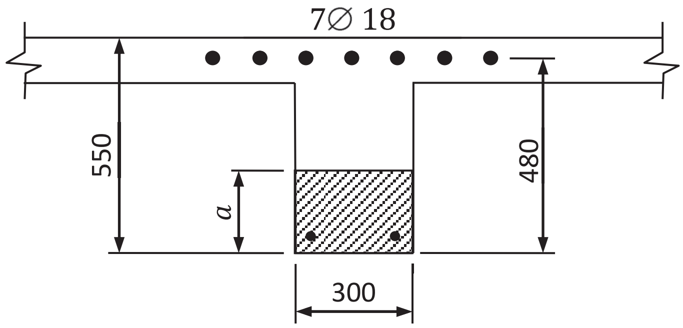
$$M_u = \phi M_n = 0.9 \cdot 519.05 = 467.15\ KN \cdot m$$

4.11.5 Analysis of the negative-moment capacity of a T-section.

Example:

Compute the negative design moment capacity of the T-section beam.

$f_c' = 20\ MPa, \qquad f_y = 400\ MPa.$



Solution:

Analyze as rectangular section because that the compression zone is within the web depth.

$A_s(7\varnothing\ 18) = 1781.3\ mm^2$

$$a = \frac{A_s f_y}{0.85\ f_c' b_w} = \frac{1781.3\ \cdot 400}{0.85\ \cdot 20 \cdot 300} = 139.71\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1781.3 \cdot 400\ \left(480 - \frac{139.71}{2}\right) \times 10^{-6} = 292.23\ KN \cdot m$$

Check for strain $\varepsilon_s \geq 0.005$

$$c = \frac{a}{\beta_1} = \frac{139.71}{0.85} = 164.36\ mm, \qquad \beta_1 = 0.85$$

$$\varepsilon_s = 0.003 \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{480 - 164.36}{164.36}\right) = 0.00576 > 0.005 \qquad OK$$

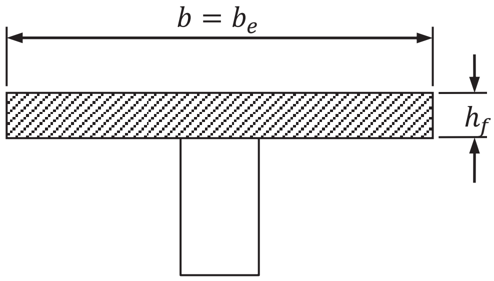
Take $\phi = 0.9$ for flexure as tension-controlled section.

$$M_u = \phi M_n = 0.9 \cdot 292.23 = 263.01\ KN \cdot m$$

4.11.6 Design of T-section.

The design of a T-section beam involves the choice of the cross section and the reinforcement required. The flange thickness and width are usually established during the design of the floor slab. The size of the beam stem is influenced by the same factores that affect the size of a rectangular beam. In the case of a continuous T-beam, the concrete compressive stresses are most critical in the negative-moment regions, where the compression zone is in the beam stem (web).

Design Procedure:



1. Check if the depth of the compression block within the thickness of the flange.

Let $a = h_f$, then compute $\bar{M}_{nf} -$ the total moment capacity of the flange.

$$\bar{M}_{nf} = 0.85 f_c' b h_f \left(d - \frac{h_f}{2} \right)$$

Case I:

$$a \le h_f \qquad or \qquad \bar{M}_{nf} \ge \frac{M_u}{\phi},$$

Design as rectangular section.

Case II:

$$a > h_f \qquad or \qquad \bar{M}_{nf} < \frac{M_u}{\phi},$$

Design as T-section. **GO to step 2.**

2. $M_n = M_{nf} + M_{nw}$, $\qquad A_s = A_{sf} + A_{sw}$,

$T_f = C_f \quad \Longrightarrow \quad A_{sf} f_y = 0.85 f_c' (b - b_w) h_f, \quad from\ where$

$$A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y}, \qquad and$$

$$M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2} \right) = 0.85 f_c' (b - b_w) h_f \left(d - \frac{h_f}{2} \right)$$

3. Design the web as rectangular section with $b = b_w$, where

$$M_{nw} = M_n - M_{nf} = \frac{M_u}{\phi} - M_{nf}$$

$$A_{sw} = \rho_w b_w d, \qquad \rho_w = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2 R_{nw} m}{f_y}} \right), \qquad R_{nw} = \frac{M_{nw}}{b_w d^2}$$

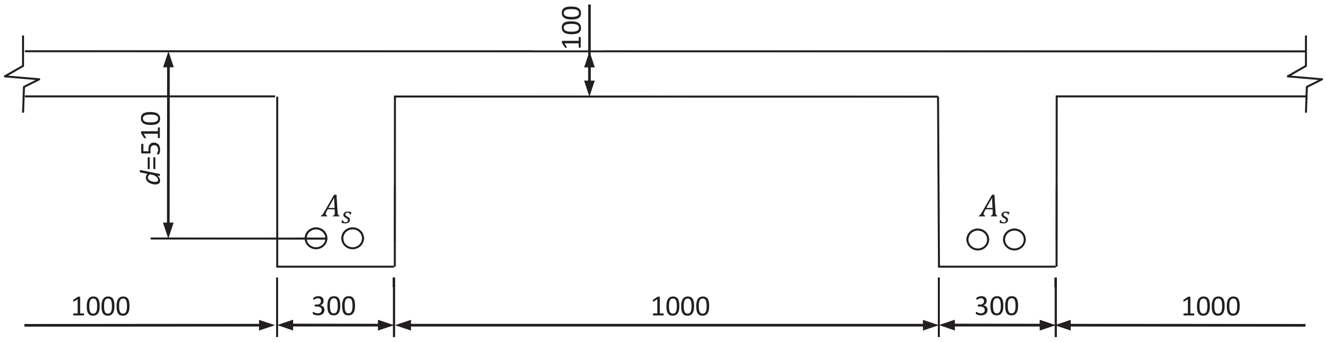
The index XX_f and XX_w in the previous notation refers to $f - flange$, $\quad w - web$.

Example:

Compute the area of steel reinforcement for the interior beam shown below. The beam has a clear span of 6 m (face to face).

Ultimate factored moment $M_u = 720\ KN \cdot m$

$f_c' = 20\ MPa, \qquad f_y = 400\ MPa.$



Solution:

From the Geometry of T-section:

$b_w = 300\ mm, \quad d = 510\ mm, \quad t = h_f = 100\ mm$

b_e is the smallest of:

$$(a)\quad b_e \le \frac{L}{4} = \frac{6000}{4} = 1500\ mm,$$

$$(b)\quad b_e \le b_w + 16h_f = 300 + 16 \cdot 100 = 1900\ mm,$$

$(c)\quad b_e \le$ Center to Center spacing between adjacent beams

$$b_e = 1300\ mm. -control$$

Take $b = b_e = 1300\ mm.$

$$\overline{M}_{nf} = 0.85 f_c' b h_f \left(d - \frac{h_f}{2}\right) = 0.85 \cdot 20 \cdot 1300 \cdot 100 \left(510 - \frac{100}{2}\right) \times 10^{-6} = 1016.6\ KN \cdot m$$

$$\overline{M}_{nf} = 1016.6\ KN \cdot m > \frac{M_u}{\phi} = \frac{720}{0.9} = 800\ KN \cdot m \quad \Longrightarrow \quad a < h_f$$

The section will be designed as rectangular section with $b = 1300\ mm.$

$$R_n = \frac{M_u}{\phi b d^2} = \frac{720}{0.9 \cdot 1300 \cdot 510^2} = 2.366\ \ MPa, \qquad m = \frac{f_y}{0.85 f_c'} = \frac{400}{0.85 \cdot 20} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2R_n m}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \cdot 2.366 \cdot 23.53}{400}}\right) = 0.0064,$$

$A_s = \rho b d = 0.0064 \cdot 1300 \cdot 510 = 4243.2\ mm^2$

Check for $A_{s,min}$

$$A_{s,min} = 0.25 \frac{\sqrt{f_c'}}{f_y} b_w d \ge \frac{1.4}{f_y} b_w d$$

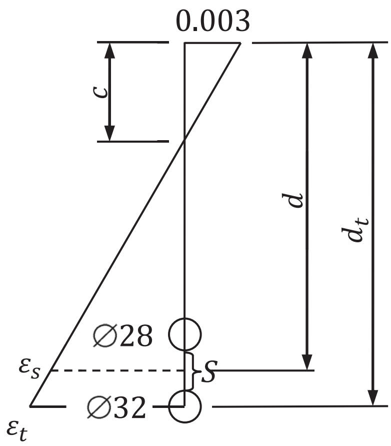
$$A_{s,min} = 0.25 \frac{\sqrt{20}}{400} 300 \cdot 510 = 428\ mm^2$$

$$A_{s,min} = \frac{1.4}{400} 300 \cdot 510 = 534\ mm^2 \quad -control$$

$A_s = 4243.2\ mm^2 > A_{s,min} = 534\ mm^2 \ \ - OK$

Use 3 ∅ 32 + 3 ∅ 28 in two layers with

$A_s = 24.127 + 18.473 = 42.6\ cm^2 > \ A_{s,req} = 42.43\ cm^2\ - OK$



Check for strain:

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{4260\ \cdot 400}{0.85 \cdot 20 \cdot 1300} = 77.1\ mm$$

$$c = \frac{a}{\beta_1}, \qquad\qquad \beta_1 = 0.85$$

$$c = \frac{77.1}{0.85} = 90.71\ mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 510 + \frac{25}{2} + \frac{32}{2} = 538.5\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{538.5 - 90.71}{90.71}\right) = 0.0148 > 0.005 \qquad OK$$

Check for bar placement in one layer:

$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 3 \times 32}{2} = 52\ mm > d_b = 32\ mm, \quad > 25\ mm \qquad OK$$

Example:

Repeat the previous example using $M_u = 930\ KN \cdot m$.

<u>**Solution:**</u>

$$\bar{M}_{nf} = 1016.6\ KN \cdot m < \frac{M_u}{\phi} = \frac{930}{0.9} = 1033.3\ KN \cdot m \quad \Rightarrow \quad a > h_f$$

The section will be designed as T-section section.

$$A_{sf} f_y = \ 0.85 f_c'(b - b_w) h_f, \quad from\ where \qquad A_{sf} = \frac{0.85 f_c'(b - b_w) h_f}{f_y},$$

$$A_{sf} = \frac{0.85 f_c'(b - b_w) h_f}{f_y} = \frac{0.85 \cdot 20(1300 - 300) \cdot 100}{400} = 4250\ mm^2$$

$$M_{nf} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) = 4250 \cdot 400\left(510 - \frac{100}{2}\right) \times 10^{-6} = 782\ KN \cdot m$$

$$M_{nw} = M_n - M_{nf} = \frac{M_u}{\phi} - M_{nf} = \frac{930}{0.9} - 782 = 251.3\ KN \cdot m$$

Here the web will be designed as rectangular section with $b = b_w = 300\ mm$ to resist $M_{nw} = 251.3\ KN \cdot m$

$$R_{nw} = \frac{M_{nw}}{b_w d^2} = \frac{251.3 \cdot 10^6}{300 \cdot 510^2} = 3.22\ MPa,$$

$$m = \frac{f_y}{0.85 f_c{}'} = \frac{400}{0.85 \cdot 20} = 23.53$$

$$\rho_w = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2R_{nw}m}{f_y}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \cdot 3.22 \cdot 23.53}{400}}\right) = 0.009,$$

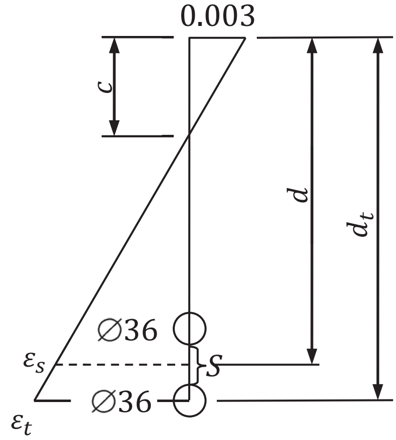
$$A_{sw} = \rho_w b_w d = 0.009 \cdot 300 \cdot 510 = 1377\ mm^2,$$

$$A_s = A_{sf} + A_{sw} = 4250 + 1377 = 5627\ mm^2$$

$$A_s = 5627\ \ mm^2 > A_{s,min} = 534\ mm^2\ \ -OK$$

Use 6 ∅ 36 in two layers with $A_s = 61.07\ cm^2 >\ A_{s,req} = 56.27\ cm^2$ – OK

$$A_{sw,provided} = A_{s,provided} - A_{sf} = 6107 - 4250 = 1857\ mm^2$$



Check for strain:

$$a = \frac{A_{sw} f_y}{0.85\ f_c' b_w} = \frac{1857\ \cdot 400}{0.85 \cdot 20 \cdot 300} = 145.65\ mm$$

$$c = \frac{a}{\beta_1}, \qquad \beta_1 = 0.85$$

$$c = \frac{145.65}{0.85} = 171.35\ mm$$

$$d_t = d + \frac{S}{2} + \frac{d_b}{2} = 510 + \frac{25}{2} + \frac{36}{2} = 540.5\ mm$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) = 0.003\left(\frac{540.5 - 171.35}{171.35}\right) = 0.00646 > 0.005 \qquad OK$$

Check for bar placement in one layer:

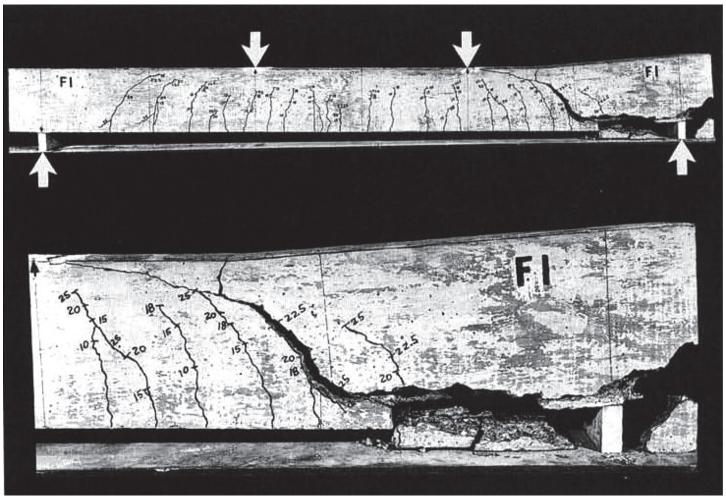
$$S_b = \frac{300 - 40 \times 2 - 10 \times 2 - 3 \times 36}{2} = 46\ mm > d_b = 36\ mm, \quad > 25\ mm \qquad OK$$

CHAPTER 5 SHEAR IN BEAMS

1.1. INTRODUCTION

When a simple beam is loaded, bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters.

The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.



Shear failure of reinforced concrete beam

By the traditional theory of homogeneous, elastic, uncracked beams, we can calculate shear stresses, v, using equation

$$v = \frac{VQ}{Ib}$$

where $V -$ total shear at the section considered,

$Q -$ statical moment about the neutral axis of that portion of cross-section lying between a line through the point in question parallel to the neutral axis and nearest face, upper or lower, of the beam,

$I -$ moment of inertia of cross-section about the neutral axis,

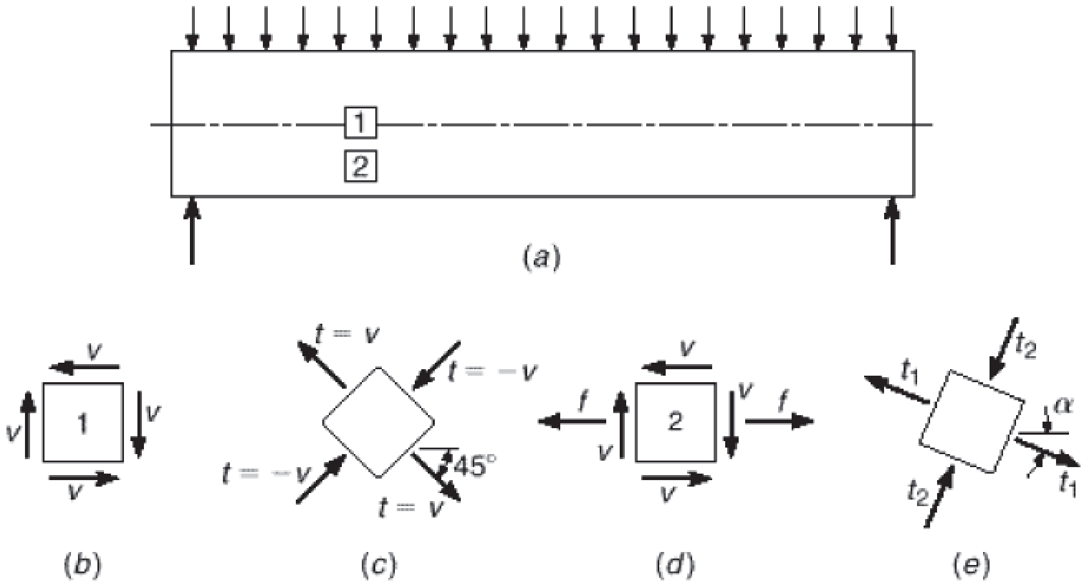
$b -$ width of beam at the given point.

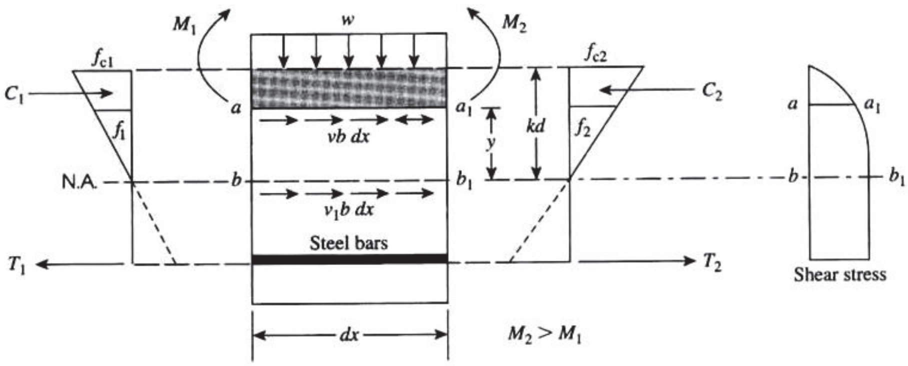
The tensile stresses are equivalent to the principal stresses. Such principal stresses are traditionally called **diagonal tension stresses**. When the diagonal tension stresses reach the tensile strength of concrete, a diagonal crack develops. This brief analysis explains the concept of diagonal tension and diagonal cracking. The actual behavior is more complex, and it is affected by other factors. For the combined action of shear and normal stresses at any point in a beam, the maximum and minimum diagonal tension (principal stresses) f_p are given by the equation

$$f_p = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + v^2}$$

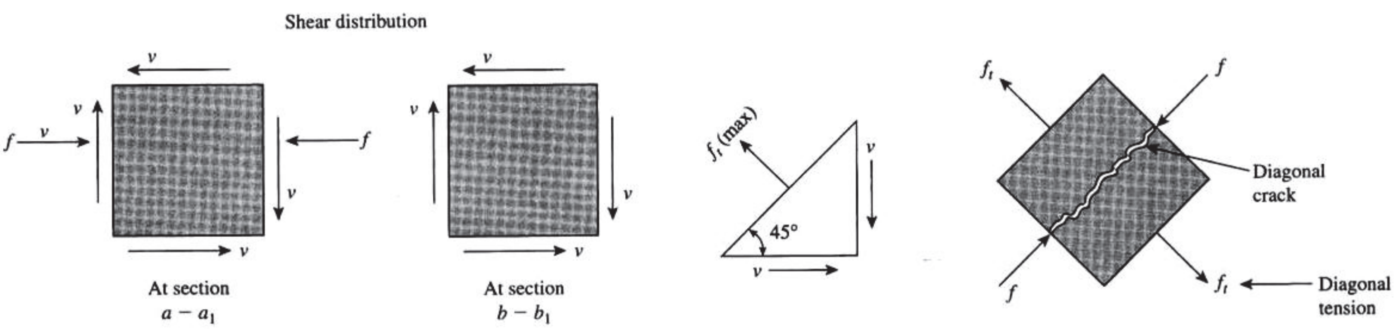
f – intensity of normal stress due to bending,

v – shear stress.

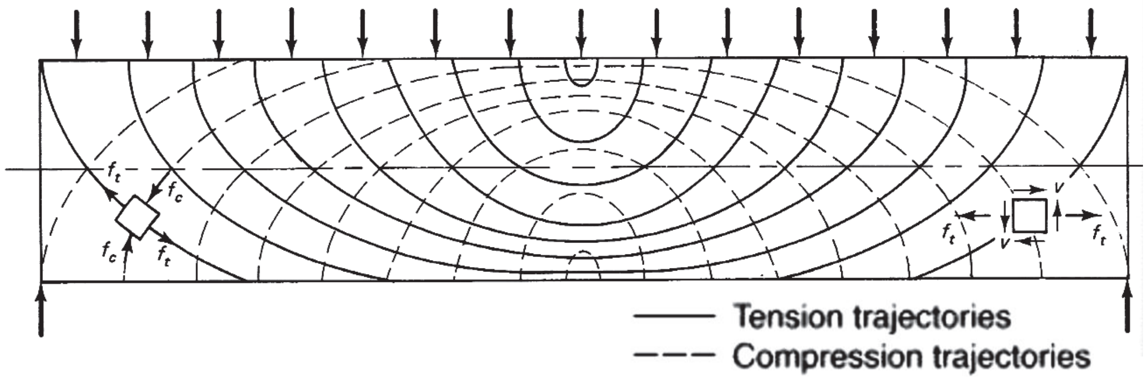




(a) Forces and stresses along the depth of the section,

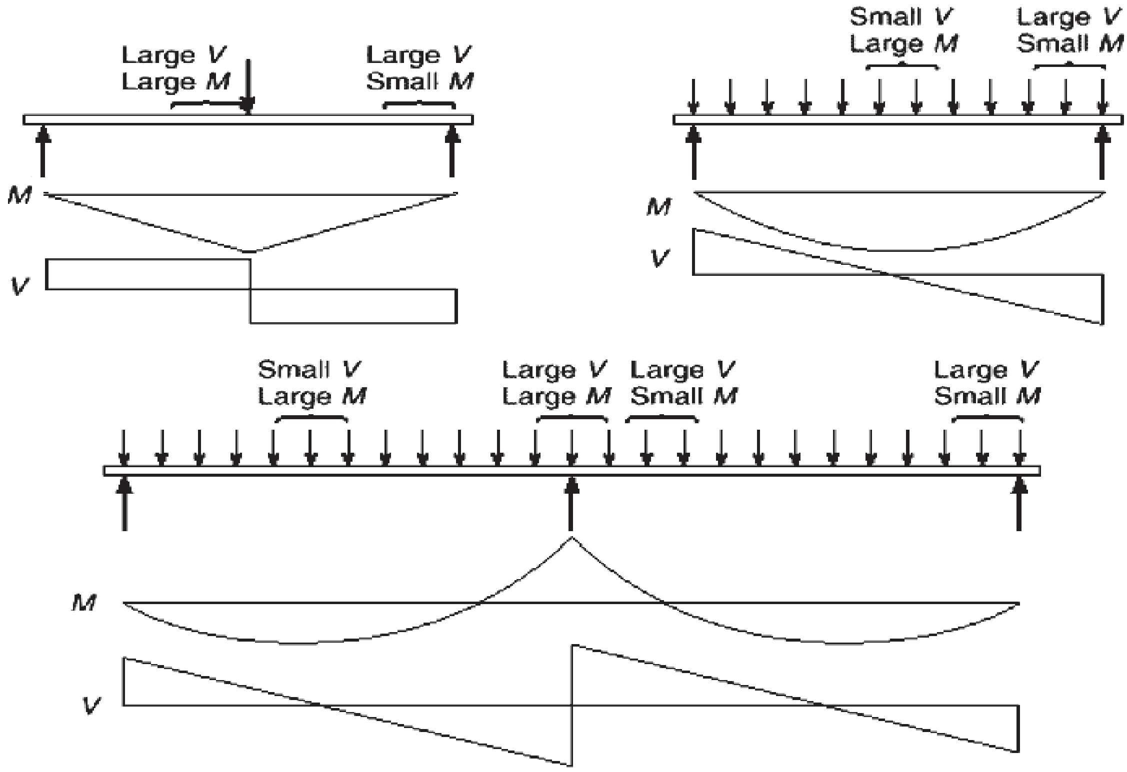


(b) normal and shear stresses, (c) pure shear, (d) diagonal tension.



Trajectories of principal stresses in a homogeneous isotropic beam.

1.2. CRITICAL SECTIONS FOR SHEAR DESIGN



Typical Locations of critical combinations of shears and moment

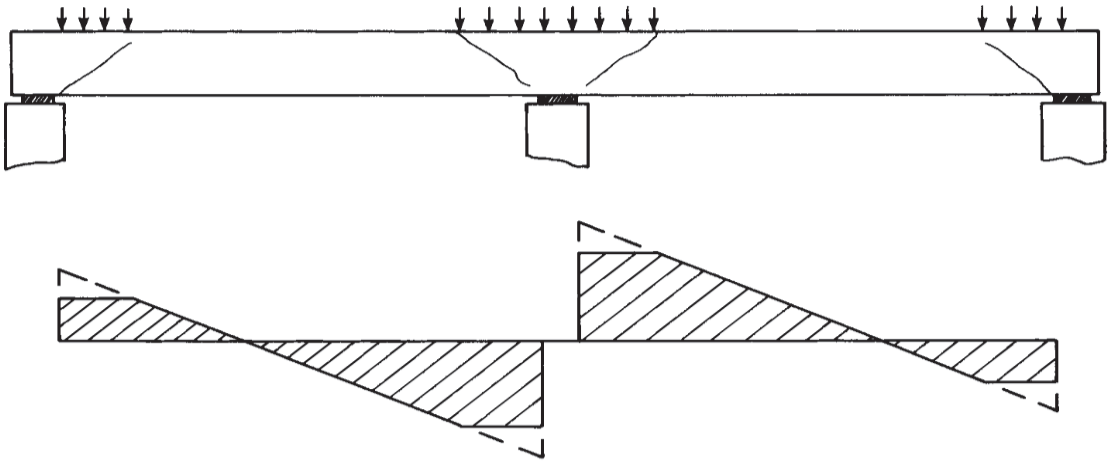
In a beam loaded on the top flange and supported on the bottom as shown in the figure below, the closest inclined cracks that can occur adjacent to the supports will extend outward from the supports at roughly 45°. Loads applied to the beam within a distance d from the support in such a beam will be transmitted directly to the support by the compression fan above the 45° cracks and will not affect the stresses in the stirrups crossing the cracks shown. As a result, ACI Code Section 11.1.3.1 states:

For nonprestressed members, sections located less than a distance d from the face of the support may be designed for the same shear, V_u, as that computed at a distance d.

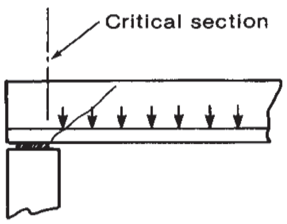
This is permitted only when

1. the support reaction, in the direction of the applied shear, introduces compression into the end regions of a member,
2. the loads are applied at or near the top of the beam, and
3. no concentrated load occurs within d from the face of the support.

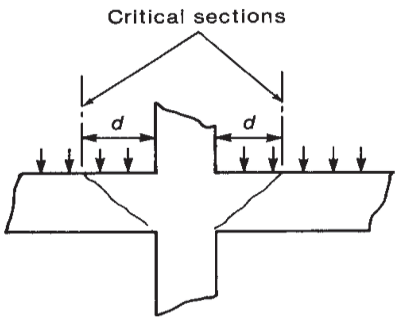
Thus, for the beam shown below, the values of V_u used in design are shown shaded in the shear force diagram.



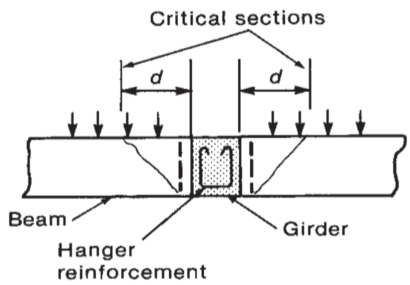
This allowance must be applied carefully because it is not applicable in all cases. There are shows five other typical cases that arise in design. If the beam was loaded on the lower flange, as indicated in Fig. a, the critical section for design would be at the face of the support, because loads applied within d of the support must be transferred across the inclined crack before they reach the support.



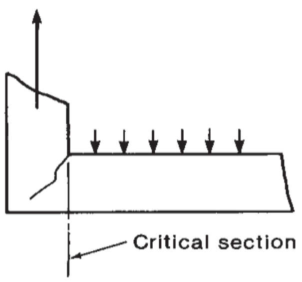
(a) Beam loaded on tension flange.



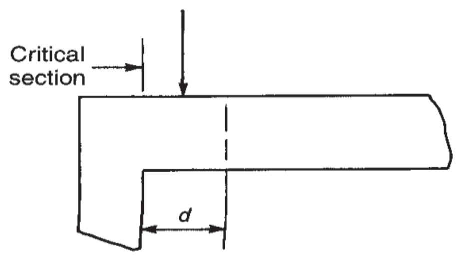
(b) Beam column joint.



(c) Beam supported by shear.



(d) Beam supported by tension force.



(e) Beam with concentrated load close to support.

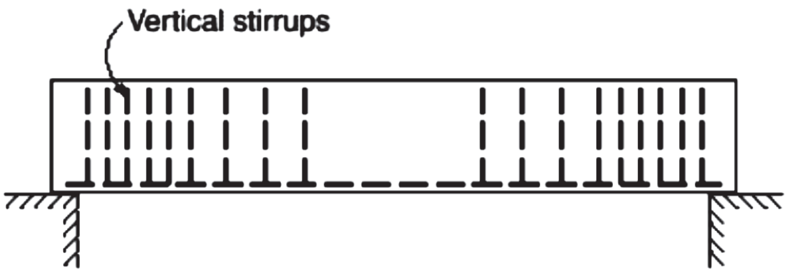
A typical beam-to-column joint is shown in Fig. b. Here the critical section for design is d away from the section as shown.

If the beam is supported by a girder of essentially the same depth, as shown in Fig. c, the compression fans that form in the supported beams will tend to push the bottom off the supporting beam. The critical shear design sections in the supported beams normally are taken at the face of the supporting beam. The critical section may be taken at d from the end of the beam if hanger reinforcement is provided to support the reactions from the compression fans.

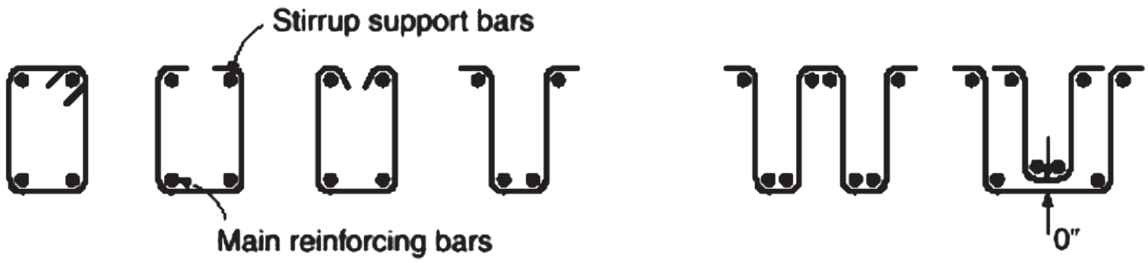
Generally, if the beam is supported by a tensile force rather than a compressive force, the critical section will be at the face of the support, and the joint must be carefully detailed, because shear cracks will extend into the joint, as shown in Fig. d.

Occasionally, a significant part of the shear at the end of the beam will be caused by a concentrated load acting less than d from the face of the column, as shown in Fig. e. In such a case, the critical section must be taken at the support face.

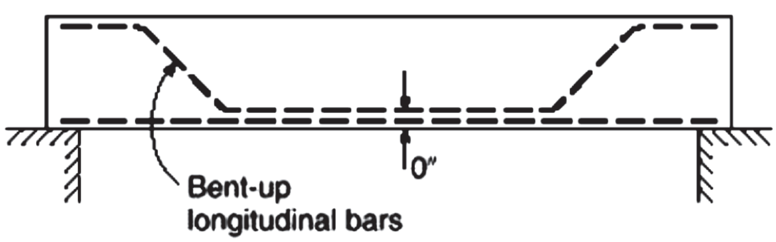
1.3. TYPES OF WEB REINFORCEMENT



(*a*)



(*b*) (*c*)



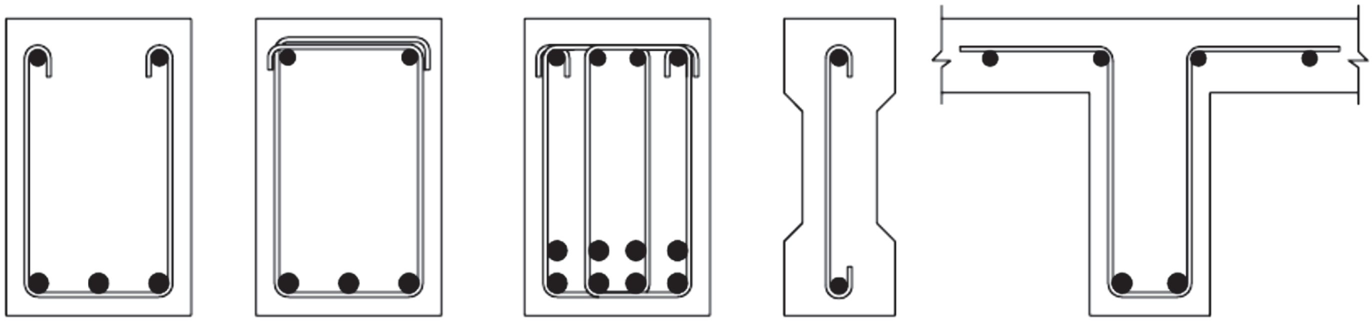
(*d*)

a) Vertical ***Stirrups,***

b) U-shaped bars single stirrups.

c) Multiple-leg stirrups

d) Bent-up longitudinal (inclined) bars



The ACI Code defines the types of shear reinforcement as:

11.4.1.1 — Shear reinforcement consisting of the following shall be permitted:

(a) Stirrups perpendicular to axis of member;

(b) Welded wire reinforcement with wires located perpendicular to axis of member;

(c) Spirals, circular ties, or hoops.

11.4.1.2 — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

(a) Stirrups making an angle of 45 degrees or more with longitudinal tension reinforcement;

(b) Longitudinal reinforcement with bent portion making an angle of 30 degrees or more with the longitudinal tension reinforcement;

(c) Combinations of stirrups and bent longitudinal reinforcement.

1.4. DESIGN PROCEDURE FOR SHEAR

Design of cross section subjected to shear shall be based on:

$$\phi V_n \geq V_u$$

where V_u − the factored shear force at the section,

V_n − the nominal shear strenght,

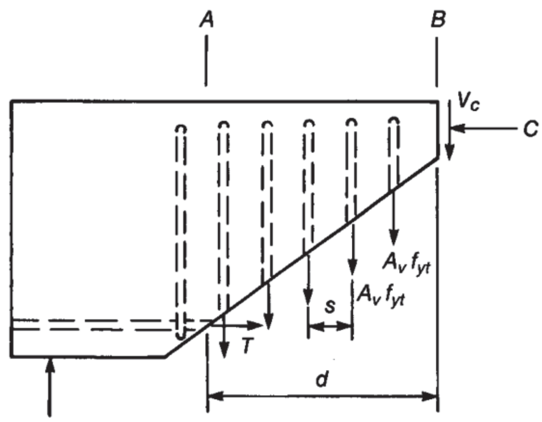
$$V_n = V_c + V_s,$$

V_c − the nominal shear strenght provided by concrete,

V_s − the nominal shear strenght provided by shear reinforcement (stirrups),

The figure shows a free body between the end of a beam and an inclined crack. The horizontal projection of the crack is taken as d, suggesting that the crack is slightly flatter than 45°. If s is the stirrup spacing, the number of stirrups cut by the crack is d/s. Assuming that all the stirrups yield at failure, the shear resisted by the stirrups is

$$V_s = \frac{A_v f_{yt} d}{s}$$



ACI Code 11.2.1 states, for members subject to shear and flexure only

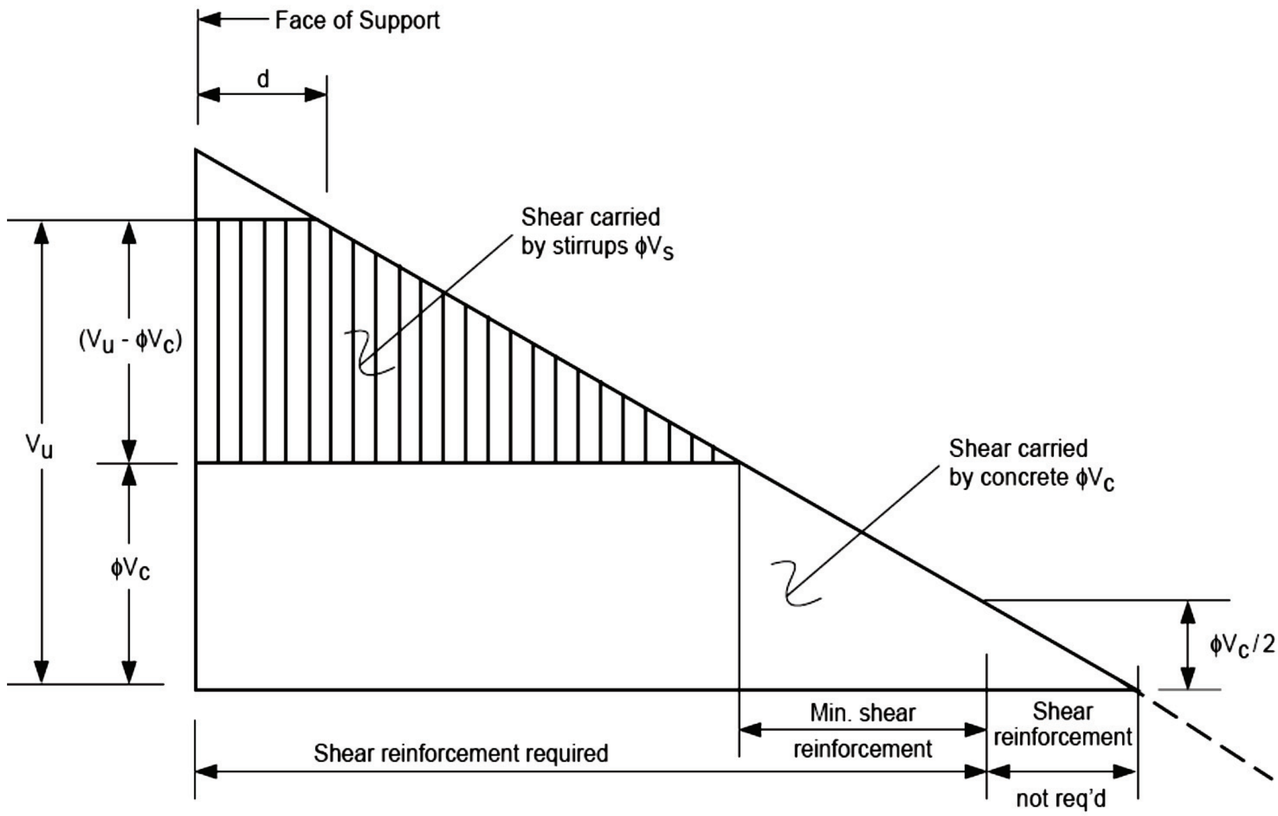
$$V_c = \frac{1}{6}\lambda\sqrt{f_c'}b_w d = 0.17\lambda\sqrt{f_c'}b_w d, \qquad \lambda = 1.0 \ for\ Normal - weight\ concrete$$

V_c shall be permitted to be computed by the more detailed calculation

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d \le 0.29\sqrt{f_c'}b_w d, \qquad where \qquad \frac{V_u d}{M_u} \le 1$$

To simplify the calculations the formula $V_c = 0.17\lambda\sqrt{f_c'}b_w d$ will be used.

Shear conditions and cases (Items):



Check for dimensions:

The ACI Code, 11.4.7.9, states that V_s shall not be taken greater than $0.66\sqrt{f_c'}b_w d$.

$$So, \qquad if \;\; V_s > V_{s,max} - The\ section\ must\ be\ enlarged\ (Dimenstions\ are\ not\ enough)$$

$$where \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c, \qquad V_{s,max} = \frac{2}{3}\sqrt{f_c'}\,b_w d$$

__Case I:__

$$V_u \le \frac{1}{2}\phi V_c \qquad - No\ shear\ reinforcement\ is\ required$$

__Case II:__

$$\frac{1}{2}\phi V_c < V_u \le \phi V_c \quad - Minimum\ shear\ reinforcement\ is\ required\ \left(A_{v,min}\right)\ except:$$

- footings and solid slabs,
- Hollow-core units with total untopped depth not greater than 315 mm and hollow-core units where V_u is not greater than $0.5\phi V_{cw}$;
- Concrete joist construction;
- Beams with h not greater than 250 mm;
- Beam integral with slabs with h not greater than 600 mm and not greater than the larger of 2.5 times thickness of flange, and 0.5 times width of web;
- Beams constructed of steel fiber-reinforced, normalweight concrete with f_c' not exceeding 40 MPa, h not greater than 600 mm, and V_u not greater than $0.17\sqrt{f_c'}\,b_w d$.

For these cases no shear reinforcement is required unless $V_u > \phi V_c$.

Minimum shear reinforcement, $A_{v,min}$

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} = 0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}} \;\ge\; \frac{1}{3}\frac{b_w s}{f_{yt}} = 0.35\frac{b_w s}{f_{yt}},$$

$$or\ in\ the\ form \qquad \left(\frac{A_{v,min}}{s}\right) \ge \frac{1}{3}\frac{b_w}{f_{yt}} \ge \frac{1}{16}\sqrt{f_c'}\frac{b_w}{f_{yt}},$$

$$Here \qquad s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$$

where $\quad s -$ step of stirrups (spacing between stirrups),

$\qquad f_{yt} -$ yield stress of stirrups

__Case III:__

$$\phi V_c < V_u \le \phi\left(V_c + V_{s,min}\right)$$

$$\frac{A_{v,min}}{s} = \frac{V_{s,min}}{f_{yt} d} \Longrightarrow \qquad V_{s,min} = \left(\frac{A_{v,min}}{s}\right) f_{yt} d$$

then, $V_{s,min}$ is the maximum of

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'}b_w d \qquad and \qquad V_{s,min} = \frac{1}{3}b_w d$$

Minimum shear reinforcement is provided $(A_{v,min})$ with

$$s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$$

Case IV:

$$\phi(V_c + V_{s,min}) < V_u \le \phi(V_c + V_s') \quad - stirrups\ are\ required$$

$where \qquad V_{s,min} < V_s \le V_s', \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c \qquad V_s' = \frac{1}{3}\sqrt{f_c'}b_w d$

$and \qquad \frac{A_v}{s} = \frac{V_s}{f_{yt}d}.$

$here \qquad s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$

Case V:

$$\phi(V_c + V_s') < V_u \le \phi(V_c + V_{s,max}) \quad - stirrups\ are\ required$$

$where \qquad V_s' < V_s \le V_{s,max}, \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c\ , \qquad V_s' = \frac{1}{3}\sqrt{f_c'}b_w d,$

$V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d \qquad and \qquad \frac{A_v}{s} = \frac{V_s}{f_{yt}d}.$

$here \qquad s_{max} \le \frac{d}{4} \qquad or \qquad s_{max} \le 300\ mm$

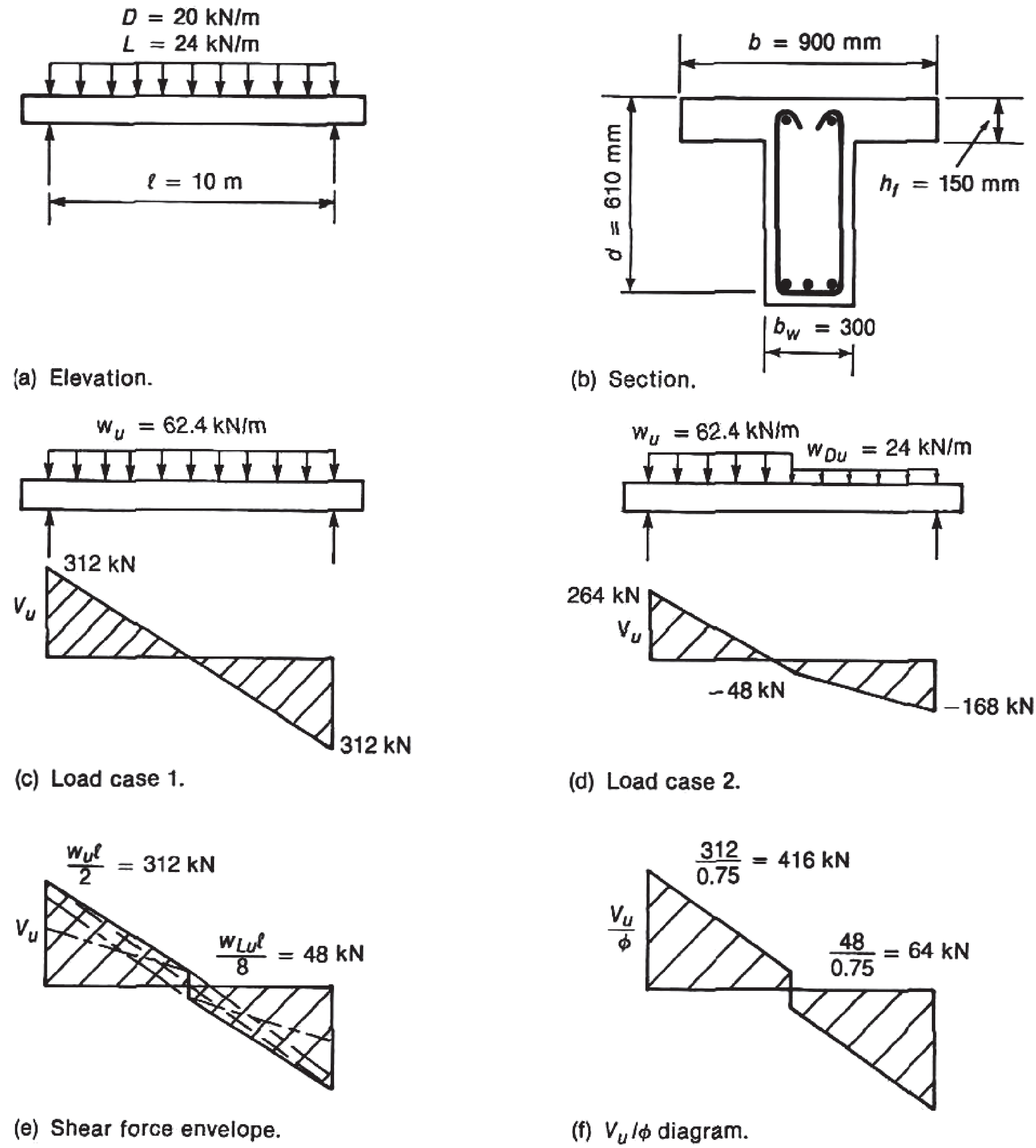
Example:

The Figure shows the elevation and cross section of a simply supported T-beam. This beam supports a uniformly distributed service (unfactored) dead load of $20\ KN/m$, including its own weight, and a uniformly distributed service live load of $24\ KN/m$. Design vertical stirrups for this beam. The concrete strength is $25\ MPa$, the yield strength of the flexural reinforcement is $420\ MPa$, and the yield strength of the stirrups is $300\ MPa$.

The support reactions act usually at the center of supports with full span center to center of supports, in this example, we have no information about the support width, so we assumed that the shear calculations will be done for the given clear span with end reactions at the face of supports for the following all examples.

Important Note:

In a normal building, the dead and live loads are assumed to be uniform loads. Although the dead load is always present over the full span, the live load may act over the full span, or over part of the span. Full uniform load over the full span gives the maximum shear at the ends of the beam. Full uniform load over half the span plus dead load on the remaining half gives the maximum shear at midspan. The maximum shear forces at other points in the span are closely approximated by a linear shear-force envelope resulting from these cases.



(a) Elevation.

(b) Section.

(c) Load case 1.

(d) Load case 2.

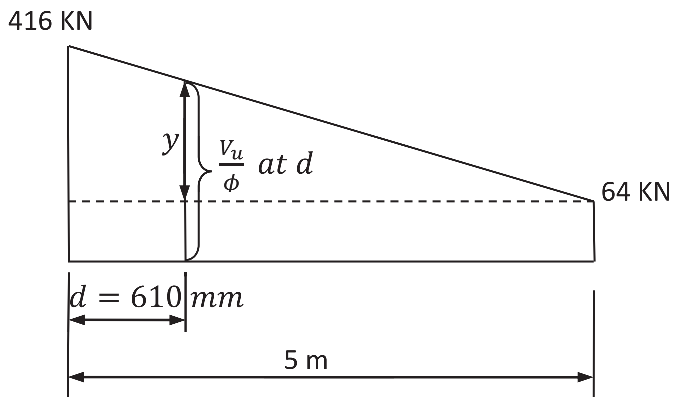
(e) Shear force envelope.

(f) V_u/ϕ diagram.

Solution:

Critical section at $d = 610\,mm$ from the face of support.

V_u at $d = 610\,mm.$



$$\frac{416 - 64}{5} = \frac{y}{5 - 0.61} \quad \longrightarrow y = 309\,KN$$

$$V_n = \frac{V_u}{\phi} = y + 64 = 309 + 64 = 373\,KN$$

$$V_c = \frac{1}{6}\lambda\sqrt{f_c'}b_w d = \frac{1}{6}\cdot 1\cdot\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 152.5\,KN.$$

Check for section dimensions:

$$V_s = V_n - V_c = 373 - 152.5 = 220.5\,KN.$$

$$V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d = \frac{2}{3}\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 610\,KN$$

$$V_s = 220.5\,KN < V_{s,max} = 610\,KN \quad - \textit{the section is large enough.}$$

OR

$$V_{n,max} = V_c + V_{s,max} = \frac{1}{6}\sqrt{f_c'}b_w d + \frac{2}{3}\sqrt{f_c'}b_w d = \left(\frac{1}{6} + \frac{2}{3}\right)\sqrt{f_c'}b_w d = \frac{5}{6}\sqrt{f_c'}b_w d = 5V_c$$

$$V_{n,max} = 5\cdot 152.5 = 762.5\,KN$$

$$\frac{V_u}{\phi} = 373\,KN < V_{n,max} = 762.5\,KN \quad - \textit{the section is large enough}$$

Find the maximum stirrups spacing:

$$if \quad V_s < V_s' = \frac{1}{3}\sqrt{f_c'}b_w d \quad then \quad s_{max} \le \frac{d}{2} \quad or \quad s_{max} \le 600\,mm$$

$$V_s' = \frac{1}{3}\sqrt{f_c'}b_w d = \frac{1}{3}\sqrt{25}\cdot 300\cdot 610\cdot 10^{-3} = 305\,KN$$

$$V_s = 220.5\,KN < V_s' = 305\,KN \quad then$$

$$s_{max} \le 600\,mm, \qquad s_{max} \le \frac{d}{2} = \frac{610}{2} = 305\,mm - control$$

$$V_n = 373\,KN > V_c = 152.5\,KN \quad or$$

$$V_u = \phi V_n = 0.75\cdot 373 = 279.75\,KN > \phi V_c = 0.75\cdot 152.5 = 114.375\,KN$$

Try minimum shear reinforcement:

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} \quad but\ not\ less\ than$$

$$A_{v,min} = \frac{1}{3}\frac{b_w s}{f_{yt}}, \qquad - control \qquad \left(\frac{1}{16}\sqrt{f_c'} = \frac{5}{16} < \frac{1}{3}\right)$$

Use stirrups U-shape (double-leg stirrups) $\varnothing 10$ with $A_v = 2\cdot 78.5 = 157.1\,mm^2$

$$s = \frac{3A_v f_{yt}}{b_w} = \frac{3 \cdot 157.1 \cdot 300}{300} = 471.3\ mm > \ s_{max} = 305\ mm, \quad take\ s = s_{max} = 305\ mm$$

$$V_{s(2\varnothing\,10)} = \frac{A_v f_{yt} d}{s} = \frac{157.1 \cdot 300 \cdot 610}{305} \cdot 10^{-3} = 94.26\ KN$$

$$V_s = 220.5\ KN > V_{s(2\varnothing\,10)} = 94.26\ \ KN, \quad find\ "s" – Case\ IV$$

Alternative step is to calculate $V_{s,min}$

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'} b_w d = \frac{1}{16}\sqrt{25} \cdot 300 \cdot 610 \cdot 10^{-3} = 57.2\ KN$$

$$V_{s,min} = \frac{1}{3} b_w d = \frac{1}{3} 300 \cdot 610 \cdot 10^{-3} = 61\ KN \qquad -control$$

$$\phi\left(V_c + V_{s,min}\right) < \ V_u \leq \ \phi(V_c + V_s')$$

$$0.75(152.5 + 61) = 160.13\ KN < \ V_u = 279.75\ KN < \ 0.75(152.5 + 305) = 343.13\ KN$$

Or $V_s = 220.5\ KN > V_{s,min} = 61\ KN -$ Case IV

Compute the stirrups spacing required to resist the shear forces.

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} \quad \Longrightarrow \quad s = \frac{A_v f_{yt} d}{V_s} = \frac{157.1 \cdot 300 \cdot 610}{220.5 \cdot 10^3} = 130.4\ mm.$$

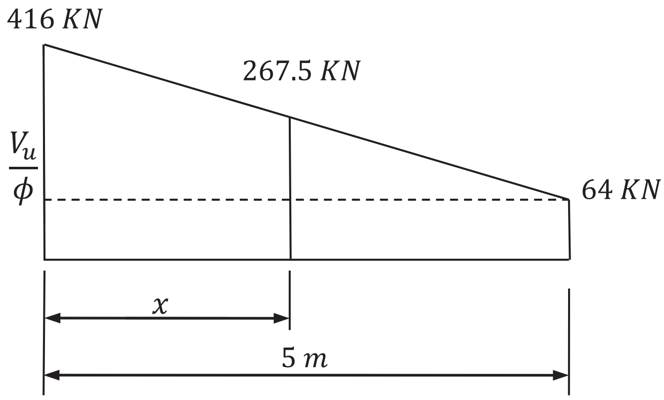
Take U-shape (double-leg stirrups) $\varnothing\ 10@125\ mm \ < s_{max} = 305\ mm.$

Changing "s" to $s_2 = 2s_1 = 2 \cdot 125 = 250\ mm$ for another region.

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} = \frac{\left(\frac{V_u}{\phi} - V_c\right)}{f_{yt} d} \quad \Longrightarrow \quad \frac{V_u}{\phi} = \frac{A_v f_{yt} d}{s} + V_c$$

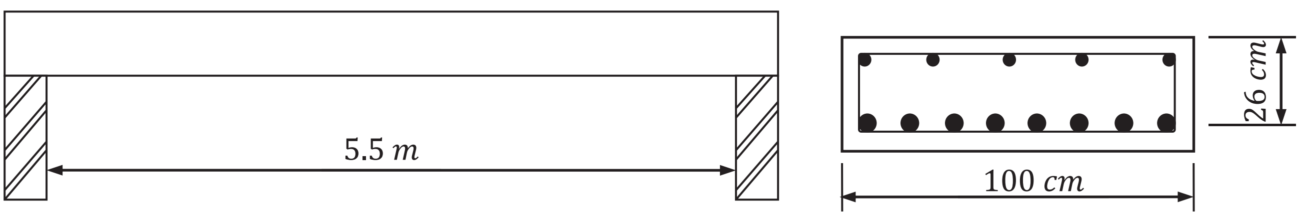
$$\frac{V_u}{\phi} = \frac{157.1 \cdot 300 \cdot 610}{250 \cdot 10^3} + 152.5 = 267.5\ KN$$

$$\frac{416 - 64}{5} = \frac{267.5 - 64}{5 - x} \quad \longrightarrow x = 2.1\ m$$



Example:

The simply supported beam shown below is loaded by a service dead load of 40 KN/m, and a uniformly distributed service live load of 25 KN/m. Design vertical stirrups for this beam. The concrete strength is 25 MPa, and the yield strength of the stirrups is 412 MPa.

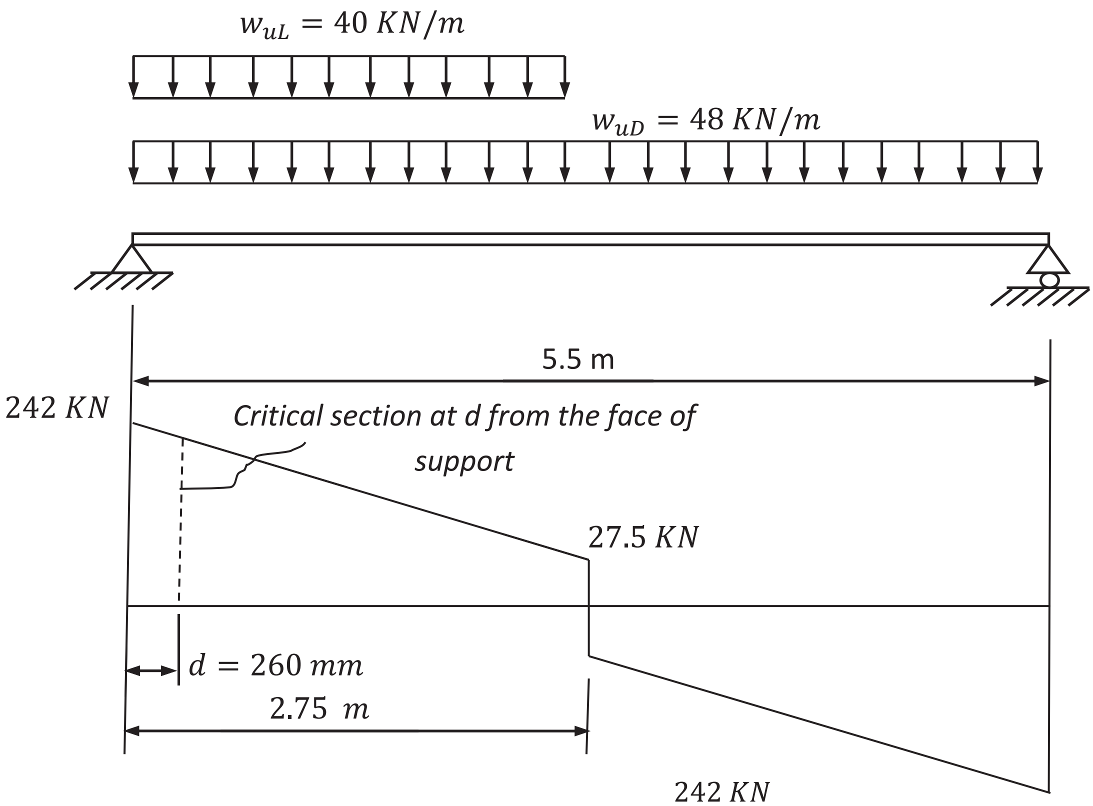


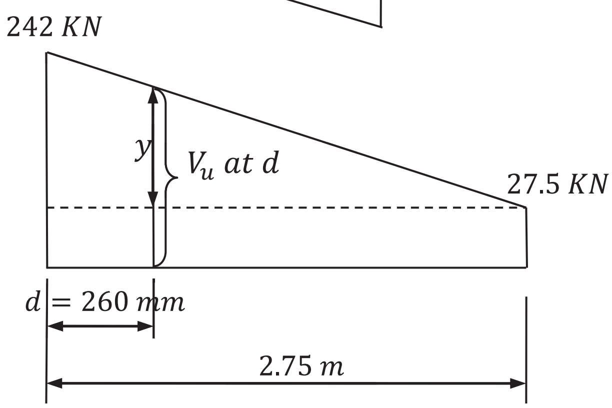
Solution:

$w_{uD} = 1.2 \cdot 40 = 48 \quad KN/m,$ $\qquad\qquad$ $w_{uL} = 1.6 \cdot 25 = 40 \quad KN/m$

$$V_u \; at \; face \; of \; support = \frac{wl}{2} = \frac{(48 + 40) \cdot 5.5}{2} = 242 \; KN,$$

$$V_u \; at \; midspan = \frac{w_{uL} l}{8} = \frac{40 \cdot 5.5}{8} = 27.5 \; KN,$$





Critical section at $d = 260 \; mm$ from the face of support.

V_u at $d = 260 \; mm.$

$$\frac{242 - 27.5}{2.75} = \frac{y}{2.75 - 0.26} \quad \longrightarrow y = 194.22 \; KN$$

$$V_u = y + 27.5 = 194.22 + 27.5 = 221.72 \; KN$$

$$V_c = \frac{1}{6} \lambda \sqrt{f_c'} b_w d = \frac{1}{6} \cdot 1 \cdot \sqrt{25} \cdot 1000 \cdot 260 \cdot 10^{-3} = 216.67 \; KN.$$

Check for section dimensions:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{221.72}{0.75} - 216.67 = 79 \; KN.$$

$$V_{s,max} = \frac{2}{3} \sqrt{f_c'} b_w d = \frac{2}{3} \sqrt{25} \cdot 1000 \cdot 260 \cdot 10^{-3} = 866.67 \; KN$$

$V_s = 79\ KN < V_{s,max} = 866.67\ KN \quad -\textit{the section is large enough.}$

Check for $V_{s,min}$:

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} \qquad \textit{but not less than}$$

$$A_{v,min} = \frac{1}{3}\frac{b_w s}{f_{yt}}, \qquad -control \qquad \left(\frac{1}{16}\sqrt{f_c'} = \frac{5}{16} < \frac{1}{3}\right)$$

$$V_{s,min} = \frac{1}{16}\sqrt{f_c'}b_w d = \frac{1}{16}\sqrt{25}\cdot 1000\cdot 260\cdot 10^{-3} = 81.25\ KN$$

$$V_{s,min} = \frac{1}{3}b_w d = \frac{1}{3}1000\cdot 260\cdot 10^{-3} = 86.67\ KN \qquad -control$$

$$\phi V_c < V_u \le \phi(V_c + V_{s,min})$$

$$0.75(216.67) = 162.5\ KN < V_u = 221.72\ KN < 0.75(216.67 + 86.67) = 227.51\ KN$$

Or $V_s = 79\ KN < V_{s,min} = 86.67\ KN$ – Case III

$$\frac{A_{v,min}}{s} = \frac{1}{16}\sqrt{f_c'}\frac{b_w}{f_{yt}} \qquad \textit{but not less than} \qquad \frac{A_{v,min}}{s} = \frac{1}{3}\frac{b_w}{f_{yt}},$$

$$\frac{A_{v,min}}{s} = \frac{1}{16}\sqrt{25}\frac{1000}{412} = 0.7585$$

$$\frac{A_{v,min}}{s} = \frac{1}{3}\times\frac{1000}{412} = 0.80906 \quad -control$$

Use stirrups 2U-shape (4-leg stirrups) $\varnothing\, 8\ mm$ with $A_v = 4\cdot 50.27 = 201.1\ mm^2$

$$\frac{201.1}{s} = \ = 0.80906 \qquad \Longrightarrow \qquad s = 248.6\ mm$$

$$s_{max} \le 600\ mm, \qquad s_{max} \le \frac{d}{2} = \frac{260}{2} = 130\ mm - control$$

Take 2U-shape (4-leg stirrups) $\varnothing\, 8@125\ mm < s_{max} = 130\ mm$