Concrete Structures

Analysis and Design

Second Edition

Emphasizing American Concrete Institute
(ACI 318-02)
Inch-Pound and SI Units

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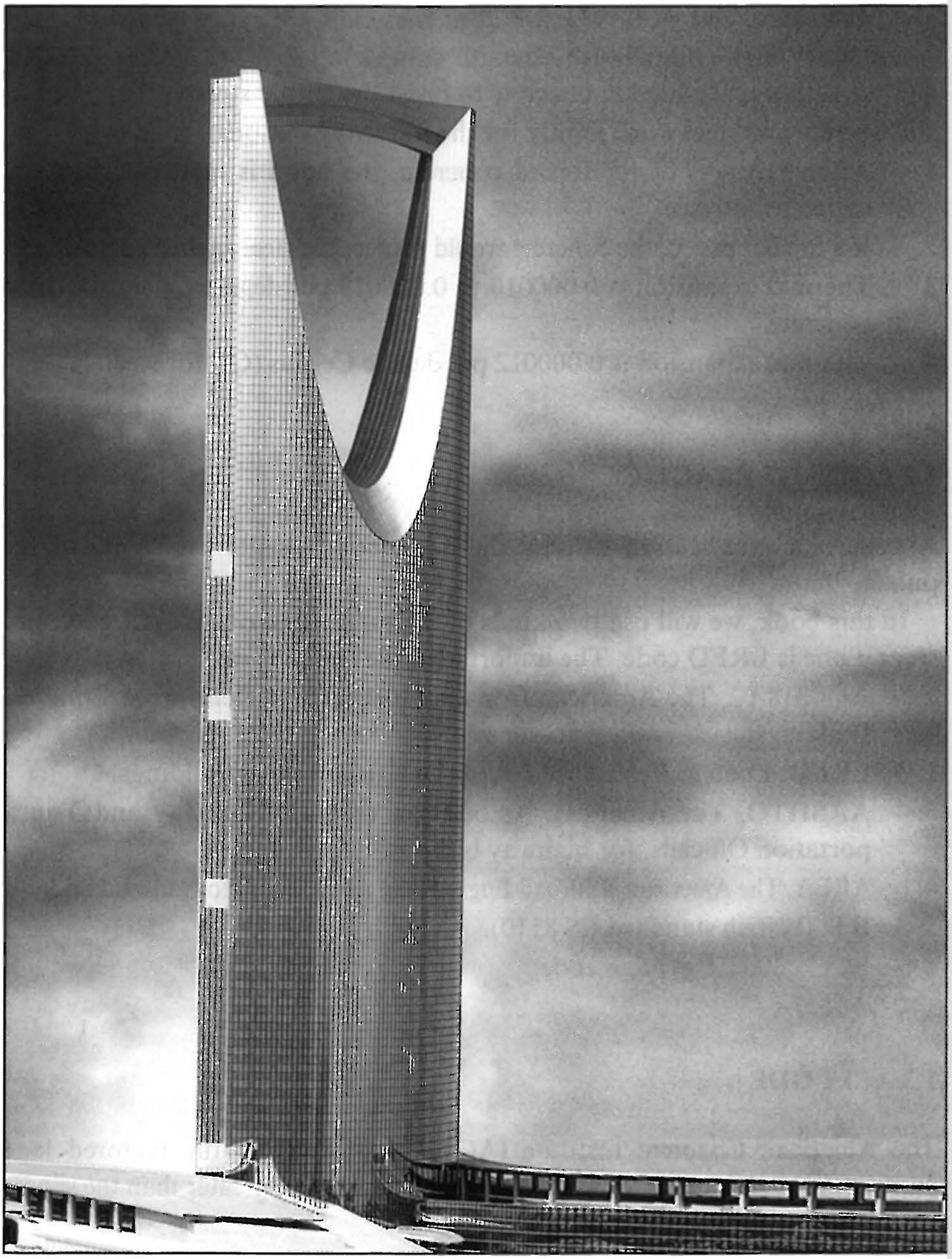


CHAPTER 1

1.1 OVERVIEW

There are three most common types of structures such as: reinforced concrete, steel and wood that using will be extensive at the civil engineering and architecture branch.

Reinforced concrete structures can build bridges, buildings, water tanks, roads, retaining walls, tunnels and others.

Reinforced concrete is consisted of five materials such as: water, cement, aggregate, sand and steel. The first four materials are called plain concrete, which carry high compressive strength comparing with its tensile strength, and the fifth is embedded in concrete to resist the tensile stresses.

Concrete and steel work jointly for the following reasons:

1 - After hardness of reinforced concrete, the bond is increased between concrete and steel.
2 - If a fire happened, the concrete would protect the steel against corrosion.
3 - Thermal expansion is 0.000010 to 0.000013 per degree Celsius (C°) for concrete.
4 - Thermal expansion is 0.000012 per degree Celsius (C°) for steel.

1.2 CODE OF PRACTICE

A code is a specification helping the designer to ensure the safety of the public.

In this book, we will use two kinds of codes; the first one is ACI code and second one is LRFD code. The important codes known are:

1 - ACI 318-02, The American Concrete Institute for Reinforced Concrete Buildings.
2 - LRFD, Load & Resistance Factor Design, for Steel Buildings.
3 - AASHTO, The American Association of State Highway and Transportation Officials, for highway bridges.
4 - AREA, The American Railroad Engineering Association, for railroad bridges.
5 - B.S. (British standard BS 8110).
6 - ECC - 2000, Egyptian code.

1.3 ACI CODE

The American Concrete Institute (ACI) Code produces the factored load multiplying by the service load. The factored load must be greater than the service laod. The ACI code has used factoral load U as a combination of dead load, live load, wind load, earthquake load, lateral earth pressure and fluid pressure.

In addition, dead load D and live load L are the service loads or the effective loads: The dead load consists of structural self-weight, partitions, ceilings, and all mechanical equipements and the live load consists of furniture, people, wind, earthquake or soil pressure.

The ACI code specifies dead load D and live load L, as shear force, bending moment and axial force.

The load factors for the different cases are given in the following, as in the ACI 9.2.1 code;

$$U = 1.2D + 1.6L$$

Where D and L represent the service dead and live load respectively, and U represents the total factored load.

$$U = 1.2D + 1.6W + 1.0L$$

$$U = 0.9D + 1.6W$$

Where W is the wind load, when the live load and wind load are acting together on the structure.

$$U = 1.2D + 1.0E + 1.0L$$

Where earthquake force E is included in the design

$$U = 1.2D + 1.6(L + H)$$

Where lateral earth pressure H is involved, it is considered as live load and the above equation becomes as following:

$$U = 0.9D + 1.6H$$

$$U = 1.2D + 1.2F + 1.6L$$

Where liquid pressure F is involved, the pressure is considered as dead load, and the equation becomes as following:

$$U = 1.2D + 1.2T + 1.6L$$

If the temperature changes, shrinkages, creeps and the differential settlement T becomes as dead load D:

$$U = 1.2(D + T)$$

1.4 STRENGTH REDUCTION FACTORS

The design strength is equal to or greater than the required strength, and the ACI code specifies the nominal strength in accordance and assumptions, and also is designated by the subscript *n*.

Design strength $\geq$ Required strength
ϕ Nominal strength $\geq$ Required strength

$$\phi\ P_n \geq P_u$$

$$\phi\ M_n \geq M_u$$

$$\phi\ V_n \geq V_u$$

Where P_n, M_n and V_n are the axial compression, bending moment and shear, respectively, and the nominal strength (n) in the subscript.
Where P_u, M_u and V_u represent the required strength.

Table 1.1 Reduction factors ϕ

Nominal Strength	Reduction factor ϕ
- Flexure, with or without axial tension,	0.90
- Shear and torsion	0.75
- Bearing on concrete	0.65
- Compression member, spirally reinforced	0.70
- Columns with ties	0.65
- Bending in plain concrete	0.55

Example 1.1
Determine the required strength P_u and nominal strength P_n. If a dead load $D = 150$ KN and a live load $L = 120$ KN, assume the reduction factor $\phi = 0.65$

Solution.
Multiply the load factor by the respective service load to produce P_u

$$U = P_u = 1.2D + 1.6L$$

$$P_u = 1.2(150) + 1.6(120) = 372\,\text{KN (83.6 kips)}$$

The nominal strength is:

$$\phi\, P_n \geq P_u$$

Required $\quad P_n = \dfrac{P_u}{\phi} = \dfrac{372}{0.65} = 572.3$ KN (128.7 kips)

Example 1.2

Compute the nominal flexural strength M_n and apply factored loads to the simply supported beam as shown in Figure 1.1. Assume a concentrated load $P_u = 30$ KN and $\phi = 0.9$

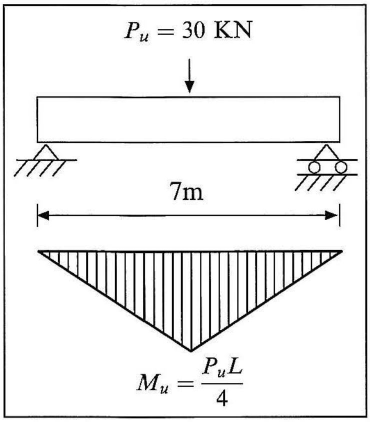


Figure 1.1

Solution.

$$M_u = \frac{P_u L}{4}$$

$$M_u = \frac{30(7)}{4} = 52.5 \text{ KN.m (38.7 ft-kips)}$$

$$\phi\, M_n \geq M_u$$

$$M_n = \frac{52.5}{0.9} = 58.33 \text{ KN.m (43 ft-kips)}$$

MECHANICAL PROPERTIES OF CONCRETE

2

CHAPTER 2

2.1 CONCRETE

Plain concrete is a mixture of fine aggregate, water, cement and coarse aggregate. All the components of the plain concrete are mixed together until they become a paste, which surrounds the voids in aggregate during its fresh concrete. The steel bars are placed into forms and a concrete paste is filled around the steel bars until it changes from a plastic to a solid state in about 24 hours, to become reinforced concrete, as shown in Fig. 2.1.

The expected outcomes of concrete properties are effected by their ingredients which are expected to give reasonable data as designed in the beginning. Compressive strength, modulus of elasticity and Poisson's ratio are also expected to give good agreement at 7,14 and 28 days tests. As a result, the good homogeneous material gives a good relation with embedded steel bars in concrete forms. Therefore, the expected outcome will be more accurate not only for good homogeneous between the composite materials, but also during the cure cycle.

Workability

The slump is the difference between height measured of the steel cone and the top of fresh concrete after the cone had lifted. The slump test is used to control the workability and quality of concrete, as shown in Figure 2.2

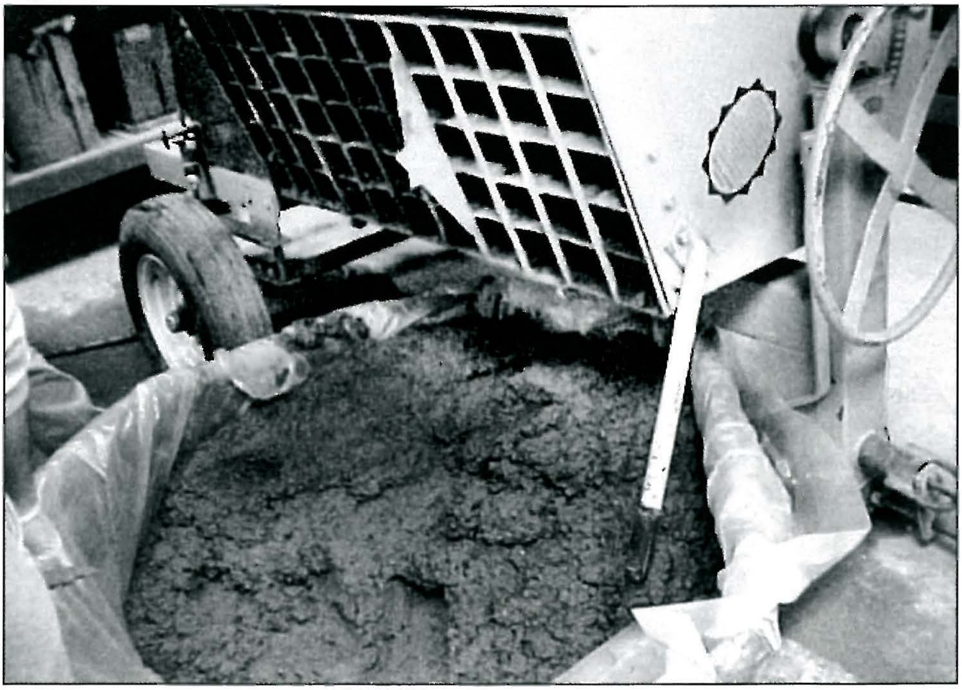


Figure 2.1 Composite material.

Figure 2.2 Slump test.

2.2 COMPRESSIVE STRENGTH

Compressive strength f_c' depends upon water, the cement ratio and the quality of the cure cycle. According to the ACI code, the compressive strength of concrete f_c' is obtained from the standard test cylinder 6 - in. (150 mm) diameter by 12- in (300 mm) high measured at 7, 14 and 28 days of age before testing. After 28 days of water curried or placed in a constant temperature room to obtain 100 percent of humidity. Then, the preparation starts by replacing the specimen on the MTS (Material test system), as shown in Fig. 2.3. In this test, the concrete is subjected to compressive stresses and not to tensile stresses: therefore, a specimen is used to determine the concrete compressive strength by many shapes such as: cylinder 150×300 mm (ACI code), Prism $70 \times 70 \times 350$ mm (France) and cube $150 \times 150 \times 150$ mm (Germany, Egypt, Great Britain).

Table 2.1 shows the value of compressive strength (Wayne State University-Structure Lab.).

Figure 2.3 MTS Machine and specimens (Wayne State University- Structure Lab.)

Typical stress - strain relationship for concrete cylinder produced by compressive strength test, is shown in Fig. 2.4.

The shape of curve depends on the age of specimen, the composite of concrete material, MTS machine and loading.

The ACI code defines that the maximum concrete strain, is 0.003, and for high - compressive strength f'_c, between 8000 to 12,000 psi (55.12 to 82.7 MPa). Nonprestressed structures are: 3500 to 6000 psi (24.11 to 41.34 MPa). For greater than 6000 psi (41.34 MPa) is used for prestressed concrete.

Table 2.1 Compressive strength (MPa)

Age	**Specimen Number**				**Mean**	**Std. Dev.**
(days)	1	2	3	4	(MPa)	
7	49.26	49.78	49.54	47.11	48.92	1.226
14	58.1	58.8	55.39	56.12	57.1	1.609
28	66.62	64.91	62.47	60.28	63.57	2.77

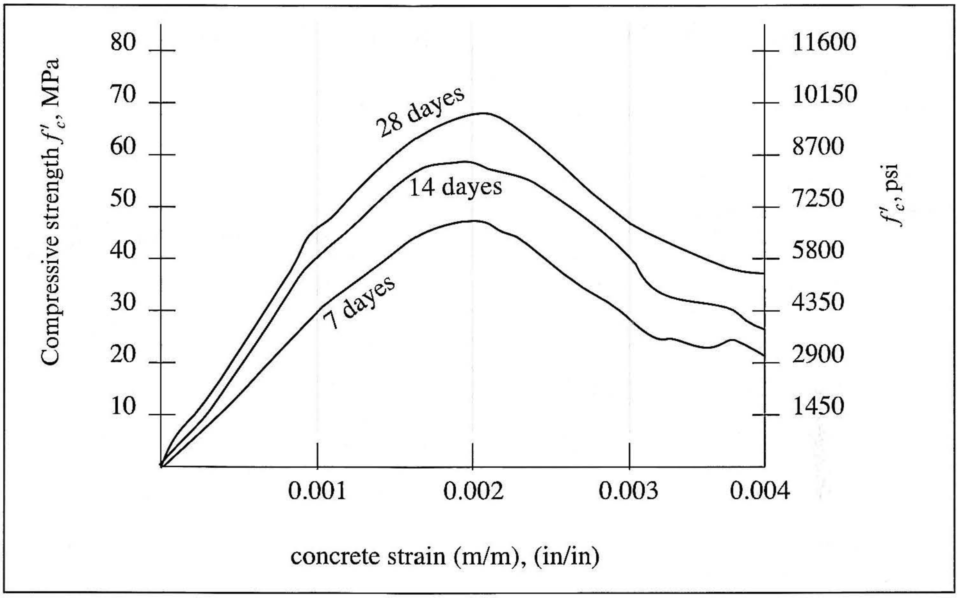


Figure 2.4 Concrete stress - strain curve

2.3 MODULUS OF ELASTICITY

The ACI code determines a value of the modulus of elasticity of concrete $E_c = w_c^{1.5}(33)\sqrt{f_c'}$ psi for normal concrete, and the slope of the stress-strain curve defines the initial modules used with the parabolic stress method. For values of w_c between 90 and 155 Ib/ft^3 (1500 and 2500 kg/m^3), the ACI code specifies modulus of elasticity E_c

$$E_c = \begin{cases} w_c^{1.5}(33)\sqrt{f_c'} & \text{psi} \\ w_c^{1.5}(0.0432)\sqrt{f_c'} & \text{MPa} \end{cases} \tag{2.1}$$

Where w_c is the unit weight of concrete between (1500 and 2500 kg/m^3), and the value of w_c when made from crushed stone is: 145 Ib/ft^3 (2353kg/m^3). Substituting Eq. (2.1) in value w_c becomes:

$$E_c = \begin{cases} 57000\sqrt{f_c'} & \text{psi} \\ 4700\sqrt{f_c'} & \text{MPa}(E_c \text{ and } f_c' \text{ in MPa}) \\ 15000\sqrt{f_c'} & \text{kgf/cm}^2\ (E_c \text{ and } f_c' \text{ in kfg/cm}^2) \end{cases} \tag{2.2}$$

For most concrete, the Poisson's ratio is equal to the transfer strain divided by the longitudinal strain; $v = (0.2 \text{ to } 0.23)$
Table 2.2 shows the values of E_c, for $w_c = 145\ \text{lb/ft}^3$

Table 2.2 Values of E_c

SI units		Inch - pound units	
f'_c (MPa)	E_c (MPa)	f'_c (psi)	E_c (psi)
20.67	21368	3000	3,122,018
24.11	23077	3500	3,372,165
27.56	24673	4000	3,604,996
31.00	26168	4500	3,823,676
34.45	27586	5000	4,030,508

Multiply MPa values by 10.2 to get kgf/cm^2

Modular Ratio, *n*

The relation: stress - strain for reinforcement steel, is a linear under the yield stress, which is compared with concrete curve. But in concrete, it is assumed as a linear it varies with its density and strength. The modulus of elasticity of the steel is:

$E_s = 29,000,000$ psi (199926000 KPa $\approx 200,000$ MPa).

$$n = \frac{E_s}{E_c} \tag{2.3}$$

Table 2.3 Values of modular ratio, *n*

SI units		Inch - pound units	
f'_c (MPa)	n	f'_c (psi)	n
20.67	$9.3 \approx 9.0$	3000	$9.2 \approx 9.0$
24.11	$8.6 \approx 8.5$	3500	$8.6 \approx 8.5$
27.56	$8.1 \approx 8.0$	4000	$8.04 \approx 8.0$
31.00	$7.6 \approx 7.5$	4500	$7.56 \approx 7.5$
34.45	$7.2 \approx 7.0$	5000	$7.2 \approx 7.0$

2.4 CONCRETE TENSILE STRENGTH

Tensile strength is low about 10 to 15% of the compressive strength, and usually is determined by using the split - cylinder test and using the same size of compressive strength.

At the end of the curing period, several experiments will be conducted on the specimens to obtain the tensile strength, as shown in Fig. 2.5.

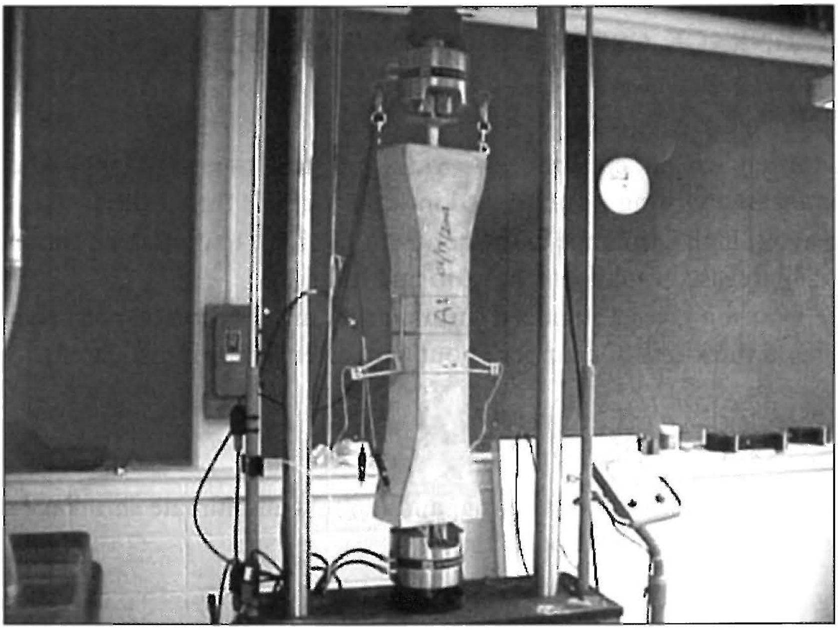
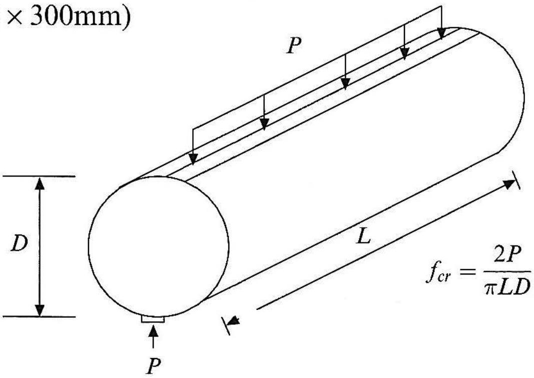


Figure 2.5 Tensile test (Wayne State University- Structure Lab.)

The difference between tensile strength and compressive strength is that the fine cracks existing in concrete, and during the tensile test, the stresses flow cracks and voids, but in compression test, the cracks and voids are able to transmit compression stresses.

The tensile test is called splitting test in the form of a 6 in. diameter by 12 in. length (150 × 300mm)



$$f_{cr} = \frac{2P}{\pi LD}$$

Where f_{cr} is splitting - cylinder tensile strength
From ACI code the modulus of rupture is:

$$f_r = 7.5\sqrt{f'_c} > 1.12 f_{cr} \qquad \text{psi} \tag{2.4}$$

or

$$f_r = 0.7\sqrt{f'_c} > 1.26 f_{cr} \qquad \text{MPa} \tag{2.5}$$

2.5 SHRINKAGE, CREEP AND TEMPERATURE

Shrinkage

For normal weight concrete, the value of shrinkage is 0.0003 when the specimen after casting is submerged in water not less than 7 days.

To avoid high shrinkage in the concrete, we have to consider proportional size of aggregate, water- cement ratio and humidity.

The Branson gives a standard shrinkage strain equation (for less than 4 in. slump and thickness of member about 6 in. after 7 days moist cured).

$$\varepsilon = \left(\frac{t}{35+t}\right)(\varepsilon_{sh})_u \tag{2.6}$$

Where t is (days) after moist curing, and $(\varepsilon_{sh})_u$ is an ultimate shrinkage strain. Branson suggests using 800×10^{-6} in/in.

Creep

The creep deformation occurs under a constant load during its life and the creep increases with early age, then decreases with time. That function is with modulus of elasticity E_c and compressive strength.

Temperature

The concrete coefficient is expanded with increasing temperature that equal to 6×10^{-6} in/in/°F (10×10^{-6}/C°) and for steel is equal to approximately (11×10^{-6}/C°).

2.6 REINFORCING STEEL

Reinforcing steel is an important material with reinforced concrete to resist tensile stresses, increase the compressive strength and to increase the bond between concrete and steel.

The size of bars under ACI code are 0.375 to 2.257 in. in diameter (9.5 to 57.3mm), and in the SI units are 6.0 mm to 57mm nominal diameter.

All reinforcement steel bars smooth or twisted are rounded, and modulus of elasticity E_s for steel is 29,000,000 (f'_c in psi) by ACI code, and in the SI units is 200,000 (f'_c in MPa).

Table 2.4 determines the nominal dimensions for number of bar, diameter, area and weight. According to ASTM and SI units.

Table 2.4 Reinforcing bar dimensions.[4]

Bar	**Diameter**		**Area**		**Nominal weight**	
Number	in	mm	in^2	mm^2	Ib/ft	kg/m
3	0.375	9.5	0.11	71	0.376	0.559
4	0.500	12.7	0.20	129	0.668	0.995
5	0.625	15.9	0.31	200	1.043	1.552
6	0.750	19.1	0.44	284	1.502	2.235
7	0.875	22.2	0.60	387	2.044	3.041
8	1.000	25.4	0.79	510	2.670	3.973
9	1.128	28.7	1.00	645	3.400	5.059
10	1.270	32.3	1.27	819	4.303	6.403
11	1.410	35.8	1.56	1006	5.313	7.906
14	1.693	43.0	2.25	1451	7.65	11.38
18	2.257	57.3	4.00	2580	13.60	20.24

The shape of steel bars are various from exterior shape and diameter, as shown in Fig. 2.6. For metric bar sizes introduced in Middle East are more convenient than American bars size because there are only 9 bars. Therefore, the amount of steel in metric calculations is higher that makes the diameter of bar restricted by reducing the number of bars.

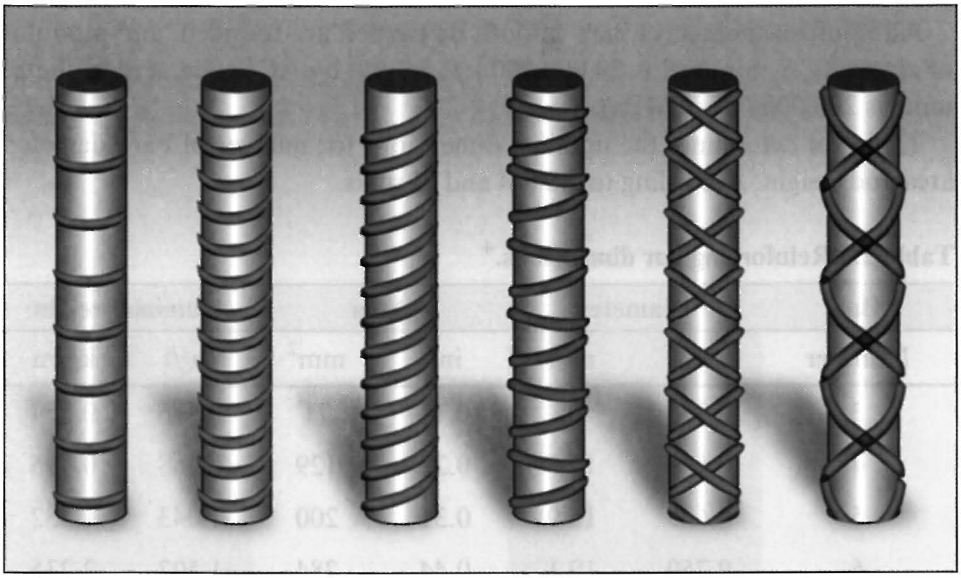


Figure 2.6 Reinforcing steel bars. (Courtesy of Concrete Reinforcing Steel Inst.)

The stress-strain relationship for steel, shown in Fig. 2.7 is depended on ACI code, for designing concrete structures.

The value of modulus of elasticity E_s for all Grades of steel is equal to 29000 ksi (200 GPa, 204×10^4 kg/cm^2). To compute yield point at the stress side, when the strain increases, the yield stress is reduced immediately, as shown in Figure.

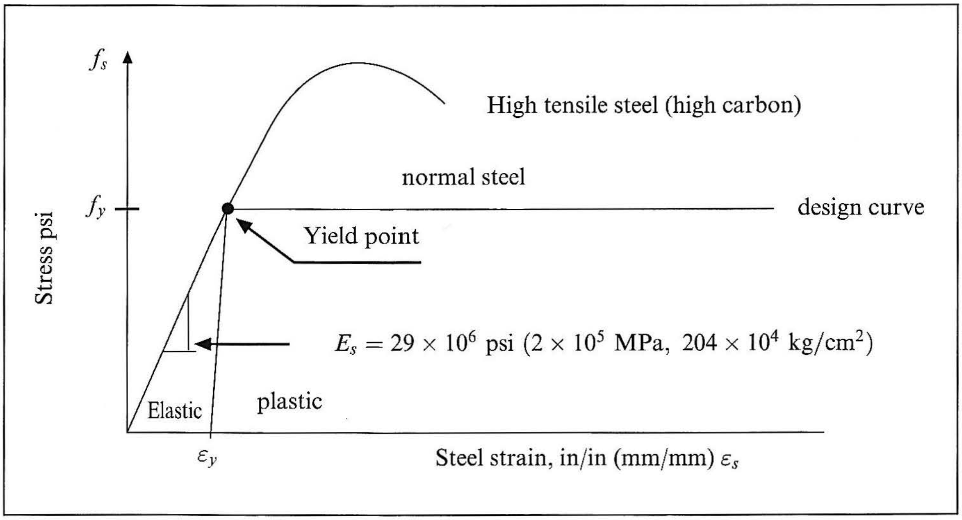


Figure 2.7 Stress - Strain curve for steel.

Table 2.5 Area of cross- section of U. S. bars (in^2)

Bar No.	Nominal Diameter (in)	Number of bars										
		1	2	3	4	5	6	7	8	9	10	Weight lb/ft
3	0.375	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10	0.376
4	0.500	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	0.668
5	0.625	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10	1.043
6	0.750	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40	1.502
7	0.875	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00	2.044
8	1.000	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90	2.670
9	1.128	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	3.400
10	1.270	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70	4.303
11	1.410	1.56	3.12	4.68	6.24	7.80	9.39	10.92	12.48	14.04	15.60	5.313
14	1.693	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.0	20.25	22.50	7.650
18	2.257	4.00	8.00	12.00	16.00	20.0	24.0	28.00	32.0	36.00	40.00	13.60

* Number 3 and 4 are generally used in stirrups
* Number 14 and 18 are generally used in columns.

Table 2.6 Area of cross- section of SI bars (mm^2)

ϕ mm	Number of bars										
	1	2	3	4	5	6	7	8	9	10	Weight mg/m
6	28.3	56.6	84.8	113	141	170	198	226	254	283	0.222
8	50.3	101	151	201	251	302	352	402	452	503	0.395
10	78.5	157	236	314	393	471	550	628	707	785	0.617
12	113	266	339	452	565	679	792	905	1020	1130	0.888
14	154	308	462	616	770	924	1080	1230	1390	1540	1.21
16	201	402	603	804	1005	1206	1407	1608	1810	2010	1.58
18	254	509	763	1020	1270	1530	1780	2040	2290	2540	2.00
20	314	628	942	1260	1570	1880	2200	2510	2830	3140	2.47
22	380	760	1140	1520	1900	2280	2660	3040	3420	3800	2.98
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910	3.85
28	616	1230	1850	2460	3080	3700	4310	4930	5540	6160	4.83
30	707	1410	2120	2830	3535	4240	4950	5660	6360	7070	5.55
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040	6.31
34	908	1820	2720	3630	4540	5450	6360	7260	8170	9080	7.13

To obtain area in cm^2 divide $mm^2/100$

Table 2.7 Minimum cross- section width for bars in single layer (in)

Bar size	Number of bars						
	2	3	4	5	6	7	8
5	7.2	8.8	10.5	12.0	13.5	15.2	16.8
6	7.3	9.0	10.6	12.4	14.1	15.9	17.6
7	7.4	9.4	11.2	13.2	15.0	17.0	19.0
8	7.5	9.5	11.4	13.4	15.5	17.5	19.4
9	7.6	9.7	12.3	14.5	16.7	19.1	21.2
10	7.9	10.3	13.2	15.6	18.1	20.6	23.2
11	8.2	11.0	13.9	16.7	19.5	22.3	25.1
14	8.8	12.1	15.5	19.0	22.4	25.8	29.0
18	10.5	15.0	19.5	24.0	28.4	33.0	37.5

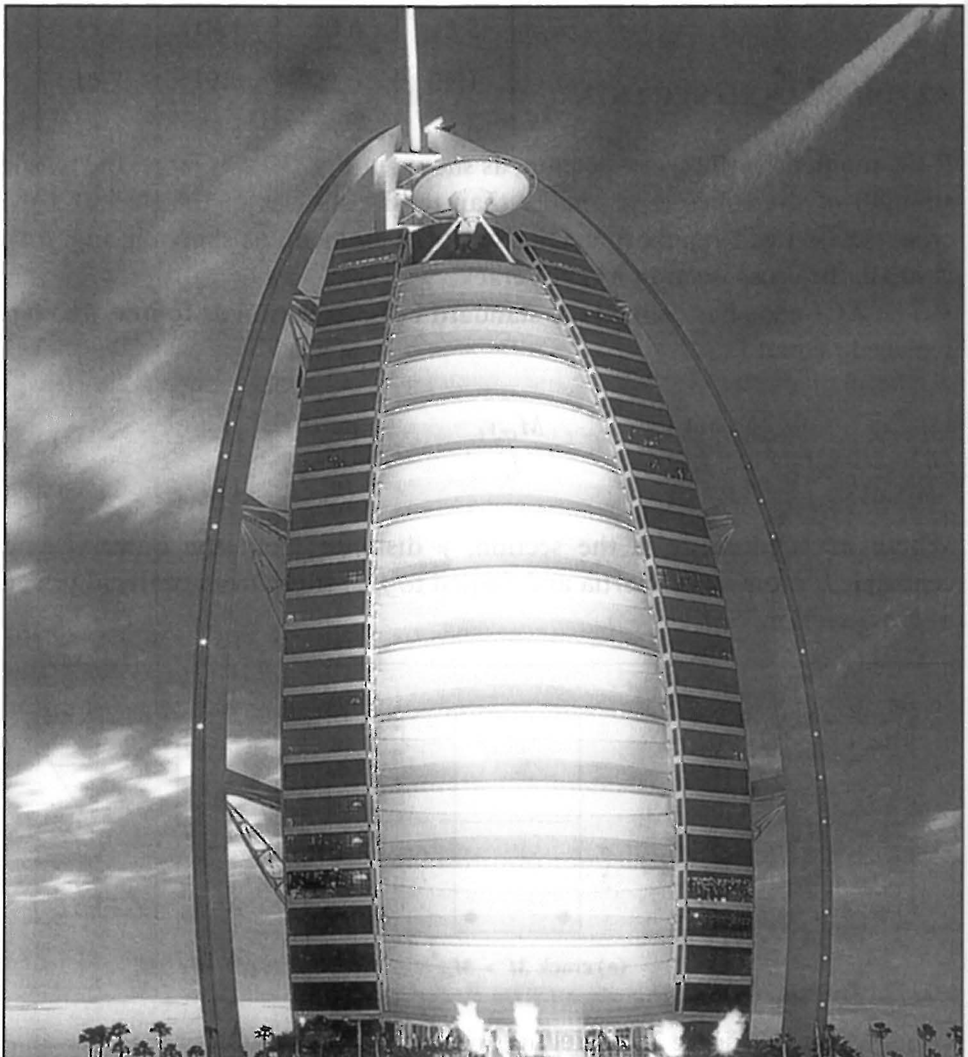
* Number 3 and 4 assumed as stirrups

Table 2.8 Properties of U. S. bars and metric bars

Metric Bar No.	U.S Bar No.	Metric diameter (mm)	U.S dimeter (in)	Metric area (mm^2)	U.S area (in^2)	Perimeter	
						mm	in
10	3	9.52	0.375	71.2	0.11	30	1.18
13	4	12.7	0.500	126.7	0.20	40	1.571
16	5	15.87	0.625	197.8	0.31	50	2.0
19	6	19.05	0.750	285	0.44	60	2.36
22	7	22.22	0.875	387.5	0.6	70	4.75
25	8	25.4	1.000	506.7	0.79	80	3.142
29	9	28.65	1.128	644.7	1.00	90	3.544
32	10	32.26	1.270	817.3	1.27	101.5	4.00
39	11	35.81	1.41	1007.2	1.56	112.5	4.430
43	14	43.0	1.693	1452.2	2.25	135	5.32
57	18	57.33	2.257	2581	4.00	180	7.1

ANALYSIS AND DESIGN OF BEAMS

3



CHAPTER 3

3.1 INTRODUCTION

Analysis and design of reinforced concrete beams are based on the following fundamental propositions:

1 - The external force should be in equilibrium with the internal stress of the concrete beam.
2 - Deflection control.
3 - The control of crack should be a perfect adhesive between surrounded steel bars and concrete to ensure that no slip will take place.
4 - Stress - strain curves are assumed in a good relationship.
5 - Design strength of the beam will be greater or equal to a required strength.

3.2 UNCRACKED SECTION

If the moment in the cross- section, as shown in Fig. 3.1a, is large, the tensile strength of the concrete is smaller than tensile stresses of the steel and the cross- section will expose to crack. But if the moment, as shown in Fig. 3.1b is small, the cross- section will not crack.

The ACI code has defined the standard beam equation as follow, and has replaced f equal f_r.

$$f = \frac{My}{I_g}, \qquad f_r = \frac{M_{cr} y_t}{I_g}, \qquad M_{cr} = \frac{I_g f_r}{y_t} \tag{3.1}$$

Where M is moment in the section, y distance from the outer end to centroid, I_g moment of inertia and f equal to f_r is stress from centroid to end of cross-section.

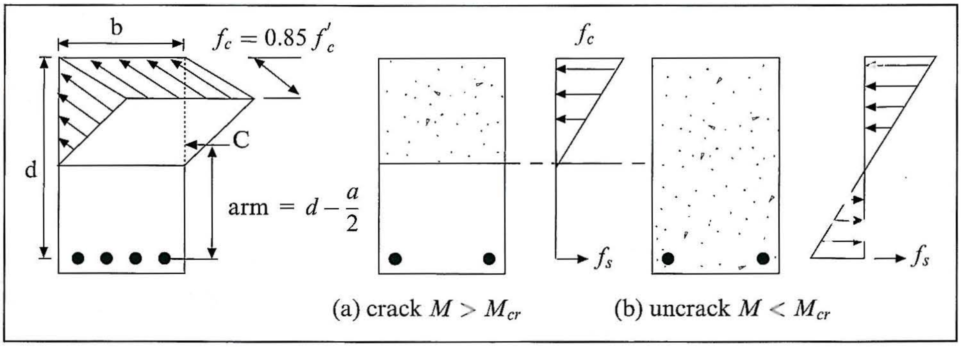


Figure 3.1 Cracking and uncracking section.

Example 3.1

Calculate the cracking moment M_{cr} and P where $f'_c = 30$ MPa and the dimensions of cross- section as shown in Fig. 3.2.

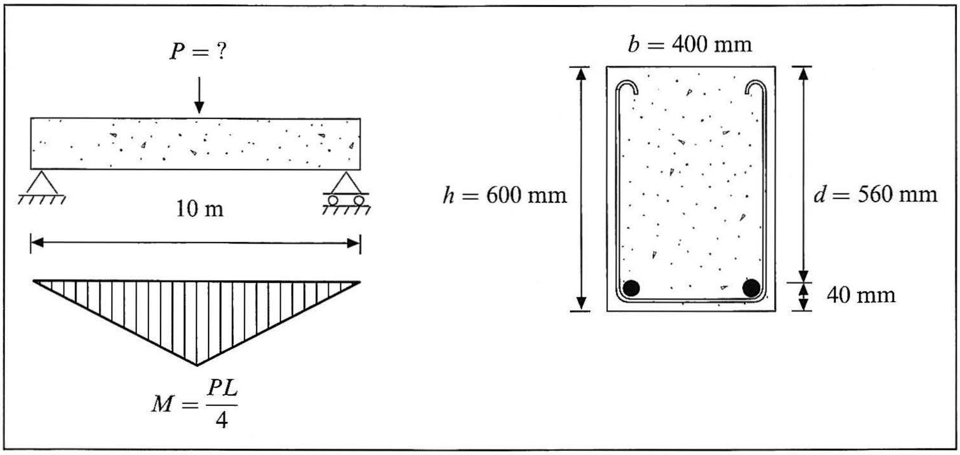


Figure 3.2 Rectangular cross-section.

Solution.

Compute I_g

$$I_g = \frac{bh^3}{12} = \frac{400 \times 600^3}{12} = 7.2 \times 10^9 \text{ mm}^4 \ (17280 \text{ in}^4)$$

From Eq. (2.5) $f_r = 0.7\sqrt{f'_c}$ (metric units)

$$f_r = 0.7\sqrt{30} = 3.83 \text{ MPa } (0.55 \text{ ksi})$$

$$y_t = \frac{h}{2} = \frac{600}{2} = 300 \text{ mm}$$

$$M_{cr} = \frac{I_g f_r}{y_t} = \frac{7.2 \times 10^9(3.83)}{300} = 9.2 \times 10^7 \text{ N.mm (92 KN.m)}$$

$$M = \frac{PL}{4} = M_{cr}$$

$$P = \frac{4 \times 92}{10} = 36.8 \text{ KN (8.27 kips)}$$

Example 3.2
Recalculate example 3.1 by using inch - pound units for cracking moment M_{cr}. If $f'_c = 4000$ psi, $b = 16$ in, $h = 24$ in. and $f_{cr} = 350$ psi.

Solution.
Compute I_g

$$I_g = \frac{bh^3}{12} = \frac{16(24)^3}{12} = 18432 \text{ in}^4$$

From Eq. 2.4

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi} > 1.12 f_{cr}$$

$$f_r = 474 \text{ psi} > 1.12(350) = 392 \text{ psi} \qquad \textbf{O.K}$$

$$M_{cr} = \frac{f_r\, I_g}{y_t} \qquad y_t = \frac{24}{2} = 12 \text{ in}$$

$$M_{cr} = \frac{0.474 \text{ ksi } (18432)}{12} = 728 \text{ in.kips}$$

$$= 60.6 \text{ ft.kips } (82.2 \text{ KN.m})$$

3.3 FLEXURAL FAILURE

When the beam of concrete is loaded to failure, there are three possible types of failure such as: balanced, ductile and brittle.

Balanced

If the section reached the compression zone which is the top surface, the strain is 0.003. At same time when the steel stain reaches ε_y. In this case, the section will be in a balanced condition or in a balanced amount of reinforcement as shown in Fig. 3.3.

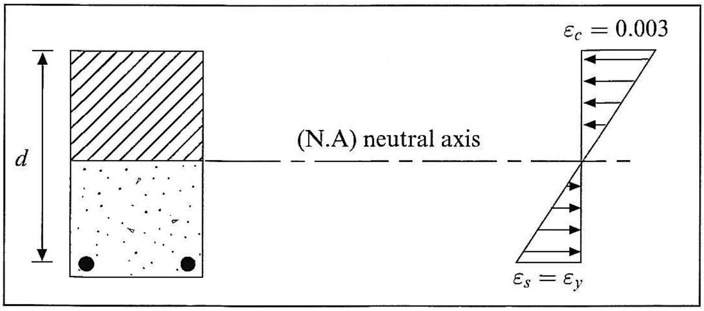


Figure 3.3 Balanced failure.

$\varepsilon_s = \varepsilon_y$ (Balanced condition)

Ductile

This type of failure is called ductile. An important thing: this failure takes place in under - reinforcement section is that tension steel reaches its yield strain ε_y before concrete section reaches its maximum strain $\varepsilon_c = 0.003$. On the other hand, the steel strain is greater than the yield strain. Ductile failure is recommended because it is noticeable when the failure cracks happen, and gives enough warning before collapsing, and in the ACI code this is the only acceptable type of failure.

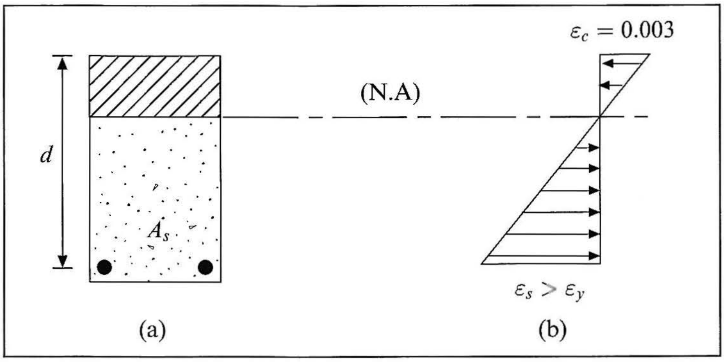


Figure 3.4 Ductile failure.

When

$$\varepsilon_s > \varepsilon_y \text{ (Tension control)}$$

$$T = A_s f_y$$

$$f_s = E_s \varepsilon_y = f_y \qquad C_s = A_s' f_y$$

$$A_s f_s = A_s f_y$$

Brittle

This failure should not be recommended, therefore the ACI code ensures that section in under- reinforced by placing limits on reinforcing steel ratio and the maximum depth of neutral axis to the total depth, because this failure occurs without any warning.

When

$$\varepsilon_s \leq \varepsilon_y \qquad \text{(Compression control)}$$

$$f_s = E_s \varepsilon_s \qquad T = A_s f_s = A_s(E_s \varepsilon_s)$$

$$A_s f_s = A_s E_s \varepsilon_s \qquad C_s = A_s' f_s' = A_s'(E_s \varepsilon_s')$$

$$f_s < f_y$$

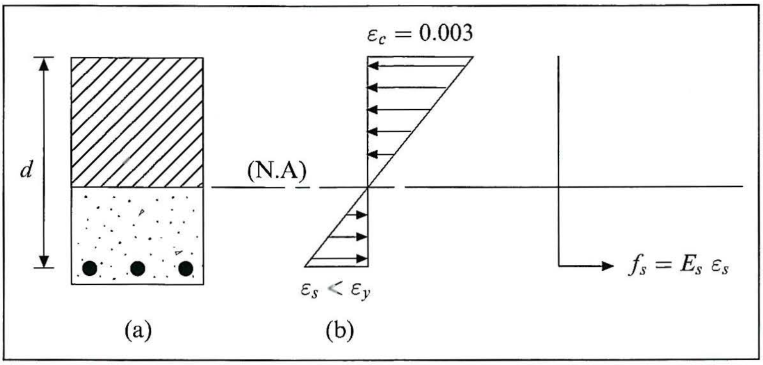


Figure 3.5 Brittle failure.

3.4 THE BALANCED RECTANGULAR SECTION

A cross- section of the reinforced concrete beam is a balanced strain, when the section is reached in top fiber of compression zone, the maximum strain ε_{cu} is 0.003 with the yield strain ε_y equal to steel strain ε_s. On the other hand, when the area of steel A_s is greater than the area steel balance A_{sb}, the internal force in concrete C is equal to the steel force T. That means, the depth of a wall increases, and the distance c is greater than c_b. Or the depth will be reduced and the distance c will be smaller than c_b. This balanced strain condition is shown in Fig. 3.6.

The reinforcement ratio ρ_b is created from the following equations, which are obtained from equilibrium and compatibility.

$$\rho = \frac{A_s}{bd} \qquad \text{(if } A_s \text{ known)}$$

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87,000}{87,000 + f_y(\text{psi})}\right) \tag{3.2}$$

For the reinforcement ratio ρ_b it may be obtained from the linearity of the strain condition:

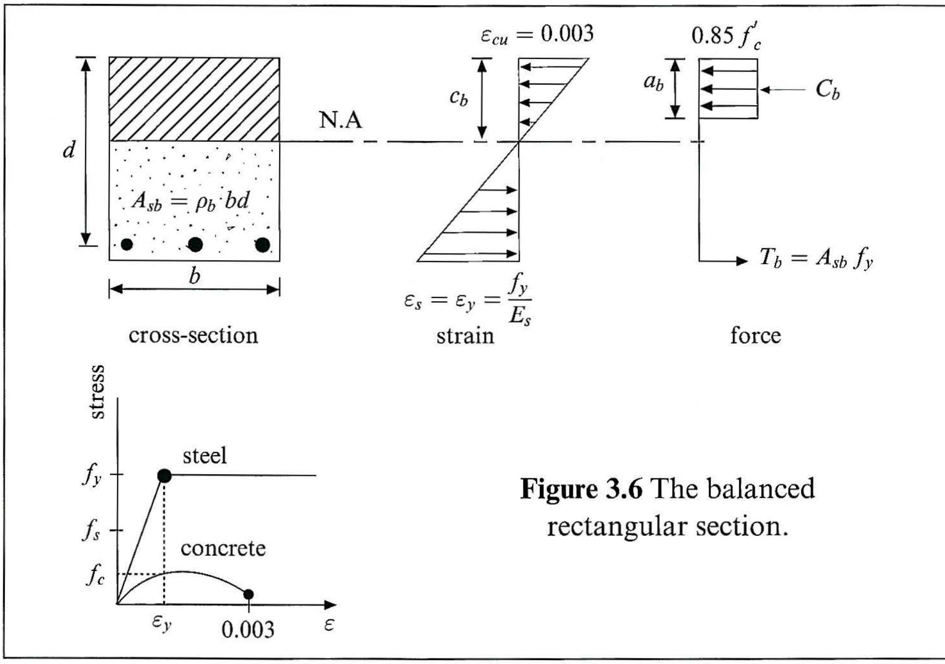


Figure 3.6 The balanced rectangular section.

$$\frac{c_b}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} \qquad \varepsilon_y = \frac{f_y}{E_s} \tag{3.3}$$

$$c_b = \frac{0.003}{0.003 + f_y/29000000}(d) = \frac{87,000}{87,000 + f_y}(d)$$

$$c_b = \frac{600}{600 + f_y}(d) \qquad \text{SI}$$

For $E_s = 200$ GPa (200 000 MPa) and f_y in MPa

From equilibrium Eq.

$$T_b = C_b$$

$$A_{sb} f_y = 0.85 f'_c \ b \ a_b$$

$$a_b = \beta_1 \ c_b$$

$$A_{sb} f_y = 0.85 f'_c \ b \ \beta_1 \ c_b$$

$$A_{sb} = \frac{0.85 \ \beta_1 f'_c \ b}{f_y} \left(\frac{87,000}{87,000 + f_y}\right) d$$

$$\rho_b = \frac{A_{sb}}{bd}$$

$$\rho_b = \frac{0.85 \ \beta_1 f'_c}{f_y} \left(\frac{87,000}{87,000 + f_y}\right)$$

$$\rho_b = \begin{cases} \dfrac{0.85\,\beta_1 f'_c}{f_y}\left(\dfrac{87{,}000}{87{,}000 + f_y}\right) & \text{Inch – Pound} \\ \\ \dfrac{0.85\,\beta_1 f'_c}{f_y}\left(\dfrac{600}{600 + f_y\ (\text{MPa})}\right) & \text{SI} \end{cases} \tag{3.4}$$

Where β_1 is the strength factor, if the compressive strength is less than or equal to 4 kips/in^2 (27.57 MPa), β_1 is 0.85 and between 4 to 8 ksi (27.5 to 55.1 MPa), the value β_1 gets from equations as shown in Fig. 3.7, and more than 8 ksi (55.1 MPa), β_1 is equal to 0.65.

Where $a = \beta_1\ c$

$\beta_1 = 0.85$ — $f'_c \le 4$ ksi (27.5 MPa)

$\beta_1 = 0.85 - 0.05\ (f'_c \text{ ksi} - 4)$ — $4 \text{ ksi} < f'_c \le 8$ ksi

$\quad = 0.85 - 0.007\ (f'_c - 30)$ — $30 \text{ MPa} < f'_c \le 58$ MPa

$\beta_1 = 0.65$ — $f'_c > 8$ ksi (58 MPa)

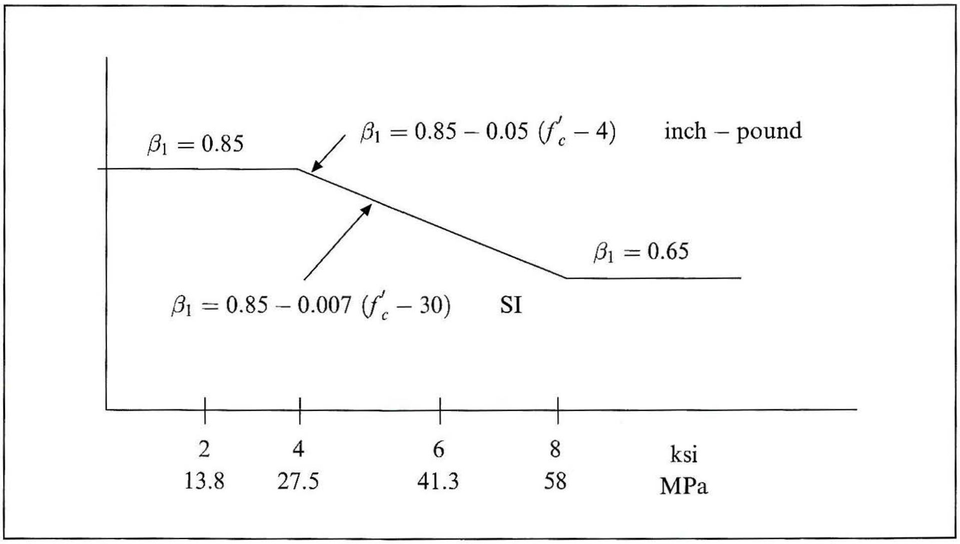


Figure 3.7 Variation of β_1 with 28 - day compressive strength[9].

Example 3.3

The dimensions of the cross- section, is shown in Fig. 3.8. Use $f_y = 350$ MPa and $f'_c = 35$ MPa. Compute β_1 and check if the steel strain ε_s exceeds the strain of steel yield ε_y.

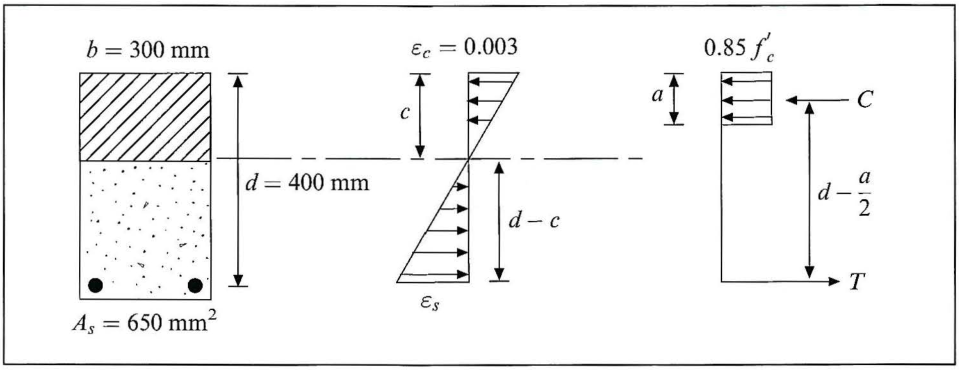


Figure 3.8 Cross-section beam.

Solution.

Equilibrium Eq. $T = C$ $\qquad A_s f_y = 0.85 f'_c\, ba$

$$650 \times 350 = 0.85\ (35)\ 300\ a$$

$$a = \frac{227,500}{8925} = 25.5 \text{ mm}$$

From Fig. (3.7) $\quad \beta_1 = 0.85 - 0.007\ (35 - 30) = 0.81 \qquad$ SI

Where $\quad c = \frac{a}{\beta_1} = \frac{25.5}{0.81} = 31.5$ mm

$$\frac{0.003}{c} = \frac{\varepsilon_s}{d - c}$$

$$\varepsilon_s = \frac{0.003\ (400 - 31.5)}{31.5} = 0.0351$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{350}{200,000} = 0.00175$$

Since $\varepsilon_s = 0.0351 > \varepsilon_y = 0.00175$, the beam is underreinforced. **O.K**

Example 3.4
Recomputed Example 3.3, where $f_y = 50$ ksi, $f'_c = 5$ ksi and the dimensions of the cross-section are $b = 12$ in., $d = 16$ in. and $A_s = 2.25$ in^2.

Solution.

$$T = C \qquad A_s f_y = 0.85 f'_c\ b\ a$$

$$2.25\ (50\ 000\ \text{psi}) = 0.85\ (5000\ \text{psi})\ 12\ \text{a}$$

$$a = \frac{112,500}{51,000} = 2.2\ \text{in}$$

$$\beta_1 = 0.85 - 0.05\ (5 - 4) = 0.8$$

$$c = \frac{a}{\beta_1} = \frac{2.2}{0.8} = 2.75\ \text{in}$$

$$\frac{0.003}{c} = \frac{\varepsilon_s}{d - c}$$

$$\varepsilon_s = \frac{0.003\ (16 - 2.75)}{2.75} = 0.0144$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{50}{29\ 000} = 0.00172$$

since $\varepsilon_s = 0.0144 > \varepsilon_y = 0.00172$ (the beam is underreinforced) **O.K**

3.5 MAXIMUM AND MINIMUM REINFORCEMENT RATIOS

Maximum Reinforcement Ratio ρ_{max}

The ACI-02 section 10.3.5 requires that the net tensile strain ε_t shall not be less than 0.004. In the previous editions of the ACI code, this limit was not stated, but was implicit in the maximum tension reinforcement ratio that was given as $\rho_{max} = 0.75\ \rho_b$. According to ACI-02, the maximum reinforcement ratio can be estimated from:

$$\frac{c_{max}}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_t} = \frac{0.003}{0.003 + 0.004} = 0.4286$$

$$a_{\max} = \beta_1\, c_{\max} = \beta_1 d\,(0.4286)$$

$$A_{s.\max} f_y = 0.85 f'_c\, b\, a_{\max}$$

$$= 0.85 f'_c\, \beta_1\, bd\,(0.4286)$$

$$A_{s.\max.} = \frac{0.364\,\beta_1 f'_c\, bd}{f_y}$$

$$\rho_{\max} = \frac{A_{s.\max}}{bd} = \frac{0.364\,\beta_1 f'_c}{f_y}$$

$$\rho_{\max} = \frac{0.364\,\beta_1 f'_c}{f_y} \tag{3.5}$$

The distance c from the top surface to the neutral axis is determined by:

$$c_{\max} = 0.43\, d \tag{3.6}$$

Example 3.5
Determine if the steel is enough to use it in the cross-section ($b = 12$ in., $d = 20.5$ in., $A_s = 6.0$ in^2., $f'_c = 4$ ksi and $f_y = 40$ ksi) as shown in Fig. 3.9.

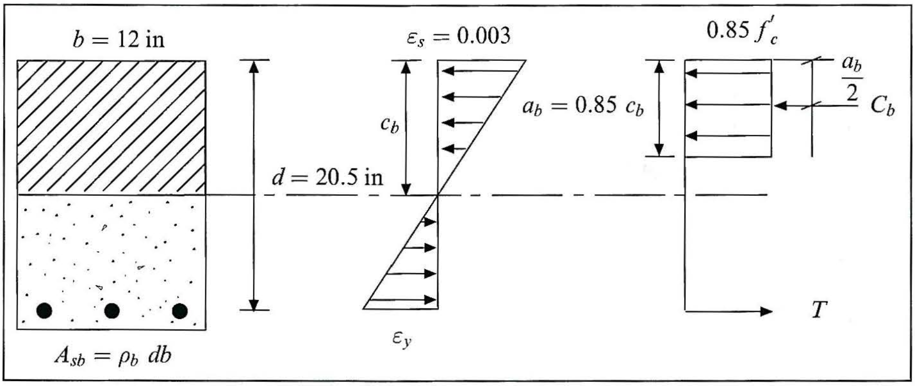


Figure 3.9

Solution.

a - Determine $\rho_{\max}$

$$\beta_1 = 0.85 \qquad \text{where } f'_c = 4 \text{ ksi}$$

$$\rho_{\max} = \frac{0.364\,\beta_1 f'_c}{f_y} = \frac{(0.364)(0.85)(4)}{(40)} = 0.031$$

$$A_{s.\max} = \rho_{\max}\, bd$$

$$A_{s.\max} = (0.031)(12)(20.5) = 7.61 \text{ in}^2$$

$$A_s = 6 \text{ in}^2 < 7.61 \text{ in}^2$$ **O.K.**

b - $0.85 f'_c\, ab = A_s f_y$

$$(0.85)(4)\, a(12) = (6)(40)$$

$$a = 5.88 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92 \text{ in}$$

$$\frac{\varepsilon_t}{0.003} = \frac{d-c}{c} = \frac{20.5-6.92}{6.92} = 1.962$$

$$\varepsilon_t = (0.003)(1.962) = 0.0059 > 0.004$$ **O.K.**

Table 3.1 Maximum reinforcement ratio $\rho_{\max}$ for tension reinforcement only (Rectangular section)

f_y (MPa)	$f'_c = 20$ MPa $\beta_1 = 0.85$	$f'_c = 25$ MPa $\beta_1 = 0.85$	$f'_c = 30$ MPa $\beta_1 = 0.85$	$f'_c = 35$ MPa $\beta_1 = 0.81$
280	0.0221	0.0276	0.0331	0.0370
350	0.0177	0.0221	0.0284	0.0296
420	0.0147	0.0184	0.0237	0.0247
f_y (kgf/cm^2)	$f'_c = 200$ kgf/cm^2 $\beta_1 = 0.85$	$f'_c = 250$ kgf/cm^2 $\beta_1 = 0.85$	$f'_c = 300$ kgf/cm^2 $\beta_1 = 0.85$	$f'_c = 350$ kgf/cm^2 $\beta_1 = 0.81$
2800	0.0221	0.0276	0.0330	0.0368
3500	0.0177	0.0221	0.0264	0.0295
4200	0.0147	0.0184	0.0220	0.0246
f_y (psi)	$f'_c = 3000$ psi $\beta_1 = 0.85$	$f'_c = 4000$ psi $\beta_1 = 0.85$	$f'_c = 5000$ psi $\beta_1 = 0.8$	$f'_c = 6000$ psi $\beta_1 = 0.75$
40000	0.0232	0.0309	0.0364	0.0410
50000	0.0186	0.0248	0.0291	0.0328
60000	0.0155	0.0206	0.0243	0.0273

Minimum Reinforcement Ratio ρ_{min}

Although the ACI code limits the minimum reinforcement ratio $\rho_{min} = 200/f_y$, this equation will not be sufficient for compressive strength f'_c and will not be greater than 5000 psi (35 MPa). For more detail about ρ_{min} (see Table 3.2).

$$\rho_{min} = \begin{cases} \dfrac{200}{f_y(\text{psi})} & \\ \dfrac{1.4}{f_y(\text{MPa})} & \text{SI} \end{cases} \tag{3.7}$$

For rectangular section. Where the minimum area of steel $A_{s.min}$ is required for tensile reinforcement, the following equation determine that for rectangular section (ACI- 10.5.1).

$$A_{s.min} = \begin{cases} \dfrac{200\, b_w d}{f_y} \leq \dfrac{3\sqrt{f'_c}}{f_y} b_w d & \\ \dfrac{1.4\, b_w d}{f_y} \leq \dfrac{\sqrt{f'_c}}{4 f_y} b_w d & \text{SI} \end{cases} \tag{3.8}$$

Where f'_c = compressive strength at 28 - day, psi (MPa)

b_w = width of web, in (mm)

d = effect depth, in (mm)

f_y = steel yield

For T-section. The ACI- 10.5.2 gives new formula for T-Section with b_w is width of the flange in tension by

$$A_{s.min} = \begin{cases} \dfrac{6\sqrt{f'_c}}{f_y} b_w d & \\ \dfrac{\sqrt{f'_c}}{2 f_y} b_w d & \text{SI} \end{cases} \tag{3.9}$$

Table 3.2 Minimum reinforcement ratio ρ_{min}

SI units		Inch - pound units	
f'_c (MPa)	$\rho_{min.}$	f'_c (psi)	$\rho_{min.}$
less than 34.5	$1.4/f_y$	less than 5000	$200/f_y$
34.5	$1.5/f_y$	5000	$215/f_y$
41.3	$1.6/f_y$	6000	$230/f_y$
48.26	$1.8/f_y$	7000	$250/f_y$
55.1	$1.95/f_y$	8000	$270/f_y$

Example 3.6
Calculate the minimum area of steel $A_{s.min}$ for the cross-section, as shown in Fig.3.10. Assume $f_y = 420$ MPa, $f'_c = 30$ MPa and $A_s = 700$ mm^2.

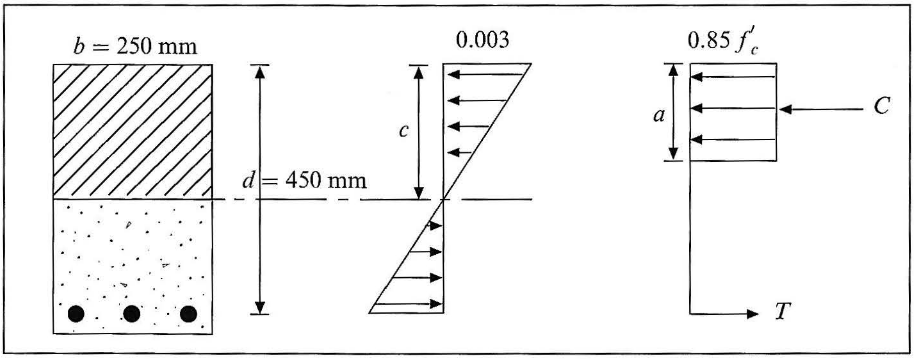


Figure 3.10

Solution.

Use $\rho_{max.} = 0.0237$ (From Table 3.1)

$$\rho_{min.} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.0033 \quad \text{(From Table 3.2)}$$

$$\rho = \frac{700}{250\ (450)} = 0.00622$$

$\rho_{min.} < \rho < \rho_{max}$ **O.K**

Minimum reinforcement from Eq. (3.8)

$$A_{s.\min.} = \frac{1.4\ b_w d}{f_y} = \frac{1.4(250)(450)}{420} = 375\ \text{mm}^2$$

$A_s = 700\ \text{mm}^2 > A_{s.\min} = 375\ \text{mm}^2$ **O.K**

3.6 CRACK CONTROL

Cracking in the reinforced concrete is resulted from the temperature change, flexural stress, the overload, the ratio of steel in the concrete and the shrinkage. The concrete exposed to higher strain that means wider opening crack, where using Grade 60 in the kind of steel. ACI code permitted an opening crack width 0.013 and 0.016 in (0.4 and 0.32 mm) and the service load steel stress is $0.60 f_y$, that result from overload factor divided by flexure strength reduction $\phi = 0.90$. On the other hand, if the opening crack reached the steel in the tension zone, the member of concrete will be in the range of deterioration by corrosion. The ACI code preparation (ACI - 10.6.4) is based on the Gergley - Lutz, and the equation for the concrete beam is:

$$w = C\ \beta\ f_s\ \sqrt[3]{d_c\ A_c} \tag{3.10.a}$$

and from Gergley-Lutz equation (3.27) is used a value of $\beta = 1.2$

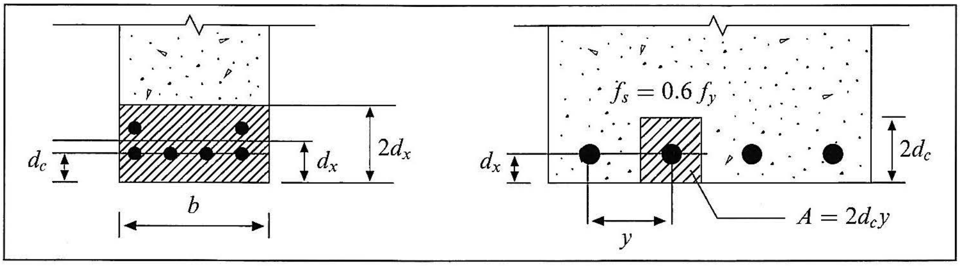


Figure 3.11

Where

w = maximum crack width at the tension fiber (mm or in).

β = distance from out-side surface to neutral axis of crack equal to 1.2 for beam and 1.3 for one-way slab.

C = experimental constant (0.076).

f_s = stress of service load in steel (MPa or ksi).

A_c = effective area in concrete under tension zone divided by number of bars A_e/m (mm^2 or in^2) where m number of steel bars.

d_c = distance from the lower fiber to the center of first layer of bars.

$$Z = \frac{w}{C\,\beta} = f_s\,\sqrt[3]{d_c\,A_c} \qquad \text{from Eq. (3.10)}$$

$$\text{Exterior } Z = \frac{13}{0.076(1.2)} = 142.54 \text{ k/in} \approx 145 \text{ k/in}$$

$$\text{Interior } Z = \frac{16}{0.076(1.2)} = 175.43 \text{ k/in} \approx 175 \text{ k/in}$$

$$Z = f_s\,\sqrt[3]{d_c\,A_c} \tag{3.10.b}$$

and $f_s = 0.6\,f_y$ (ksi) MPa

The ACI 10.6.4 limited Z is not more than 145 k/in (25.5 MN/m) for exterior exposure, and for interior exposure Z is not more than 175 k/in (30.5 MN/m), these limitations are corresponded with the maximum opening of crack.

In the ACI-02, section 10.6.4 the Z factor requirements are replaced by providing a condition for the spacing y of reinforcment closest to a surface in tension, where y shall not exceed that given by:

$$y = \frac{540}{f_s} - 2.5\,C_c \leq 12\,\left(\frac{36}{f_s}\right)$$

$$y = \frac{95000}{f_s} - 2.5\,C_c \leq 300\,\left(\frac{252}{f_s}\right) \quad \text{SI} \tag{3.11}$$

where f_s (ksi or MPa) is the reinforcement stress calculated at service load. It is permitted to take f_s as 60% of the yield strength.

For the usual case of beams with Grade 60 reinforcement and 1.5 inch clear cover to main reinforcement, with f_s = 36 ksi, the maximum bar spacing y in Fig. 3.11 is 11.25 inch.

Example 3.7

Compute the crack control Z for exterior exposure. If $f_y = 40$ ksi and 1.5 in. clear cover.

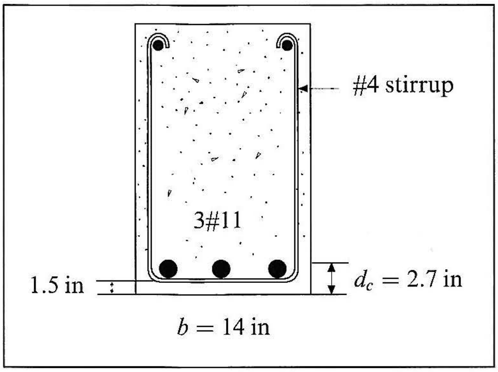


Figure 3.12

Solution.

$$Z = f_s \sqrt[3]{d_c A_c} = 0.6\,(40)\,\sqrt[3]{d_c A_c}$$

$$d_c = 1.5\,(\text{cover}) + 0.50\,(\text{stirrup}) + \frac{1}{2}(1.41)\,\#11\text{ bar} = 2.7\text{ in}$$

$$A_c = \frac{\text{area of concrete }(A_e)}{\text{number of bars (m)}} = \frac{2\,(2.7)\,14}{3} = 25.2\text{ in}^2$$

$$f_s = 0.6\,f_y = 0.6\,(40) = 24\text{ ksi}$$

and

$$Z = f_s \sqrt[3]{d_c A_c} = 24\,\sqrt[3]{2.7\,(25.2)} = 98\text{ k/in} < 145\text{ k/in} \qquad \textbf{O.K}$$

Example 3.8

For the cross-section in Fig. 3.13 determine the crack control, $f_y = 400$ MPa and the dimensions of the beam (see Fig. 3.13.)

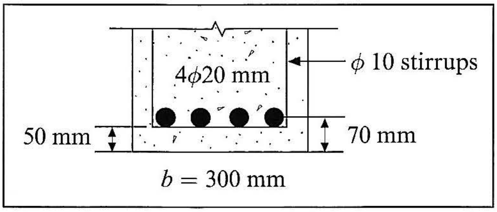


Figure 3.13

Solution.

Use Eq. (3.10.b) to solve Z by SI units

$$Z = f_s \sqrt[3]{d_c A_c}$$

$$d_c = 50 \text{ mm (cover)} + 10 \text{ mm stirrup} + \frac{1}{2}(20\text{mm}) = 70 \text{ mm}$$

$$A_c = \frac{2(70)300}{4} = 10,500 \text{ mm}^2$$

$$f_s = 0.6 f_y = 0.6\ (400) = 240 \text{ MN/m}^2$$

and

$$Z = 240 \sqrt[3]{70\ (10,500)} = 21,660 \frac{\text{MN.mm}}{\text{m}^2} = 21.66 \text{ MN/m}$$

$$Z = 21.66 \text{ MN/m} < 25.5 \text{ MN/m}$$ **O.K**

3.7 SINGLY REINFORCED BEAMS

A rectangular section beam with tension steel only is one that has been nominal strength taking into consideration, the reinforcement in the tension area. The rectangular section is also called singly reinforced section and the reinforced that place in the compression area, to increase the strength of the cross-section in that area.

The ACI 10.2.5 neglected the tensile strength in axial and flexural calculations. Thus the important dimensions in this section are depth d, width b and area of steel A_s. The depth is defined from the top surface in cross-section to the center of the layer of steel in the tension zone, as shown in Fig. 3.14 and the width is the whole width of cross-section.

The steel of area is an actual number required for cross - section. The nominal strength M_n can be expressed as follows.

$$M_n = C\ (d - \frac{a}{2}) \tag{3.12}$$

From equilibrium (Fig.3.14.):

$$C = T \tag{3.13}$$

$$0.85 f'_c\ ba = A_s f_y$$

Where M_n is the nominal moment, C is the compressive force acting on the compression area and T is the tension force acting on the tension reinforcement.

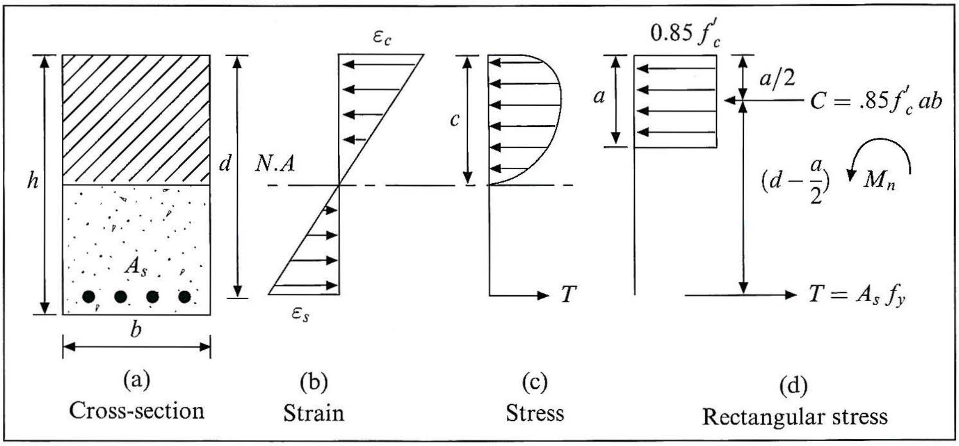


Figure 3.14 Whitney compressive stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{3.14}$$

Substitute $A_s = \rho\ bd$ and multiplying both top and bottom by d

$$a = \frac{(\rho bd) f_y (d)}{(0.85 f'_c)\ b(d)} = \frac{\rho f_y d}{0.85 f'_c} \tag{3.15}$$

From moment equilibrium:

$$M_n = A_s f_y \ (d - \frac{a}{2}) \tag{3.16}$$

Or substituting Eq. (3.13) into Eq. (3.16) to give

$$M_n = 0.85 f'_c\ ba\ (d - \frac{a}{2}) \tag{3.17}$$

When Eq. (3.15) substituted into Eq. (3.17) gives

$$M_n = 0.85 f'_c \left(\frac{\rho f_y d}{0.85 f'_c}\right) b \left(d - \frac{\rho f_y d}{2(0.85 f'_c)}\right)$$

$$= \rho f_y db \left(d - \frac{\rho f_y d}{1.7 f'_c}\right)$$

$$M_n = \rho f_y\ bd^2 \left(1 - \frac{\rho f_y}{1.7 f'_c}\right) \tag{3.18}$$

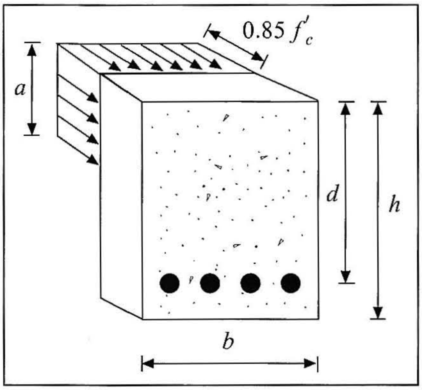
$$\phi\ M_n \geq M_u \tag{3.19}$$

Where M_u is a factored moment (required flexural strength) and $\phi\ M_n$ is designed strength where $\phi = 0.9$.

$$\phi\ M_n = \phi\,\rho\,f_y\ bd^2\left(1 - \frac{\rho\,f_y}{1.7\,f'_c}\right) \tag{3.20}$$

Where ρ should be between the maximum and the minimum range of its value

$$\rho_{\max} > \rho > \rho_{\min}$$



Example 3.9

Assuming that $b = 12$ in, $d = 20$ in, $f'_c = 4000$ psi and $f_y = 50000$ psi. Determine the nominal moment M_n.

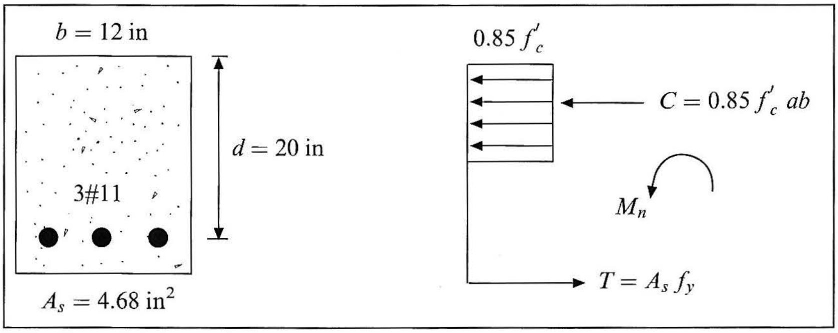


Figure 3.15

Solution.

From equilibrium equation:

$$T = C$$

Or $\quad A_s\,f_y = 0.85\,f'_c\ ba$

$$4.68\ (50) = 0.85\ (4)\ 12\ a$$

$$a = \frac{4.68(50)}{0.85\ (4)\ 12} = 5.73 \text{ in}$$

From Eq. (3.17)

$$\begin{aligned} M_n &= C\left(d - \frac{a}{2}\right) \\ &= 0.85\ (4)\ 12\ (5.73)\ \left(20 - \frac{5.73}{2}\right) \\ &= 233.8\ (17.13) = \frac{4006.1}{12} = 333.8 \text{ ft.kips} \end{aligned}$$

Example 3.10

A rectangular beam has $b = 350$ mm, $d = 550$ mm, $f_y = 350$ MPa, $f'_c = 25$ MPa and $A_s = 2640$ mm^2. Calculate the nominal moment strength M_n.

Solution.

Reinforcement ratio is:

$$\rho = \frac{A_s}{bd} = \frac{2640}{350 \times 550} = 0.0137$$

From Eq. (3.18) the nominal moment strength M_n is

$$\begin{aligned} M_n &= \rho\, f_y\, bd^2 \left(1 - \frac{\rho\, f_y}{1.7\, f'_c}\right) \\ &= 0.0137\ (0.350)\ (350)\ (550)^2 \left(1 - \frac{0.0137\ (0.350)}{1.7\ (0.025)}\right) \\ &= 507670.6\ (0.887) = \frac{450393.4}{1000} = 450 \text{ KN.m} \end{aligned}$$

By using Eq. (3.13) the M_n is:

$$2640\ (0.350) = 0.85\ (0.025)\ (350)\ \text{a}$$

$$a = \frac{2640\ (0.350)}{0.85\ (0.025)\ (350)} = 124.2 \text{ mm}$$

$$M_n = T\left(d - \frac{a}{2}\right) = 924\left(550 - \frac{124.2}{2}\right) = \frac{450419}{1000} = 450 \text{ KN.m}$$

3.8 DESIGN OF SINGLY REINFORCED BEAMS

There are two conditions of flexural failure in the design of singly reinforced beams. First, the failure occurs through yielding of tension steel. Second, the failure occurs on weakness of concrete compression zone. In section 3.3, discuss and solve the problem to find the nominal moment strength M_n, and this section should reduce the M_n by the strength factor $\phi = 0.90$ to obtain the design moment strength $\phi\ M_n$.

The reinforcement ratio ρ must be not less than $\rho_{\min}$. and not greater than $\rho_{\max}$. to obtain the area of steel A_s required for the section beam.

A rectangular beam in this design under singly reinforced must obtain the depth d and width b, also keep in mind that area of steel A_s should be between maximum area $A_{s.\max.}$ and minimum area $A_{s.\min.}$ as determined by equation (3.8) from ACI code.

For the area steel, the reinforced ratio and the design strength, must be checked during the design procedure. The following steps are required for singly reinforcement design.

1 - Select value of singly reinforcement ratio ρ, but not less than $\rho_{\min.}$ and greater than ρ_{max}.

From Eq. (3.5).

$$\rho_{\max} = \frac{0.364\ \beta_1 f_c'}{f_y}$$

Where $f_c' \leq 4$ ksi (30 MPa) $\qquad \beta_1 = 0.85$

$$4 \text{ ksi} < f_c' < 8 \text{ ksi}$$

$$\beta_1 = 0.85 - 0.05\ (f_c' \text{ ksi} - 4)$$

$$30 \text{ MPa} < f_c' < 58 \text{ MPa}$$

$$\beta_1 = 0.85 - 0.007\ (f_c' \text{ MPa} - 30)$$

$f_c' > 8$ ksi (58 MPa) $\qquad \beta_1 = 0.65$

$\rho_{\max.}$ is given in Table 3.1.

2 - Calculate the minimum depth $h_{min.}$ by using Table 3.3.

3 - From Eq. (3.20) obtain the depth d, and width b of rectangular section. After depth d has been established, add 2.5 in (65mm) from center of the first layer of steel to the fiber of section to cover the steel from fire or corrosion,

$$M_n = \rho f_y bd^2 \left(1 - \frac{\rho f_y}{1.7 f'_c}\right)$$

4 - Compute the area of steel A_s from the following equation. and it should be between the maximum and the minimum area of steel.

$$A_s = \rho\, bd$$

where ρ is computed from step 1.

5 - Check the required strength; M_u must be equal to or less than the design strength $\phi\, M_n$.

$$\phi\, M_n \geq M_u$$

6 - Check the crack control if it is not more than 145 k/in (25.5 MN/m) and 175 k/in (30.5 MN/m) for exterior exposure and interior exposure.

$$Z = f_s \sqrt[3]{d_c\, A_c} \tag{3.10.b}$$

Table 3.3 Minimum thickness of beams or one-way slabs unless deflections are computed (ACI code Table (9.5a)

Minimum Thickness, *h* (in)				
Member	Simply Support	One End Continous	Both Ends Continous	Cantliver
Solid one way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

a) length L is in inchs (m). Value should be used normal-weight with f_y = 60 ksi (414 MPa). A unit weight for concrete in the rang 90 and 120 Ib/ft^3 (1500 - 2000 kg/m^3) multiply the alue in the Table by 1.65 - 0.005w (1.65 - 0.0003w) but not less than 1.09, the w is unit weight in Ib/ft^3 (kg/m^3).

b) The value of f_y other than 60 ksi should be multiplied by (0.4 + (f_y/100,000)), $(0.4 + \frac{f_y}{690})$ SI

Example 3.11

Determine a rectangular beam size b, d and A_s that has a dead load moment $M_D = 55$ ft-kips and a live load moment $M_L = 40$ ft-kips. If $f'_c = 4000$ psi and $f_y = 50000$ psi.

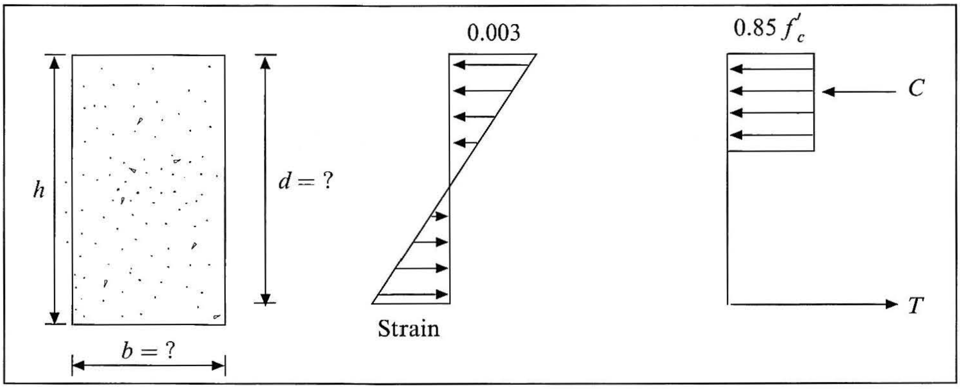


Figure 3.16

Solution.

a - Solve for ρ_b from Eq. (3.4)

$$\rho_b = \frac{0.85\ \beta_1\ f'_c}{f_y}\left(\frac{87,000}{87,000 + f_y}\right)$$

$$\rho_b = \frac{0.85\ (0.85)\ 4}{50}\left(\frac{87}{87 + 50}\right) = 0.036$$

$$\rho_{max.} = \frac{0.364\ \beta_1\ f'_c}{f_y} = \frac{0.364\ (0.85)\ (4)}{50} = 0.025$$

$$\rho_{min.} = \frac{200}{f_y} = \frac{200}{50\ 000} = 0.004 \qquad \text{From Eq. (3.7)}$$

Select value of ρ between $\rho_{min.}$ and $\rho_{max.}$

$$\rho = 0.013$$

b - Compute required moment strength M_u

$$M_u = 1.2\ M_D + 1.6\ M_L = 1.2\ (55) + 1.6\ (40) = 130 \text{ ft - kips}$$

$$M_n = M_u/\phi = 130/0.9 = 144.4 \text{ ft - kips}$$

From Eq. (3.20)

$$144.4\ (12) \quad = \rho\ f_y\ bd^2\left(1 - \frac{\rho\ f_y}{1.7\ f'_c}\right)$$

$$= (0.013)\ 50\ bd^2 \left(1 - \frac{0.013\ (50)}{1.7\ (4)}\right)$$

$$1733 = 0.587\ bd^2$$

$$bd^2 = 2952\ \text{in}^3$$

Try $b = 12$ in

$$d = \sqrt{\frac{2952}{12}} = 15.7\ \text{in} \simeq 16\ \text{in}$$

c - The required area of steel is

$$A_s = \rho\ bd = 0.013\ (12)\ (16.0) = 2.50\ \text{in}^2\ (1612.6\ \text{mm}^2)$$

From Table 2.5, use 3#9 bars, $A_s = 3$ in^2 ($4\phi 25$ mm, $A_s = 1960$ mm^2)

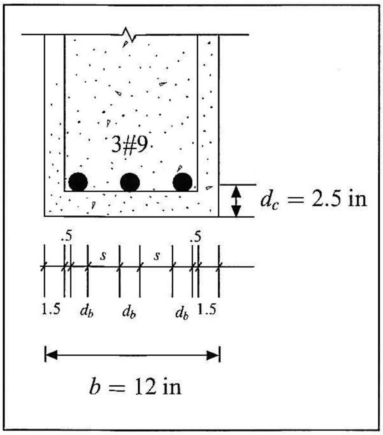
d - Total depth h is

$$h = d + 2.5\ \text{in (cover)} = 16.0 + 2.5 = 18.5\ \text{in}$$

e - Check for beam width and $s = d_b$ or 1.0 in whichever is greater

$$b = 2\ (\text{cover}) + 2\ (\#\ 4\ \text{bars stirrup}) + \sum d_b + 2\ (\text{min. bar spacing})$$

$$= 2(1.5) + 2(0.5) + 3(1.128) + 2(1.128) = 9.7\text{in} < 12\ \text{in} \quad \textbf{O.K}$$



f - Check the crack control y from Eq. (3.11)

$$y = \frac{540}{f_s} - 2.5\ C_c \leq 12\ \left(\frac{36}{f_s}\right)$$

$$C_c = 1.5\ \text{in}$$

$$f_s = 0.6\ f_y = 0.6\ (50) = 30\ \text{ksi}$$

$$y = \frac{540}{30} - 2.5\ (1.5) = 14.25 \leq 12\ \left(\frac{36}{30}\right) = 14.4 \text{ inch}$$

$$y_{\text{actual}} = \frac{1}{2}\ [b - 2\ (\text{cover}) - 2\ (\#4 \text{ bar stirrups}) - d_b]$$

$$= \frac{1}{2}\ [12 - 2\ (1.5) - 2(0.5) - 1.128] = 3.436 < 14.25$$

O.K

Example 3.12

A rectangular beam has $b = 350$ mm, $h = 650$ mm and $A_s = 2450$ mm^2 ($5\phi25$mm). Using $f'_c = 30$ MPa, $f_y = 400$ MPa, and modulus of elasticity $E_s = 200,000$ MPa. Determine the nominal moment strength and check for the maximum area of steel.

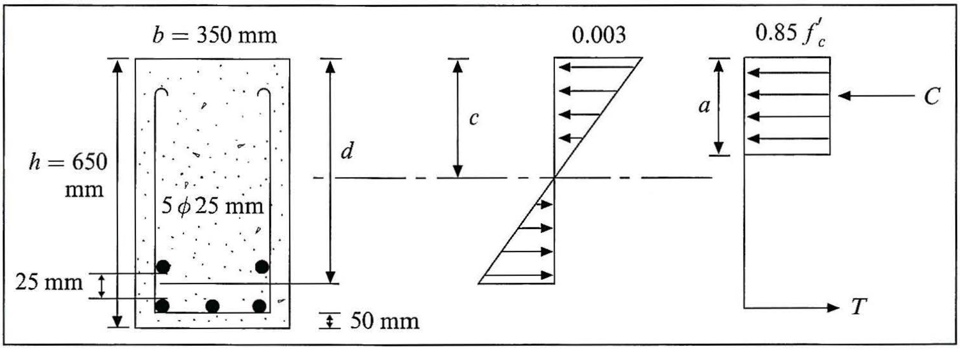


Figure 3.17

Solution

assume $\beta_1 = 0.85$ $\quad f'_c = 30$ MPa

$$\frac{c_{\max}}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_t} = \frac{0.003}{0.003 + 0.004}$$

$$c_{\max} = \frac{0.003}{0.003 + 0.004}\ d_{\text{bottom layer}}$$

$$c_{\max} = 0.4286\ d$$

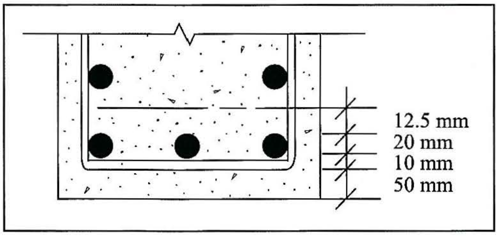
Compute d for two layers of steel

d = h - 50mm (cover) - 10mm (stirrup) - 25mm (diameter of bar) -12.5 mm (a half clear distance between two layers)

= 650 - 50 - 10 - 25 - 12.5 = 552.5 mm

Compute d for bottom layer

$$d_{\text{bottom layer}} = 650 - 50 - 10 - 12.5 = 577.5 \text{ mm}$$



$$c_{max} = 0.4286 \times 577.5 = 247.5 \text{ mm}$$

$$a_{max} = \beta_1 (c) = 0.85 \ (247.5) = 210.4 \text{ mm}$$

$$C_{max} = T_{max}$$

$$C_{max} = 0.85 f'_c \ ba_{max}$$

$$= 0.85 \ (0.03) \ (350) \ (210.4) = 1877 \text{ KN}$$

$$A_{s.max} = \frac{1877 \ (1000)}{400} = 4694 \text{ mm}^2$$

$$A_s = 2450 \text{ mm}^2 < A_{s,max.} = 4694 \text{ mm}^2 \qquad \textbf{O.K}$$

The actual nominal moment strength M_n

$$T = C$$

$$A_s f_y = 0.85 f'_c \ ba$$

$$2450 \ (0.400) = 0.85 \ (0.03) \ (350) \ a$$

$$a = \frac{980}{8.92} = 110 \text{ mm}$$

$$M_n = T\left(d - \frac{a}{2}\right) = 980 \left(0.5525 - \frac{0.110}{2}\right) = 487.5 \text{ KN.m}$$

3.9 DOUBLY REINFORCED BEAMS

Doubly reinforced beams are used for steel in the compression and tension zone in order to help necessary moment in the compression. The steel in compression also used to improve section ductility that reduces long-term deflection.

The analysis of singly reinforced beam is the same as that for doubly reinforced beam except d', A_s' and f_c', where d' is the distance from the center of the top steel to the surface of extreme fiber and A_s' is an amount of steel in the compression zone. The minimum thickness of overall depth must be satisfied with Table 3.3 to define if the deflection is concerned or not.

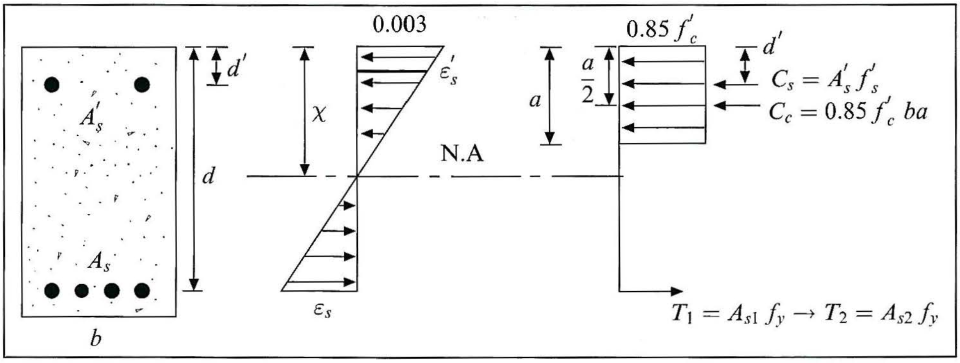


Figure 3.18 Doubly reinforced beam.

The procedure of nominal strength for doubly reinforced beam is

$$M_n = M_1 + M_2$$

Where M_n is nominal moment

$$M_1 = A_{s1} f_y \left(d - \frac{a}{2}\right) \qquad A_s = A_{s1} + A_{s2}$$

$$M_2 = A_s' f_y \left(d - d'\right) \qquad A_{s2} = A_s'$$

$$M_n = C_c \left(d - \frac{a}{2}\right) + C_s \left(d - d'\right)$$

From equilibrium equation, the total tension force is equal to the compression force.

$$T = C = C_c + C_s \tag{3.21}$$

$$C_c = 0.85 f_c' ba$$

$$C_s = A_s' f_s' = A_s'(f_s' - 0.85 f_c') \tag{3.22}$$

Where C_c is the compression force in the concrete, and C_s is the compression force in the steel.

If $C_c + C_s \neq T$, the distance χ was assumed small or large value, try to increase or decrease the distance χ until achieve the equilibrium equation correctly ($T = C_c + C_s$).

The initial assuming of χ distance, using the ratio between χ and a.

$$\chi = \frac{a}{\beta_1}$$

Figure 3.18 illustrates the strain triangle to calculate.

$$\varepsilon_s^{'} = \frac{(\chi - d')}{\chi} \, (\varepsilon_c)$$

$$\varepsilon_y = \frac{f_y}{E_s}$$

If $\begin{cases} \varepsilon_s^{'} > \varepsilon_y & \textbf{O.K} \\ \varepsilon_s^{'} < \varepsilon_y & \text{The beam does not comply with ACI code} \end{cases}$

Check area steel A_s

$$\text{Max. } A_s = A_s^{'} + \rho_{\text{max.}}\, bd$$

If $\begin{cases} A_{s,\text{max.}} \geq A_s & \textbf{O.K} \\ A_{s,\text{max.}} < A_s & \textbf{n.g} \end{cases}$

The stirrups are required to be used around the steel bars in beams.

Example 3.13

A cross - section beam has $b = 10$ in (254 mm), $d = 16$ in (406 mm), $A_s = 4.68$ in^2 (3019 mm^2), $A_s^{'} = 0.62$ in^2 (400 mm^2), $f_c^{'} = 3$ ksi (20.69 MPa) and $f_y = 50$ ksi (344.7 MPa). Calculate the nominal moment.

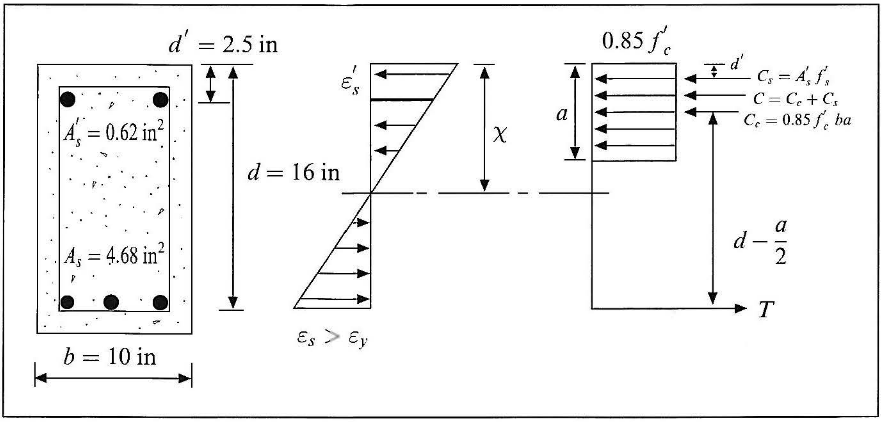


Figure 3.19

Solution.

Assume $f_s' = f_y$

From equilibrium equation

$$T = C = C_c + C_s$$
$$T = A_s f_y = 4.68\ (50) = 234 \text{ kips (1040 KN)}$$
$$234 = C_c + C_s$$
$$C_s = A_s' f_y = 0.62\ (50) = 31 \text{ kips (138 KN)}$$

Determine the compression of concrete

$$C_c = 234 - 31 = 203 \text{ kips}$$

Compute a

$$C_c = 0.85 f_c' ba = 25.5\ a$$
$$a = \frac{203}{25.5} = 8 \text{ in. (200 mm)}$$

Compute χ distance

$$\chi = \frac{a}{\beta_1} = \frac{8}{0.85} = 9.4 \text{ in. (240 mm)}$$

Determine the strain in compression steel

$$\varepsilon_s' = \frac{\chi - d'}{\chi}\ \varepsilon_c = \frac{9.4 - 2.5}{9.4}\ (0.003) = 0.0022$$
$$\varepsilon_y = \frac{f_y}{E_s} = \frac{50}{29000} = 0.0017$$
$$\varepsilon_s' > \varepsilon_y$$ **O.K**

f_s' is equal to f_y as assumed in the begining.

Compute M_n

$$M_n = M_1 + M_2$$
$$M_1 = C_c\left(d - \frac{a}{2}\right) = 203\left(16 - \frac{8}{2}\right)\frac{1}{12} = 203 \text{ ft-k}$$
$$M_2 = C_s\ (d - d') = 31\ (16 - 2.5)\ \frac{1}{12} = 34.87 \text{ ft-k}$$
$$\phi\ M_n = \phi\ (M_1 + M_2) = 0.9\ (203 + 34.87) = 214 \text{ ft-k}$$

Or, compute M_n by the following equation

$$\phi\ M_n = \phi\left(C_s\ (d - d') + C_c\left(d - \frac{a}{2}\right)\right)$$

3.10 DESIGN OF DOUBLY REINFORCED BEAMS

As mentioned previously, the section has compression and tension reinforced known as doubly reinforced beam, when the section moment exceeds the maximum moment that needs more steel bars in the compression zone.

The calculation procedure for design moment of cross-section of beam with doubly reinforced is illustrated by a reinforcement yield or does not yield at failure.

There are two types of solving the example for doubly reinforced beams. Type (1), if the reinforcement yield.

Since the compression steel is strained at its yield point assume

$$\varepsilon_s' > \varepsilon_y \quad \text{and} \quad \varepsilon_s > \varepsilon_y \qquad f_s' = f_y$$

Where ε_s', f_s' are the strain and stress in the compression steel.

$$T = C_c + C_s$$

$$C_s = A_s' f_s'$$

$$A_s = \frac{T}{f_y}$$

$$C_c = 0.85 f_c' ba$$

The obtain a value of a

$$\chi = \frac{a}{\beta_1}$$

Check to ensure the assumed value of ε_s'

$$\varepsilon_s' = \frac{\chi - d'}{\chi} (\varepsilon_c)$$

If ε_s' is greater than ε_y as assumed above

$$M_n = C_c \left(d - \frac{a}{2}\right) + C_s \, (d - d')$$

Type (2), if the reinforcement is not yield.

Assuming $\varepsilon_s' > \varepsilon_y$ and $f_s' = f_y$

Since the procedure is the same in the reinforcement yield, until check its strain to know if the strain is satisfied or unsatisfied.

$$T = C_c + C_s$$

$$C_s = A_s' f_s'$$

$$C_c = T - C_s$$

Check if ε_s' greater than ε_y

$$\varepsilon_s' = \frac{\chi - d'}{\chi}\ (\varepsilon_c)$$

If $\quad \varepsilon_s'$ less than ε_y

That means compression steel is not yield with the value of χ

Try greater value of χ and obtain a, then repeat the calculations until achieving the value of $\varepsilon_s' > \varepsilon_y$.

Then, continue the calculations to obtain A_s', A_s and M_n

Where A_{s2} is additional steel, and A_s' is comperssion steel.

$$f_s' = \varepsilon_s'\ E_s$$

Top. $\quad A_s' = \dfrac{C_s}{(f_s' - 0.85\, f_c')}$ and $T_2 = C_s$

Bot. $\quad A_{s2} = \dfrac{T_2}{f_y}$ and $A_{s1} = \dfrac{T_1}{f_y}$

$$A_s = A_{s1} + A_{s2}$$

Then, M_n is equal to:

$$M_n = C_c\left(d - \frac{a}{2}\right) + C_s\ (d - d')$$

$\phi\ M_n$ = Multiply 0.90 by value of M_n

Example 3.14

A doubly - reinforced concrete section has $b = 400$ mm, $d = 600$ mm, $f_y = 400$ MPa, $f_c' = 35$ MPa, $E_s = 200000$ MPa and the nominal moment required $M_n = 1400$ KN.m. Calculate the A_s' and A_s.

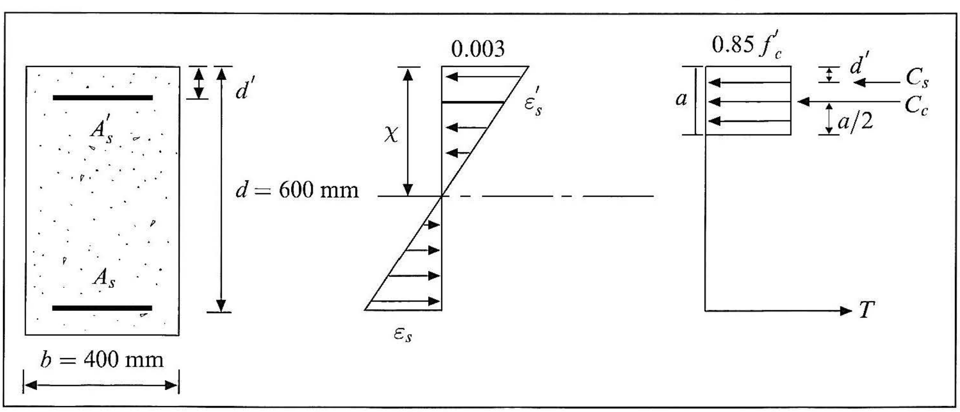


Figure 3.20

Solution.

Determine the maximum distance χ_{max}

$$\chi_{max} = \frac{0.003\ (0.600)}{0.003 + 0.004} = 0.26\ (260\ \text{mm})$$

$$a_{max} = \beta_1\ \chi_{max.} = 0.81\ (0.26) = 0.21\ \text{m}\ (210\ \text{mm})$$

$$C_c = 0.85 f'_c\ ba = 0.85\ (0.035)\ 400\ (210) = 2499\ \text{KN}$$

$$\varepsilon'_s = \frac{260 - 50}{260}\ (0.003) = 0.0024$$

Since $\varepsilon'_s > \varepsilon_y = 0.002$, compression steel yield and $f'_s = f_y$

From singly reinforced beam M_n

$$M_{n1} = C\left(d - \frac{a}{2}\right)\frac{1}{1000}$$

$$= 2499\left(600 - \frac{210}{2}\right)\frac{1}{1000} = 1237\ \text{KN.m}$$

$$M_n = M_{n1} + M_{n2}$$

$$M_{n2} = M_n - M_{n1} = 1400 - 1237 = 163.0\ \text{KN.m}$$

$$M_{n2} = C_s\ (d - d')$$

$$\text{required } C_s = \frac{M_{n2}}{(d - d')} = \frac{163\ (1000)}{600 - 50} = 296\ \text{KN}$$

$$C_s = A'_s\ (f_y - 0.85 f'_c)$$

$$A'_s = \frac{C_s}{(f_y - 0.85 f'_c)} = \frac{296}{(0.4 - 0.03)} = 800\ \text{mm}^2$$

From Table 2.6, use 4 ϕ 16 mm bars, $A'_s = 804\ \text{mm}^2$

$$T = C_c + C_s = 2499 + 296 = 2795\ \text{KN}$$

Compute for required A_s

$$A_s = \frac{T}{f_y} = \frac{2795}{0.4} = 6987.5\ \text{mm}^2$$

Use 10 ϕ 30 mm bars, $A_s = 7070\ \text{mm}^2$

Example 3.15

A rectangular reinforced beam with $f_c' = 3500$ psi, $f_y = 50000$ psi and an architect allows the dimensions of beam are $b = 13$ in, $d = 26$ in and maximum moment $M_u = 380$ ft-k. Investigating if the tension steel enough or add steel in the compression zone. If so calculate for A_s' and A_s.

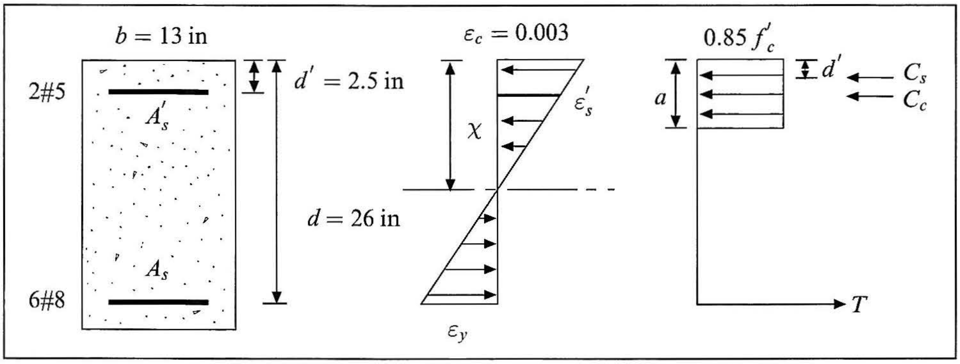


Figure 3.21

Solution. Design tension reinforcement only

From Eq. (3.2) the ρ_b is:

$$\rho_b = \frac{0.85\ (3.5)\ 0.85}{50} \left(\frac{87}{87 + 50}\right) = 0.032$$

For deflection, the ACI code limited 0.35 ρ_b

$$\rho = 0.35\ (0.032) = 0.011$$

Area steel with tension only is:

$$A_s = \rho\ bd = 0.011\ (26 \times 13) = 3.8\ \text{in}^2$$

$$T = A_s f_y = 3.8\ (50) = 190\ \text{k}$$

From equilibrium

$$T = C$$

$$190 = 0.85(3.5)13a$$

$$a = \frac{190}{38.67} = 5\ \text{in}$$

$$x = \frac{5}{0.85} = 5.88\ \text{in}$$

$$M_n = T\left(d - \frac{a}{2}\right) = 190\left(26 - \frac{5}{2}\right)\frac{1}{12} = 372 \text{ ft.k}$$

$$\phi M_n = 0.90\ (372) = 334.8 \text{ ft.k} < M_u = 380 \text{ ft.k} \qquad \textbf{n.g}$$

The section needs more strength, that means, a design section as a compression steel.

Check for strain ε_s' from the triangular, as shown in Fig. 3.21.

$$\frac{\varepsilon_s'}{\chi - 2.5} = \frac{\varepsilon_c}{\chi}$$

$$\varepsilon_s' = \frac{\varepsilon_c\ (\chi - 2.5)}{\chi} = \frac{0.003\ (5.88 - 2.5)}{5.88} = 0.0017245$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{50}{29000} = 0.0017241$$

$\varepsilon_s' > \varepsilon_y$ compression steel yield

$M_n = 380 - 334.8 = 45.2$ ft. k

Lever arm = 26 - 2.5 = 23.5 in

T (arm) $= M$

$$T_2 = \frac{45.2\ (12)}{23.5} = 23 \text{ k}$$

$$T_2 = C_s$$

$$A_{s2} = \frac{T_2}{f_y} = \frac{23}{50} = 0.46 \text{ in}^2$$

$$A_s = A_{s1} + A_{s2}$$

$$A_s = 3.8 + 0.46 = 4.26 \text{ in}^2$$

$$C_s = 23 \text{ k}$$

$$\varepsilon_s' = 0.0017245$$

$$f_s' = (0.0017245)\ (29\,000) = 50 \text{ ksi}$$

$$C_s = (f_s' - 0.85 f_c')\ A_s'$$

$$A_s' = \frac{23}{(50 - 0.85\ (3.5))} = 0.49 \text{ in}^2$$

use 2#5 bars, $A_s' = 0.62$ in^2 (compression)

and 6#8 bars, $A_s = 4.74$ in^2 (tension)

3.11 ANALYSIS OF FLANGED SECTIONS

The T, I and L - sections are used as members of reinforced concrete structures that means the beam and floor slab can act as one unit in the structure. As a result, the top portion of the flange is called T-section, and the portion under the flange or T-shape is called the stem as shown in Fig. 3.22

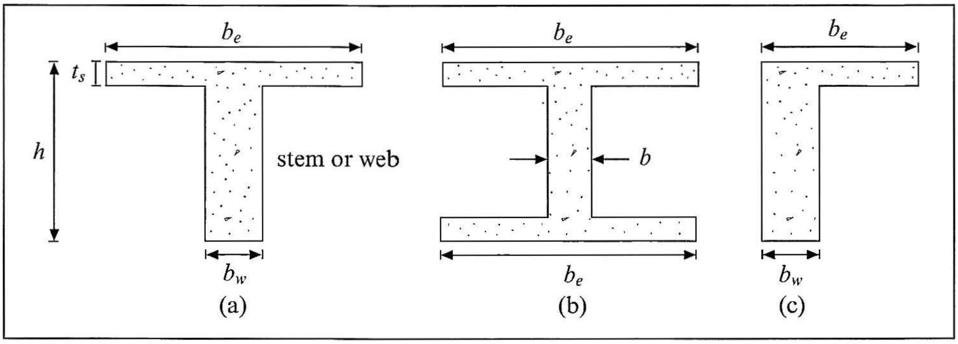


Figure 3.22 Flange section; (a) T-section, (b) I-section, (c) L-section.

The flange of T-beam has been produced by the slab thickness t_s and the b_w is the width of the stem or web that joined with T-section.
The flange of T-beam is produced from precast concrete, and used as a member of structure not only to carry a large compression force, but also to produce a large distance of the internal position that result of compression stresses closed to compression surface.

A flange is usually placed to carry enough compression to avoid brittle failure in compression zone that confirmed by a neutral axis and the depth of T-beam should be determined by a thickness of slab.

Figure 3.23a shows the locations of neutral axis that means when the neutral axis within flange thickness the section may be analyzed as a rectangular beam. If the neutral axis position is outside the flange as shown in Fig. 3.23b. Analysis as a different method.

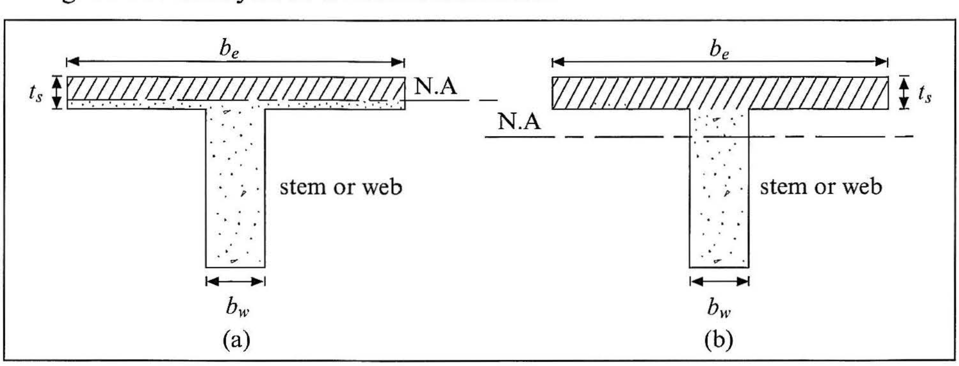


Figure 3.23 Neutral axis locations.

Effective Width b_e

The ACI code limits the effective width of T-section and L-section as the smallest of the following.

a) For T-section

$$b_e = \frac{1}{4}\text{(beam span } L\text{)}$$

$$b_e = b_w + 2\ (8)\ t_s$$

$$b_e = \text{from center to the next center of beams } (l)$$

Where t_s is the slab thickness and L is the length of the beam

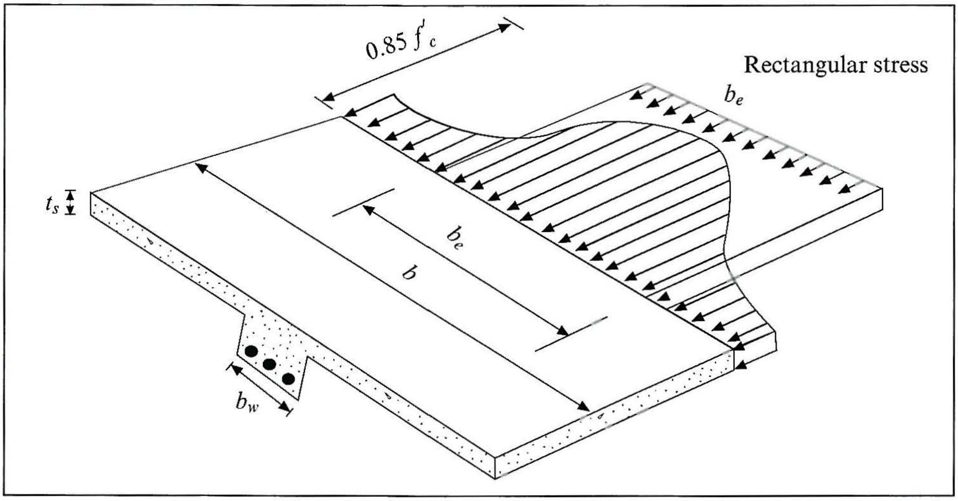


Figure 3.24 Stress distribution for T - section.

b) For L - section

The effective width b_e should be taken as the smallest of the following.

$$b_e = b_w + \frac{1}{12}\text{(beam span L)}$$

$$b_e = b_w + 6\,t_s$$

$$b_e = b_w + \frac{1}{2}\ L_{\text{clear}}$$

Where t is the slab thickness and L_{clear} is the clear distance between interior face of two beams

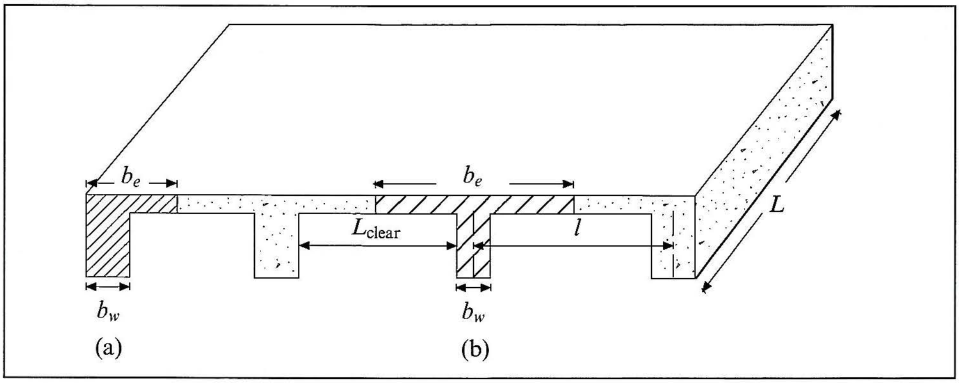


Figure 3.25 (a) L - section, (b) T - section.

The analysis of T-section, when the position of neutral axis occurs in two cases: (1) the distance is equal to or within the flange; and (2) the neutral axis is outside the flange.

Case 1: The neutral axis is equal to or less than t_s.

When the neutral axis is width t_s the section may be analyzed as singly reinforced beam and the A_s is equal to or less than:

$$A_s \leq \frac{0.85 f_c' \, b_e \, t}{f_y} \tag{3.23}$$

$$A_s f_y = 0.85 f_c' \, b_e \, t \tag{3.24}$$

Where b_e is the effective width of T-section which replaced by b in a rectangular section.

$$C = 0.85 f_c' \, b_e \, a \tag{3.25}$$

$$T = A_s f_y$$

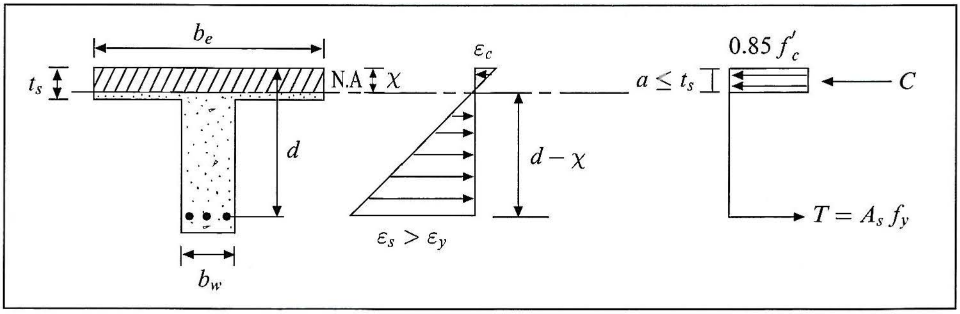


Figure 3.26 Neutral axis within the flange

Case 2: The neutral axis outside the flange.

Since the neutral axis is outside the flange, the section may be divided into two internal moments M_f and M_w with M_f resulting from the moment of the flange and M_w from the moment of the rectangular beam.

$$M_n = M_f + M_w \tag{3.26}$$

$$M_f = 0.85 f_c' A_f \left(d - \frac{t_s}{2}\right) \tag{3.27}$$

$$M_w = 0.85 f_c' A_w \left(d - \frac{a}{2}\right) \tag{3.28}$$

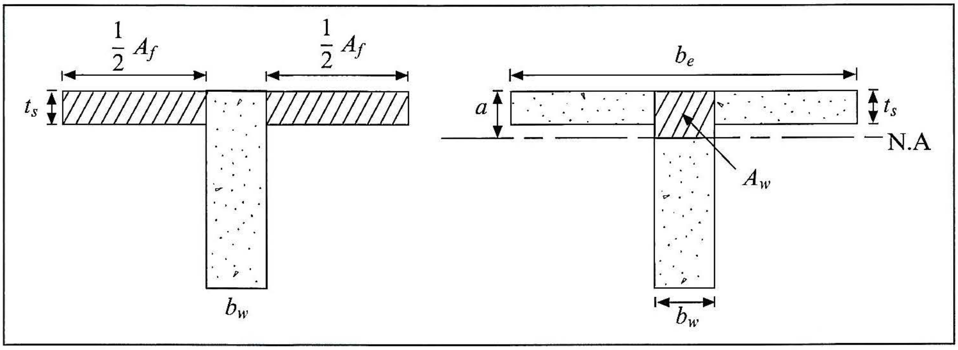
From Eq. (3.27) and (3.28), the Eq. (3.26) becomes

$$M_n = 0.85 f_c' A_f \left(d - \frac{t_s}{2}\right) + 0.85 f_c' A_w \left(d - \frac{a}{2}\right) \tag{3.29}$$

Where

$$A_w = b_w a \tag{3.30}$$

$$A_f = t_s (b_e - b_w) \tag{3.31}$$



and

$$a = \frac{T - (0.85 f_c' A_f)}{0.85 f_c' b_w} \tag{3.32}$$

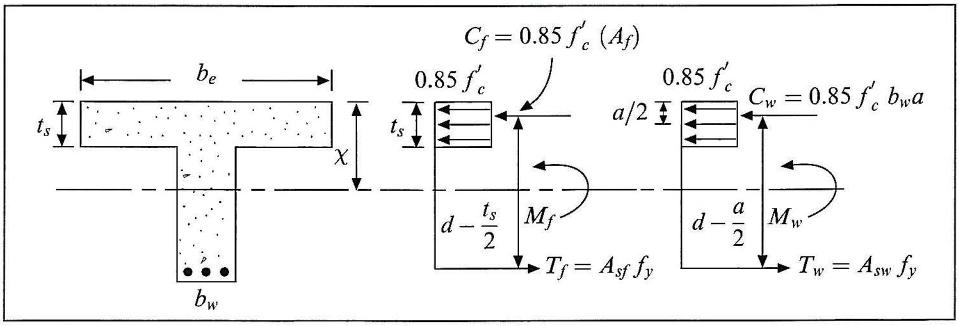


Figure 3.27 Distribution force.

3.12 DESIGN OF FLANGED SECTIONS

The design purposes for T-beam as a part of continuous beams will depend on the dimensions of the stem and flange to resist the positive moment that becomes flange in compression or resists the negative moment where the flange will not be effected. The following examples will give a clear evidence for dealing with both cases of T-section.

Example 3.16

The section shown in Fig. 3.28 which is required to design the nominal moment strength M_n of the floor system, consists of 4 in, effective depth $d = 22$ in. and the beam has a web width 12 in. Use $f'_c = 3000$ psi and $f_y = 60000$ psi. Check cracks when $Z \leq 145$ kips/in.

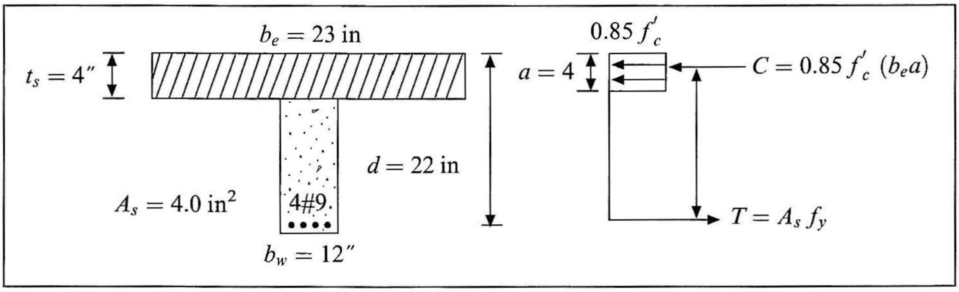


Figure 3.28

Solution. Calculate the steel tension as a rectangular section.

Assume $a = t_s = 4$ in.

$$C = T$$

$$C = 0.85 f'_c\ b_e a$$

$$= 0.85\ (3)\ 23\ (4) = 234.6\ \text{k}$$

$$A_s = \frac{(T \text{ or } C)}{f_y} = \frac{234.6}{60} = 3.91\ \text{in}^2$$

Use 4#9 bars, $A_s = 4.0$ in^2

$$T = f_y\ A_s = 60\ (4.0) = 240\ \text{k}$$

$$a = \frac{240}{0.85\ (3)\ (23)} = 4\ \text{in}$$

O.K

As a rectangular section

$$M_n = 240\left(22 - \frac{4}{2}\right)\frac{1}{12} = 400 \text{ ft-k}$$

Check the crack control $Z < 145$ kips/in

$$Z = f_s\sqrt[3]{d_c\,A_c}$$

$$d_c = 2.5 \text{ in (for one layer)}$$

$$A_c = 2\,(2.5)\,\frac{b_w}{4\ (\#\text{ of bars})}$$

$$= 5 \times \frac{12}{4} = 15 \text{ in}^2$$

$$f_s = 0.6\,(60) = 36 \text{ ksi}$$

$$Z = 36\,\sqrt[3]{2.5 \times 15} = 120.5 \text{ kips/in} < 145 \text{ kips/in} \qquad \textbf{O.K}$$

Example 3.17

The T-beam section as shown in Fig. 3.29 has $b_w = 300$ mm, $t_s = 95$ mm of slab supported by 7m span with 2.5m center to center, $d = 500$ mm, dead load moment is 85 KN.m, live load moment is 170 KN.m, $f'_c = 35$ MPa and $f_y = 400$ MPa. Determine the required area of steel A_s.

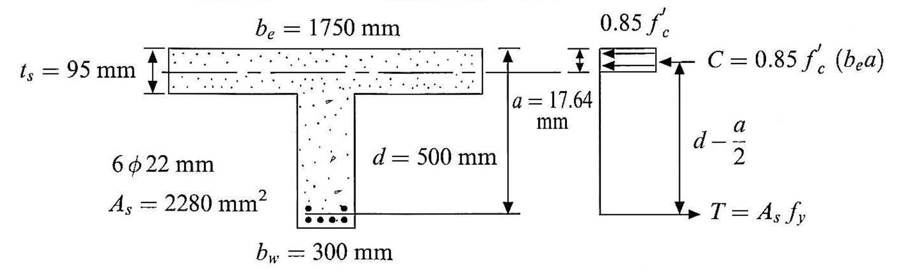


Figure 3.29

Solution. From case 1 the smallest of the effective width is

$$b_e = \frac{L}{4} = \frac{7000 \text{ mm}}{4} = 1750 \text{ mm}$$

$$M_u = 1.2\,D + 1.6\,L$$

$$= 1.2\,(85) + 1.6\,(170) = 374 \text{ KN.m}$$

$$M_n = \frac{M_u}{\phi}$$

Where $\phi = 0.9$

$$M_n = \frac{374}{0.9} = 415.6 \text{ KN.m}$$

$$M_n = T\left(d - \frac{a}{2}\right)$$

Assume $a = t_s = 95$ mm

$$M_n = T\left(500 - \frac{95}{2}\right)$$

$$T = \frac{415.6}{\left(500 - \frac{95}{2}\right)} = \frac{415.6}{(0.5 - 0.0475)} = 918.45 \text{ KN}$$

$$A_s = \frac{T}{f_y} = \frac{918.45}{0.4} = 2296 \text{ mm}^2$$

T is approximate because the value of a is assumed.

$$T = C$$

$$918.45 \text{ KN} = A_c\,(0.85)\,(0.035)$$

$$A_c = 30872 \text{ mm}^2$$

Where A_c is the area of concrete between the effective width b_e and the distance of a

$$A_c = b_e a$$

$$a = \frac{30872}{1750} = 17.64 \text{ mm}$$

Using the acual value of a to recalculate for M_n and A_s

$$M_n = T\left(d - \frac{a}{2}\right)$$

$$415.6 = T\left(0.5 - \frac{0.01764}{2}\right)$$

$$T = 846 \text{ KN}$$

$$\text{required } A_s = \frac{T}{f_y} = \frac{846}{0.4} = 2115\ \text{mm}^2$$

From Table 2.6, use 6 ϕ 22 mm

$$A_s = 2280\ \text{mm}^2$$

Example 3.18

The floor in Fig.3.30 consists of 5 in. thickness of slab, $A_s = 10.16\ \text{in}^2$, $f'_c = 3000$ psi, $f_y = 40000$ psi, $b_w = 12$ in. and $b_e = 30$ in. What is the nominal moment strength?

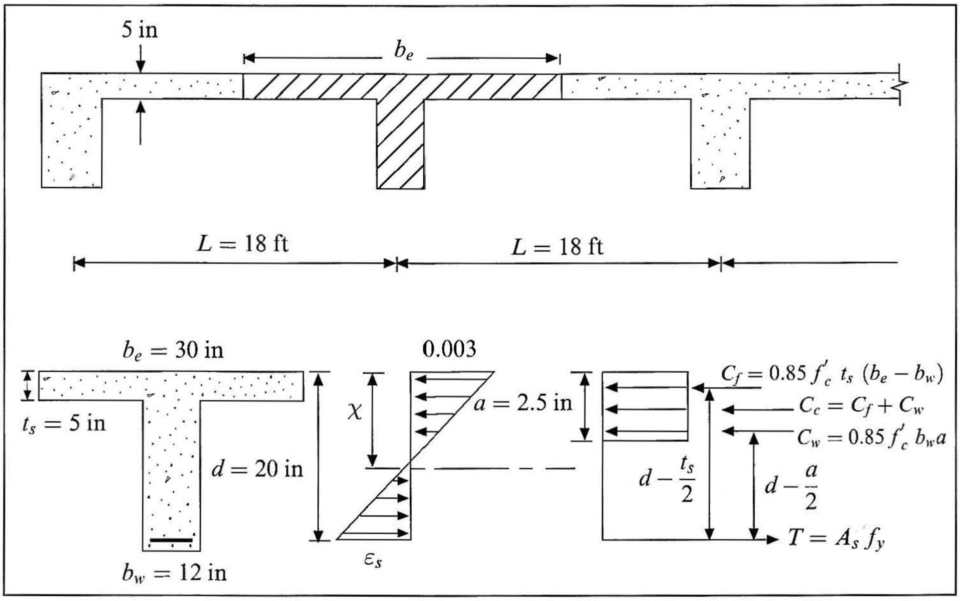
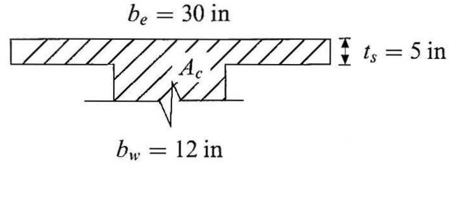


Figure 3.30

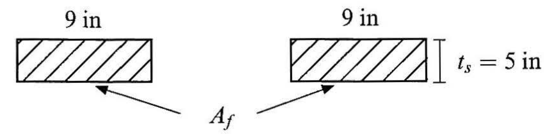
Solution. Calculate for a distance a

$$0.85 f'_c\ A_c = A_s\ f_y$$

$$A_c = \frac{10.16\ (40)}{0.85\ (3)} = 160\ \text{in}^2$$

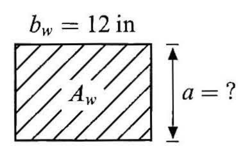


$$A_f = (30 - 12)\ t_s = 18\ (5) = 90\ \text{in}^2$$



$$A_w = A_c - A_f = 160 - 90 = 70\ \text{in}^2$$

$$a\ b_w = A_w$$



$$a = \frac{70}{12} = 5.83\ \text{in}$$

$$C_f = 0.85\, f'_c\ A_f = 0.85\ (3)\ (90) = 229.5\ \text{kips}$$

$$C_w = 0.85\, f'_c\ A_w = 0.85\ (3)\ 70 = 178.5\ \text{kips}$$

Using Eq. (3.29) to solve for M_n

$$M_n = C_f\left(d - \frac{t}{2}\right) + C_w\left(d - \frac{a}{2}\right)$$

$$= 229.5\left(20 - \frac{5}{2}\right) + 178.5\ (20 - 2.915)$$

$$= 4016.25 + 3050 = \frac{7066.25}{12} = 589\ \text{ft-k}$$

Example 3.19

An I-section beam has $f'_c = 4000$ psi, $f_y = 60000$ psi, $h = 24$ in. and other details shown in Fig 3.31. Determine the maximum area $A_{s,\max.}$ and balanced area A_{sb} according to ACI code.

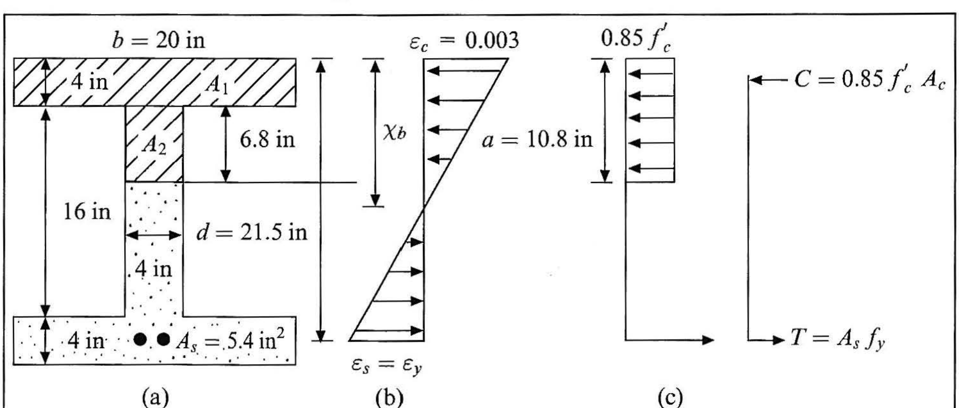


Figure 3.31 I - section beam[9].

$$\chi_b = \left(\frac{87,000}{87,000 + f_y}\right) d$$

$$= \frac{87}{87 + 60}(21.5) = 12.7 \text{ in}$$

$$a = \beta_1 \, \chi_b = 0.85 \; (12.7) = 10.8 \text{ in}$$

Determine the balanced area of steel A_{sb}

$$C_c = 0.85 \, f'_c \, A_c$$

Where $A_c = A_1 + A_2$ (as shown in Fig.3.31)

$$A_c = 20 \times 4 + 6.8 \times 4 = 107.2 \text{ in}^2$$

$$C_c = 0.85\,(4)\,107.2 = 364.5 \text{ kips}$$

$$C_c = T = A_{sb} \, f_y$$

$$A_{sb} = \frac{364.5}{60} = 6.07 \text{ in}^2$$

$$\chi_{\max} = \frac{0.003}{0.003 + 0.004} d = 0.4286 \, d$$

$$= (0.4286)\,(21.5) = 9.21 \text{ in}$$

$$a_{\max} = \beta_1 \, \chi_{\max} = 0.85 \; (9.21) = 7.83 \text{ in}$$

$$C_c = 0.85 \, f'_c \, A_c$$

$$A_c = A_1 + A_2$$

$$= 20 \times 4 + 3.83 \times 4 = 95.32 \text{ in}^2$$

$$C_c = 0.85\,(4)\,(95.32) = 324 \text{ kips}$$

$$C_c = T = A_{s.\max} \, f_y$$

$$A_{s.\max} = \frac{324}{60} = 5.4 \text{ in}^2$$

PROBLEMS

3.1 A rectangular cross - section of the beam 13 in. wide by 20 in. deep as shown in Fig. 3.1. Use $f'_c = 4$ ksi, $f_y = 50$ ksi and the beam has 14 ft length. Find the cracking moment M_{cr} and concentrated load P.

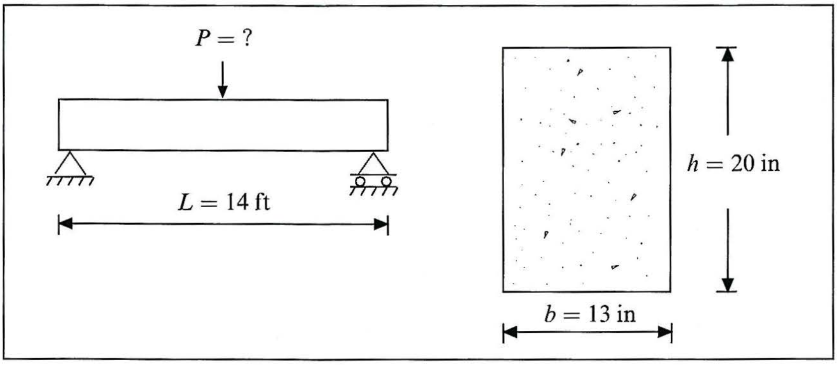


Figure P3.1

3.2 A rectangular cross-section of beam 12 in. wide by 18 in. deep as illustrated in Fig.P3.2. If $f_y = 50$ ksi, $f'_c = 3.5$ ksi and the beam has 12 ft length. Find the required value of cracking moment M_{cr} and uniform load w_u.

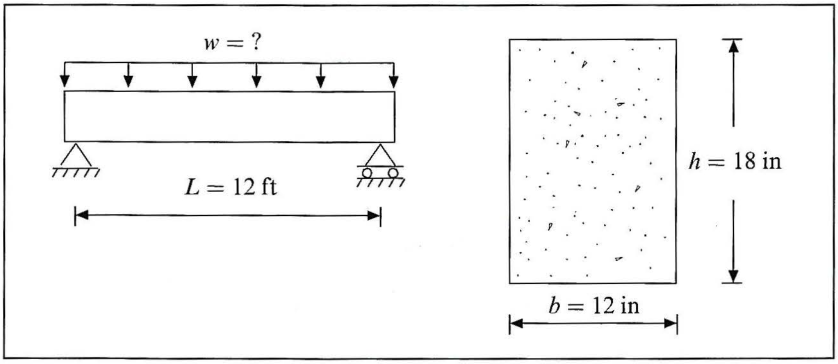


Figure P3.2

3.3 Recalculate Prob.3.1 by using SI units where $f'_c = 27.5$ MPa, $f_y = 350$ MPa, $L = 4.25$ m, $b = 330$ mm and $h = 500$ mm.

3.4 Recalculate Prob. 3.2 by using SI units where $f'_c = 25$ MPa, $f_y = 280$ MPa, $L = 3.6$ m, $b = 300$ mm and $h = 460$ mm.

3.5 Check the minimum area of steel $A_{s,\min}$. and the minimum reinforcement ratio $\rho_{\min.}$ for a rectangular cross-section of the beam as shown in Fig.P3.5. Use $f_y = 40$ ksi (280 MPa) and $f'_c = 4$ ksi (27.5 MPa).

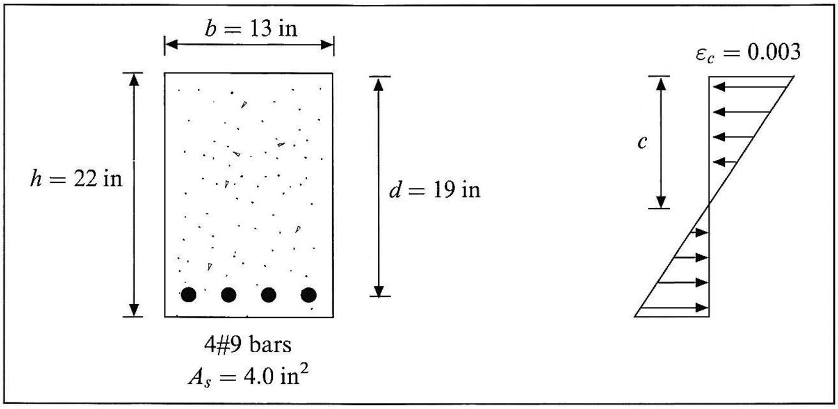


Figure P3.5

3.6 A rectangular beam has 14 in (350 mm) wide and 26 in (660 mm) deep (see Fig P3.6). If $f_y = 50$ ksi (350 MPa), $f'_c = 4$ ksi (27.5 MPa) and area of steel A_s is equal to 5.0 in^2 (3225 mm^2). Determine the maximum area of steel $A_{s,\text{max.}}$ and reinforcement ratio $\rho_{\text{max.}}$

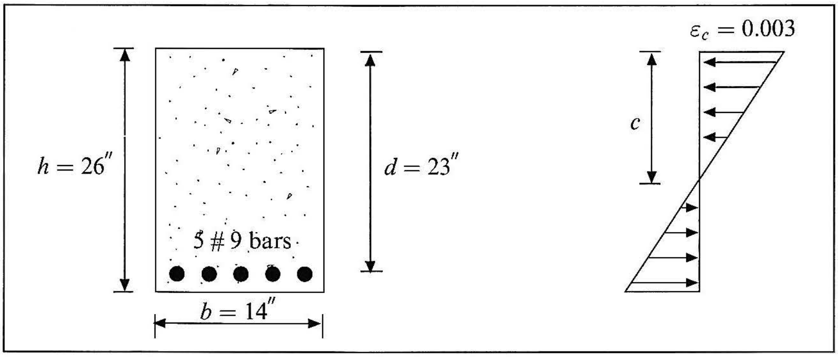


Figure P3.6

3.7 Check the minimum reinforcement ratio $\rho_{\text{min.}}$ and the minimum area of steel $A_{s,\text{min.}}$ for the cross-section of the beam, as illustrated in Fig.P3.7. Use $f_y = 45$ ksi (310 MPa), $f'_c = 3.5$ ksi (25 MPa) and $A_s = 3.0$ in^2 (1935 mm^2).

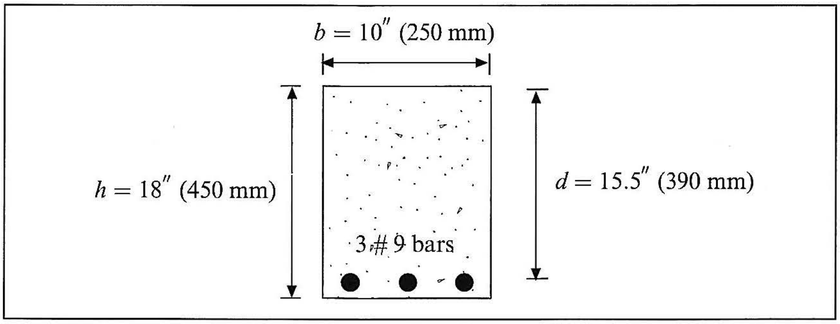


Figure P3.7

3.8 Check the crack control according to ACI code for the cross-section under exterior exposure (see Fig. P3.8). If $f'_c = 4$ ksi (27.5 MPa) and $f_y = 60$ ksi (420 MPa). Use # 3 stirrups and clear cover 1.5 in.

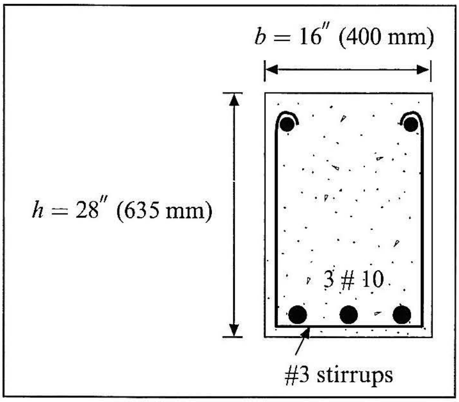


Figure P3.8

3.9 Check the crack control of the beam under exterior exposure. Use 10#8 bars, # 4 stirrups, 1.5 in. clear cover and the clear spacing between two layers is 1.0 in, $f_y = 60$ ksi and $f'_c = 3.5$ ksi.

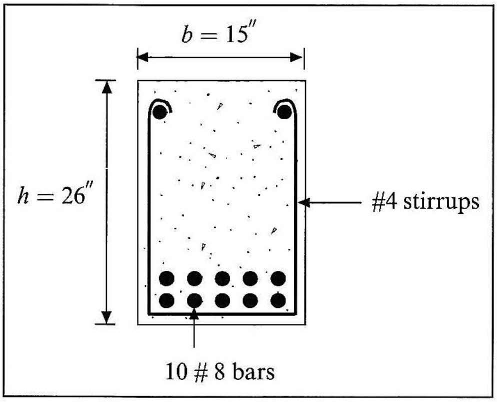


Figure P3.9

3.10 Compute the nominal moment M_n of a rectangular cross-section for each case as shown below.

Case	f_y (ksi)	f'_c (ksi)	b (in)	d (in)	Bars
1	40	3	10	17	3 # 8
2	50	4	12	19	3 # 9
3	60	5	14	22	4 # 9
4	60	4	12	18	4 # 7

3.11 Compute the nominal moment M_n of a rectangular cross-section for each case by using SI units.

Case	f_y (MPa)	f'_c (MPa)	b (mm)	d (mm)	Bars
1	280	20	250	430	3 ϕ 20 mm
2	350	27.5	300	480	4 ϕ 20 mm
3	420	35	350	560	4 ϕ 22 mm
4	420	27.5	300	450	4 ϕ 18 mm

3.12 Determine the required area of steel for a rectangular cross-section of a simply supported beam to carry uniformly distributed live and dead loads. Select reinforcement ratio between maximum and minimum ratio for each case. Check for nominal strength.

Case	b (in)	d (in)	f_y (ksi)	f'_c (ksi)	w_D (k/ft)	w_L (k/ft)	L
1	10	17	45	3	1.0	1.0	12′
2	12	19	50	4	1.25	1.2	14′
3	14	22	60	5	1.5	1.3	16′
4	16	25	60	5	2.0	1.5	18′

	b (mm)	d (mm)	f_y (MPa)	f'_c (MPa)	w_D (KN/m)	w_L (KN/m)	L (m)
5	250	430	280	20	10	10	3.6
6	300	480	350	27.5	15	12	4
7	350	560	420	35	20	15	5
8	400	650	420	35	25	20	6

3.13 Determine the required size b, d and area of steel A_s for a rectangular cross-section of a simply supported beam. Check the beam width and assume $\rho = 0.015$ for each case as following:

Case	w_D (k/ft)	w_L (k/ft)	L (ft)	f_y (ksi)	f'_c (ksi)
1	1.75	1.5	16	60	3
2	2.0	1.75	18	50	4
3	2.25	2	20	60	4.5
4	1.5	1.75	22	50	5

	w_D (KN/m)	w_L (KN/m)	L (m)	f_y (MPa)	f'_c (MPa)
5	15	10	4	350	20
6	20	15	5	420	27.5
7	25	22	6	350	35
8	30	26	7	420	35

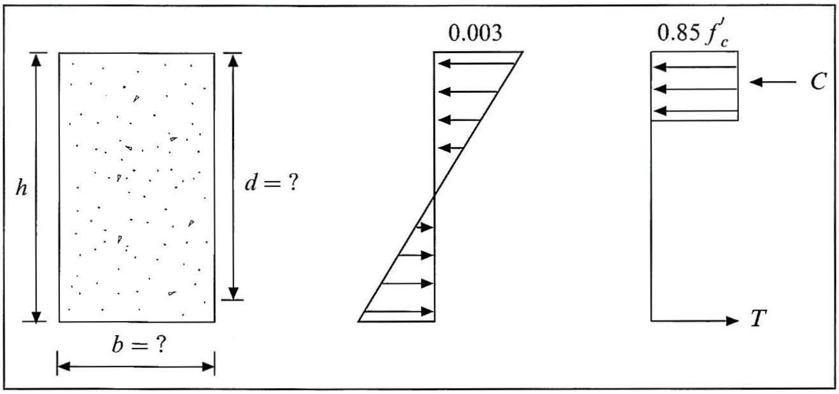


Figure P3.13

3.14 Determine the nominal moment M_n for a rectangular cross-section of the beam having both tension and compression reinforcement where $A_s = 5.08$ in^2 (3276 mm^2), $A'_s = 0.88$ in^2 (567 mm^2), $f_y = 45$ ksi (310 MPa) and $f'_c = 3.5$ ksi (25 MPa).

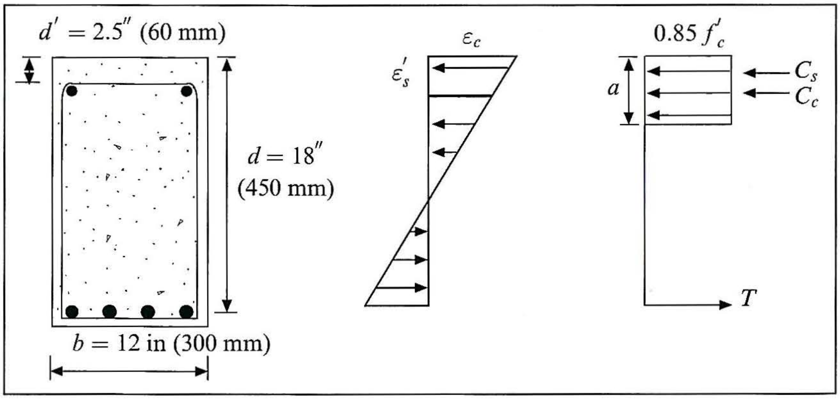


Figure P3.14

3.15 Determine the nominal moment strength M_n. If $A_s = 5.0$ in^2 (3225 mm^2), $A_s' = 1.32$ in^2 (850 mm^2), $b = 14$ in (350 mm), $d = 20$ in (500 mm) $f_c' = 4$ ksi (27.5 MPa) and $f_y = 50$ ksi (350 MPa).

3.16 What is the area of steel for tension and compression zone as shown in Fig.P3.16. If $f_y = 50$ ksi (350 MPa), $\varepsilon_c = 0.003$, $f_c' = 4$ ksi (27.5 MPa) and the nominal moment strength $M_n = 600$ ft-kips (813 KN.m).

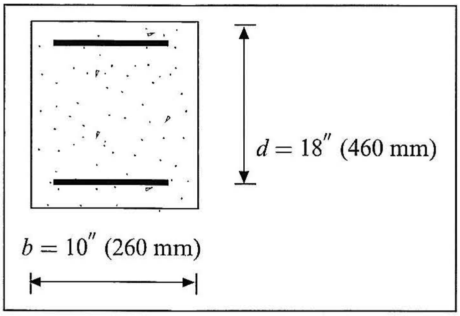


Figure P3.16

3.17 Recalculate the requirements of Prob. 3.16 and check if the compression reinforcement yield, $f_y = 60$ ksi (420 MPa) and $M_n = 330$ ft- kips (447 KN.m).

3.18 In Prob.3.16 calculate the area of steel for tension and compression zone as illustrated in Fig.P3.16. Use $f_y = 55$ ksi (380 MPa) and $M_n = 350$ ft-kips (474 KN.m).

3.19 For a rectangular beam, investigate if the tension reinforcement is adequate or add reinforcement in compression zone. If so, determine A_s' and A_s. Use $f_y = 50$ ksi (350 MPa), $\varepsilon_c = 0.003$, $f_c' = 4.5$ ksi (30 MPa) and $M_u = 290$ ft-kips (393 KN.m). For deflection, the ACI code limits 0.35 ρ_b.

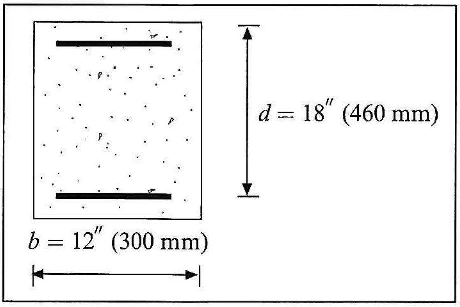


Figure P3.19

3.20 For the T-beam section shown in Fig.P3.20 it is required to determine the nominal moment strength M_n and check the crack control of the T-beam subject to the exterior exposure. Use $f_y = 60$ ksi (420 MPa), $f'_c = 3.5$ ksi (25 MPa) and $A_s = 5.08\ in^2$.

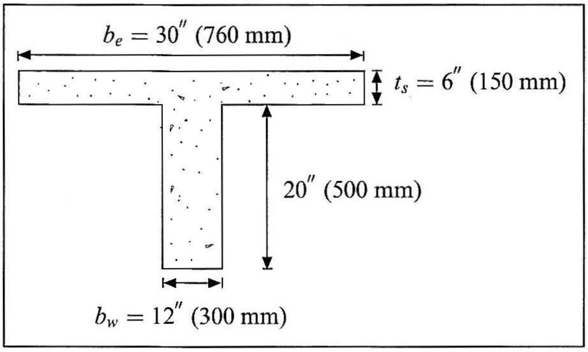


Figure P3.20

3.21 Redesign Prob.3.20 for area of steel and nominal moment M_n where $b_e = 27''$ (690 mm), $t_s = 5''$ (125 mm), $f'_c = 3$ ksi (20 MPa) and $f_y = 50$ ksi (350 MPa).

3.22 The T-beam section shown in Fig.P3.22 has $b_w = 14$ in (350 mm), $t_s = 5$ in (127 mm) of slab is supported by 10 ft (3.2 m) span with 6 ft (1.8 m) center - to - center of the beam and $M_u = 450$ ft-kips (610 KN-m). Use $f_y = 50$ ksi (350 MPa) and $f'_c = 3$ ksi (20 MPa). Determine the following.

(1) the effective width b_e

(2) area of steel A_s

(3) Check the nominal moment strength M_n

(4) Check the crack control Z

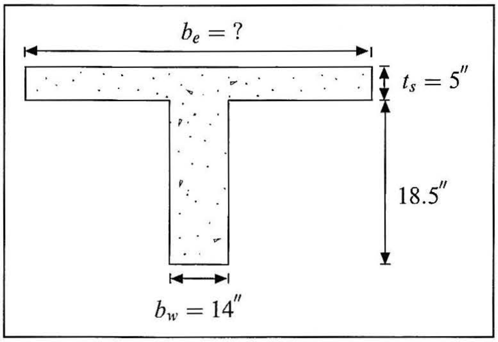


Figure P3.22

3.23 Redesign Prob.3.22 where $M_u = 350$ ft-kips (474 KN.m), $t_s = 5.5$ in. (140 mm), $L = 9$ ft (3 m) span length with 5 ft (1.6 m) center - to - center of the beam and $b_w = 15$ in (400 mm).

3.24 Redisgn Prob.3.22, if $M_u = 490$ ft-kips (664 KN.m), $t_s = 6$ in. (150 mm) $L = 12$ ft (3.6 m) span length with 6.5 ft (2 m) center - to - center of the beam and $b_w = 14$ in (360 mm).

3.25 Compute area of steel and the nominal moment strength M_n for T – beam section as illustrated in Fig.P3.25. If $f_y = 60$ ksi (420 MPa), $f'_c = 3$ ksi (20 MPa) and assume $a = 6$ in.

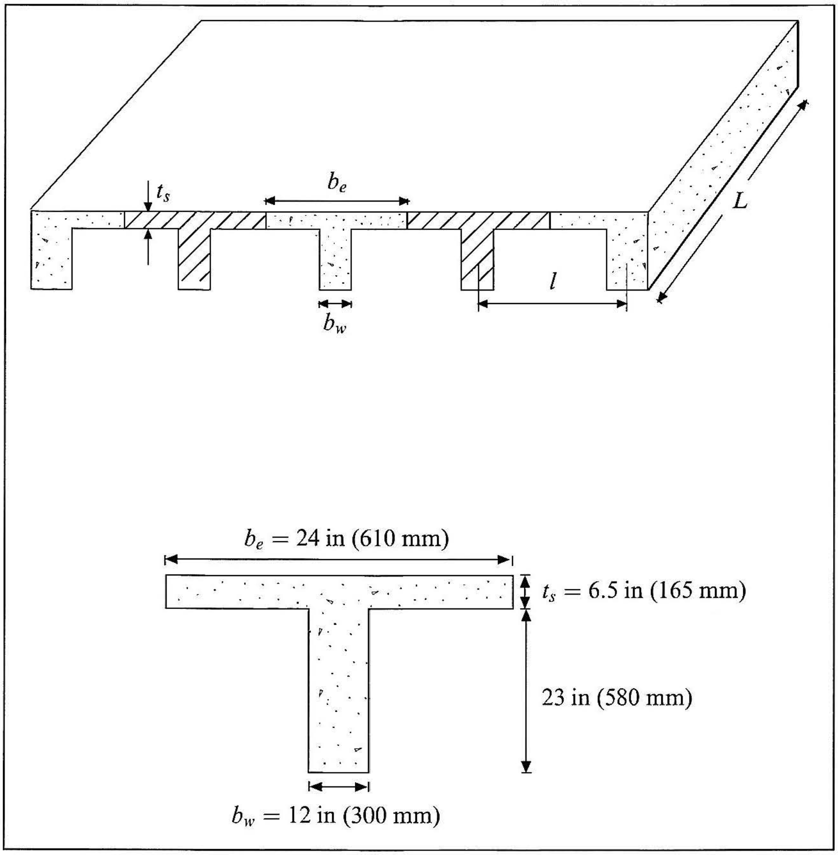
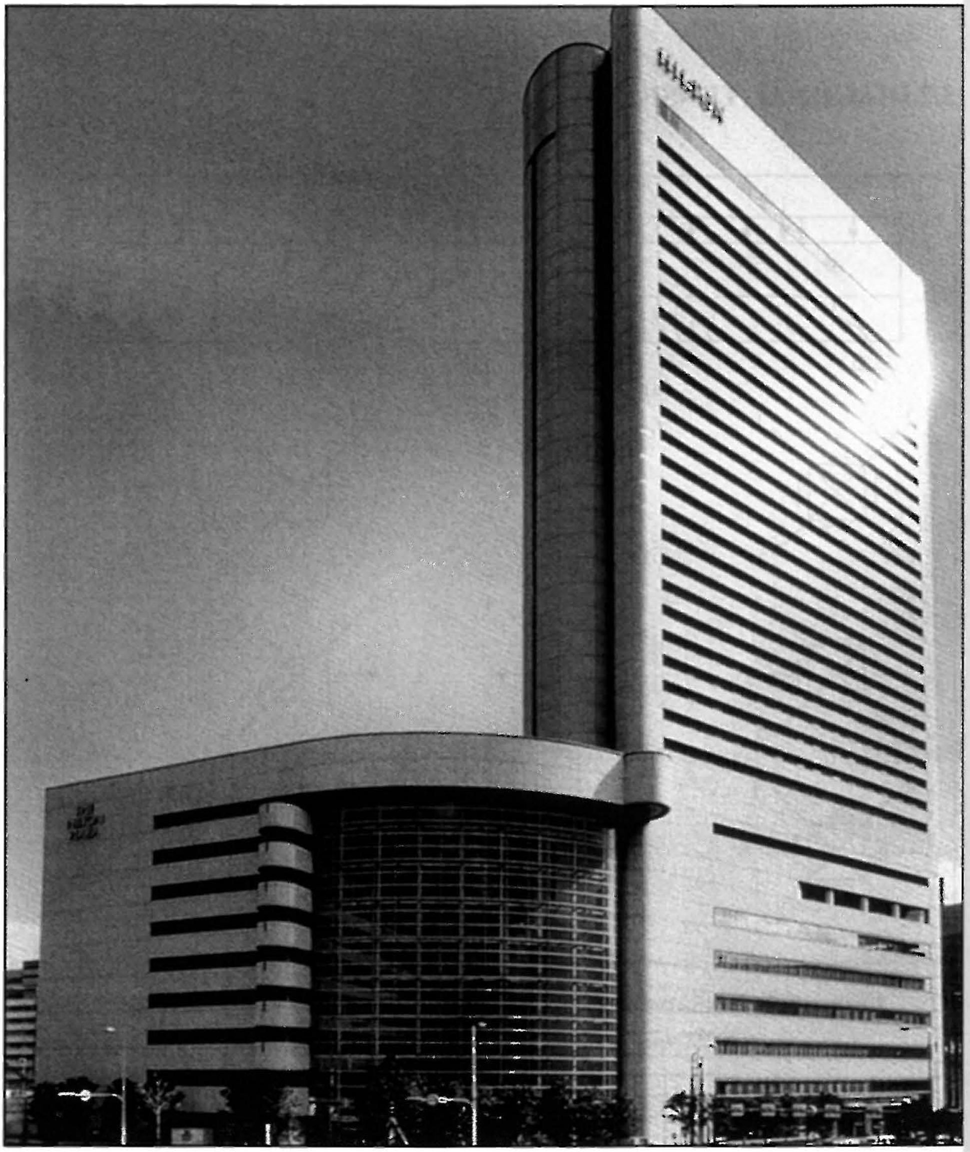


Figure P3.25

SHEAR STRENGTH 5



CHAPTER 5

5.1 INTRODUCTION

The shear strength is effected by tension and axial compression and the beam of concrete is much stronger in compression than in tension. As a result, most of failures happened namely shear failure, but the stresses created by moment are much greater than created by shear force.

The diagonal tension is effected to the shear failure from nominal flexural stress and shear stress; therefore, diagonal tension stress is more concerned than shear stress. When the moment in the beam exceeds the tensile stress, the crack will be developed at 45° that will split the concrete beam at the critical point.

5.2 DIAGONAL TENSION

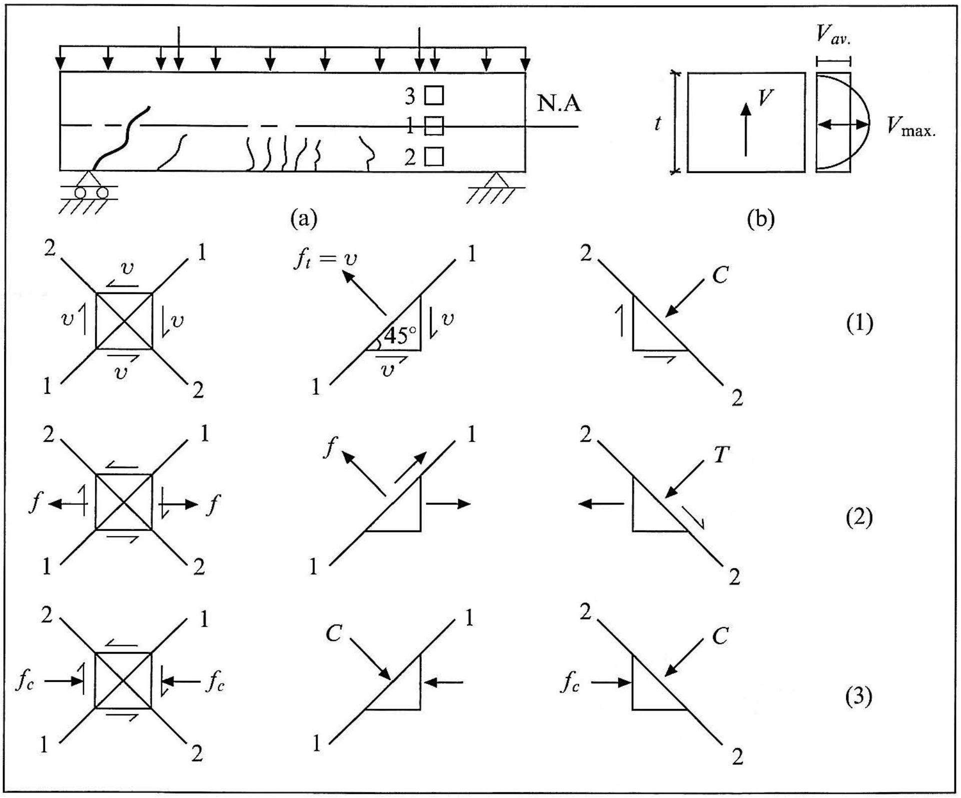


Figure 5.1 development and diagonal: (a) simply supported beam, (1) Stress at 1, (2) Stress at 2, (3) Stress at 3.

Fig.5.1a shows the diagonal cracks close to the left support and goes up with 45°, Fig.5.1 (1) shows the diagonal tension on line1-1 which decreased and the line 2-2 is increased: Fig.5.1 (2) tolerates a tension stress and Fig.5.1 (3) increases diagonal tension on line 1-1 and decreases diagonal compression on line 2-2. The tensile stress f_t in elements (1) is equal to shear v and effect at 45°.

Figure 5.1b shows shear stress on cross-section and relates between maximum shear stress and average stress. The shear stress is:

$$v = \frac{V\,M_\chi}{I_\chi\,b} \tag{5.1}$$

Where

b = the width of cross-section where shear stress is required.

I_χ = moment of inertia about χ -axis.

V = shear force at section required.

M_χ = moment of area over the required level.

v = shear stress at required section.

The ACI Code is used shear stress by dividing V by $b_w d$ simply by:

$$v = \frac{V}{b_w d} \tag{5.2}$$

Where b_w is width of a rectangular section.

Fig.5.1 (2) illustrates the element two that located below neutral axis, and tensile stress f combines with shear stress, the tensile stress is:

$$f = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + v^2} \tag{5.3}$$

and their maximum slope tension is:

$$\tan 2\,\Psi = \frac{2v}{f}$$

Where

f = principal tensile stress

Ψ = angle of f

5.3 BEAM BEHAVIOR

The primary concern with beam behavior under loading, that divides the beam for two parts. The first part, which may take an upper place of neutral axis of the beam that exposed to compression, and the second part takes lower place of neutral axis of the beam to carry tension.

It is possible to know that the opening cracks will happen in the lower part of the beam (Fig.5.2).

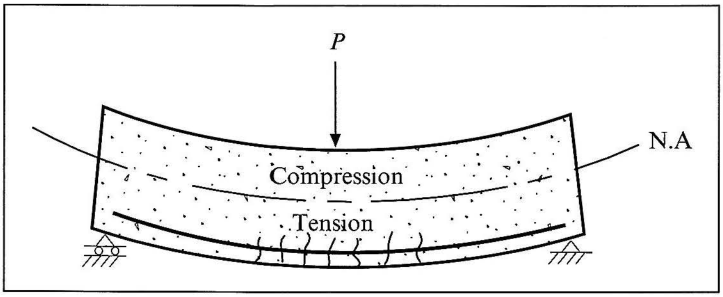


Figure 5.2 Tension and compression zone.

Fig.5.2 illustrates the flexural - shear cracks occurred between load and support that caused by load and also caused along diagonal crack during the beam loaded, but the beam can carry extra load in region of uncracked concrete.

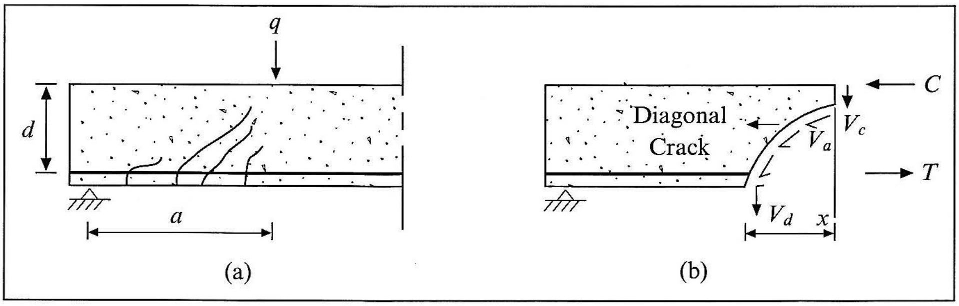


Figure 5.3 Diagonal tension cracks.

Fig.5.3b illustrates a free-body diagram from the main diagonal crack and internal force that created from loaded. This load should be equal to shear resistance V_c that created from compression part, and dowel force V_d created from bars in tension part to dowel action. The V_a is aggregate interlock. Furthermore, shear force between section x is balanced by dowel action V_d, Aggregate interlock V_a and shear resistance.

The beam failures that may occur, depend on the relation between a and b as following types;

a - Diagonal tension failure is occurred far from support and applied load. This failure is happened when the distance a is greater than $4d$ (Fig.5.4).

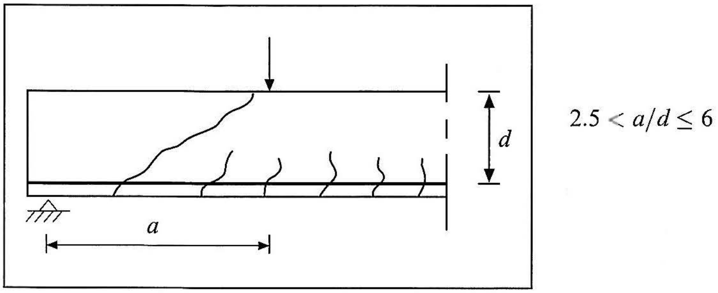


Figure 5.4 Diagonal failure.

b - Compression failure is a/d greater than or equal to 1 and smaller than or equal 2.5. That occurs when the distance of a is smaller than $4\,d$. The diagonal crack will extend until it reaches the load. Before that, the beam remains carrying more load point until the crushing failure will occur. This failure is known as shear-compression failure (Fig.5.5).

$$1 \leq a/d \leq 2.5$$

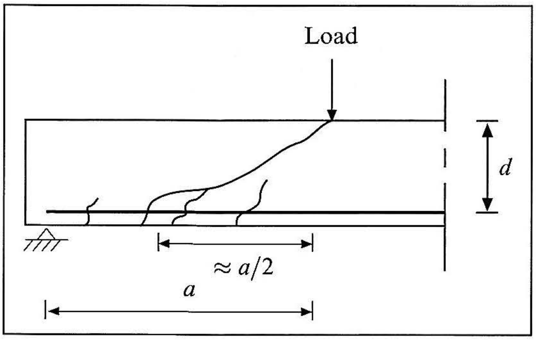


Figure 5.5 Compression failure.

c - Figure 5.6 shows diagonal crack between the support and load. The a/d is smaller than or equal to 1. That failure happens with deep beam, when the bar splitting before the shear compression happens and it is known as shear tension failure (Fig.5.6).

$$a/d \leq 1$$

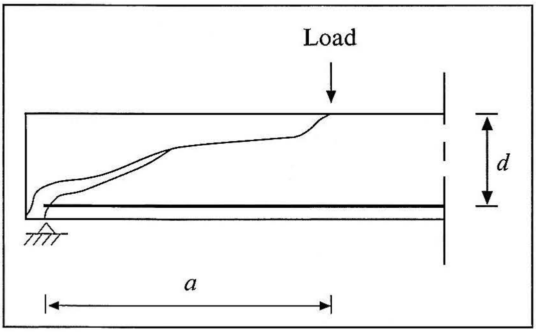


Figure 5.6 Shear-tension failure.

d - This failure is called flexure failure and the a/d is greater than 6. That will happen when the span of the beam is long and the depth is small. When the vertical crack reaches at the zone of a maximum moment and the crack will be between support of beam and a maximum moment.

$$6 < a/d$$

5.4 SHEAR STRENGTH WITHOUT STIRRUPS

It is assumed that the shear failure is happened in reinforced concrete beam with no shear reinforcement, only at the moment when the beam loading to the shear failure happened; but to achieve enough warning before the beam crushing, the minimum shear reinforcement is required. The shear strength in the beam occurs when the load creates the diagonal crack as mentioned early. The ACI code uses the cross-section area to express the nominal shear stress as:

$$v = \frac{V_c}{b_w d} \tag{5.4}$$

Where b_w is the width of beam web, V_c the nominal shear strength and v is the shear stress.

The ACI code defines equations for shear strength effected to cross-section of beam that under flexure and shear by:

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w\,\frac{V_u d}{M_u}\right) b_w d \leq 3.5\sqrt{f'_c}\,b_w d \tag{5.5}$$

$$V_c = \left(\sqrt{f'_c} + 120\ \rho_w\ \frac{V_u d}{M_u}\right) \frac{b_w d}{7} \leq 0.3\ \sqrt{f'_c}\ b_w d \qquad \text{SI} \qquad (5.6)$$

For $\frac{V_u d}{M_u}$ should be taken less than 1.0

Where

M_u = factored moment at cross-section

V_u = factored shear force at cross-section

d = depth of section

b_w = effective web width of beam

$\rho_w = \frac{A_s}{b_w d}$ (reinforcement ratio)

If the Eq. (5.5) is exceed $3.5\ \sqrt{f'_c}\ b_w d$ and $\frac{V_u d}{M_u}$ is not smaller than 1.0, the following equations will be used:

$$V_c = 2\ \sqrt{f'_c}\ b_w d \qquad \text{inch-pound} \qquad (5.7)$$

$$V_c = 0.166\ \sqrt{f'_c}\ b_w d \qquad \text{SI} \qquad (5.8)$$

Rajaopalan and Fergusan[6] suggest to use the following equations, when the reinforcement ratio ρ_w is less than 0.012.

$$V_c = (0.8 + 100\ \rho_w)\ \sqrt{f'_c}\ b_w d \leq 2\ \sqrt{f'_c}\ b_w d \qquad \text{Inch-pound} \qquad (5.9)$$

$$V_c = (0.07 + 8.3\ \rho_w)\ \sqrt{f'_c}\ b_w d \leq 0.166\ \sqrt{f'_c}\ b_w d \qquad \text{SI} \qquad (5.10)$$

If the beam is exposed to axial compression force, the shear strength V_c is given by:

$$V_c = 2\left(1 + \frac{N_u}{2000\ A_g}\right) \sqrt{f'_c}\ b_w d \qquad (5.11)$$

If the beam is exposed to axial tension force, V_c is given by:

$$V_c = 2\left(1 + \frac{N_u}{500\ A_g}\right) \sqrt{f'_c}\ b_w d \qquad (5.12)$$

The value of $\frac{N_u}{A_g}$ should be taken in psi

If the cross-section of beam is in axial compression, the ACI 11.3.2.2 is permitted to substitute Eq. (5.5) for M_u by M_m when the value of $\frac{V_u d}{M_u}$ is greater than 1.0, and value of V_c in Eq. (5.13) is not greater than V_c in Eq.(5.15).

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w\,\frac{V_u d}{M_u}\right) b_w d \tag{5.13}$$

$$M_m = M_u - N_u\,\frac{(4h - d)}{8} \tag{5.14}$$

$$V_c = 3.5\sqrt{f'_c}\,b_w d\sqrt{1 + \frac{N_u d}{500\,A_g}} \tag{5.15}$$

Where

N_u = axial force, pound

A_g = gross area, in^2.

f'_c = compression strength

h = whole depth of beam

Lightweight concrete

All the equations above use the value of shear strength V_c for normal weight concrete, but in this section, the shear strength is used for lightweight concrete, the $\sqrt{f'_c}$ is replaced by $\frac{f_{ct}}{6.7}$ and value of $\frac{f_{ct}}{6.7}$ should be less than $\sqrt{f'_c}$, or multiplied Eq. (5.5) by 0.75 to become:

$$V_c = \left[0.75\,(1.9\sqrt{f'_c}) + 2500\,\rho_w\,\frac{V_u d}{M_n}\right] b_w d \le 0.75\,(3.5)\sqrt{f'_c}\,b_w d \tag{5.16}$$

$$V_c = \left[0.75\sqrt{f'_c} + 120\,\rho_w\,\frac{V_u d}{M_n}\right]\frac{b_w d}{7} \le 0.75\,(0.3)\sqrt{f'_c}\,b_w d \quad \text{SI} \tag{5.17}$$

For "Sand-lightweight" concrete is multiplied both ends of equations by 0.85 to become:

$$V_c = \left[0.85\,(1.9\sqrt{f'_c}) + 2500\,\rho_w\,\frac{V_u d}{M_n}\right] b_w d \le 0.85\,(3.5)\sqrt{f'_c}\,b_w d \tag{5.18}$$

$$V_c = \left[0.85\sqrt{f'_c} + 120\,\rho_w\,\frac{V_u d}{M_n}\right]\frac{b_w d}{7} \le 0.85\,(0.3)\sqrt{f'_c}\,b_w d \quad \text{SI} \tag{5.19}$$

Example 5.1

For a simply supported beam shown in Fig. 5.7 has $f'_c = 3000$ psi, $f_y = 50,000$ psi and $A_s = 2$ in^2, with uniform dead load of 4 kips/ft and live load of 6.24 kips/ft. Calculate the shear strength V_c where $M_u = 250$ ft-kips.

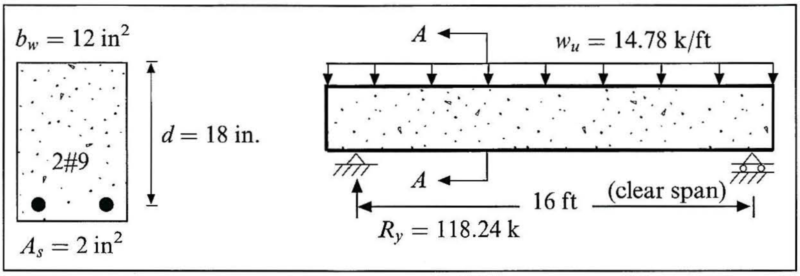


Figure 5.7

Solution.

a - Determine V_c

$$w_u = 1.2\ w_d + 1.6\ w_l$$

$$= 1.2\,(4) + 1.6\,(6.24) = 14.78 \text{ kips/ft}$$

Calculate V_u at the support when $d = 18$ in.

$$V_u = \frac{14.78 \times 16}{2} - 14.78\left(\frac{18}{12}\right) = 96.07 \text{ kips}$$

From Eq. (5.5) the value of V_c is:

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w\,\frac{V_u d}{M_u}\right) b_w d$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{2}{12\,(18)} = 0.0092$$

$$\frac{V_u d}{M_u} = \frac{96.07\,(18)}{250\,(12)} = 0.576 < 1.0 \qquad \textbf{O.K}$$

$$V_c = \left[1.9\sqrt{3000} + 2500\,(0.0092)\,(0.576)\right]\frac{12 \times 18}{1000}$$

$$= 25.34 \text{ kips} < 3.5\sqrt{f'_c}\,b_w d$$

$$3.5\ \sqrt{3000}\ (12 \times 18)\ \frac{1}{1000} = 41.4 \text{ kips}$$

$$V_c = 25.34 \text{ kips} < 41.4 \text{ kips} \qquad \textbf{O.K}$$

b - Caculate V_c by using SI units, when $f'_c = 21$ MPa (3 ksi), $f_y = 344.7$ MPa (50 ksi), $b_w = 304.8$ mm (12 in) and $d = 457.2$ mm (18 in).

From Eq. (5.8) V_c is:

$$V_c = 0.166\ \sqrt{f'_c}\ b_w d = 0.166\ \sqrt{21}\ (304.8 \times 457.2)$$

$$= 106000 \text{ N} = 106.0 \text{ KN } (23.83 \text{ kips})$$

c - Determine V_c for "Sand-lightweight" concrete by using Eq. (5.18)

$$V_c = \left[0.85\left(1.9\ \sqrt{f'_c}\right) + 2500\ \rho_w\ \frac{V_u d}{M_u}\right]\ b_w d$$

$$= 22 \text{ kips} < 35.2 \text{ kips} \qquad \textbf{O.K}$$

Example 5.2

A rectangular beam in Example 5.1 has, $A_s = 4.0$ in^2 and the beam subject to axial tension force with $N_d = -5$ kips and $N_l = -8.6$ kips. Determine shear strength V_c.

Solution.

a - Determine factored loads for tension and V_c.

$$N_u = 1.2\ (-5) + 1.6\ (-8.6) = -19.76 \text{ kips}$$

When the beam has tension force use Eq. (5.12)

$$V_c = 2\left(1 + \frac{N_u}{500\ A_g}\right)\ \sqrt{f'_c}\ b_w d$$

$$= 2\left(1 + \frac{(-19,760)}{500\ (20.5 \times 12)}\right)\ \sqrt{3000}\ (12 \times 18)\ \frac{1}{1000} = 19.86 \text{ kips}$$

b - Compute shear strength V_c by using $N_u = 19.76$ kips in compression force, From Eq. (5.11), the V_c is:

$$V_c = 2\left(1 + \frac{N_u}{2000\ A_g}\right)\ \sqrt{f'_c}\ b_w d$$

$$= 2\left(1 + \frac{19,760}{2000\ (12 \times 20.5)}\right)\ \sqrt{3000}\ (12 \times 18)\ \frac{1}{1000} = 24.61 \text{ kips}$$

5.5 SHEAR STRENGTH WITH STIRRUPS

If the shear force is greater than the shear strength of concrete, the stirrups are necessary to cover the area of steel around all the bars in cross-section, as shown in Fig.5.8: That will prevent diagonal cracks to occur or to growth. The most common types of bars size are no. 3 and no.4 (ϕ 8 and ϕ 10 mm), and the common spacing of stirrups is 4 in (100 mm), but at the both ends of the beam, the distance will be closer, because its critical section exists at $\frac{1}{2}d$ of the beam.

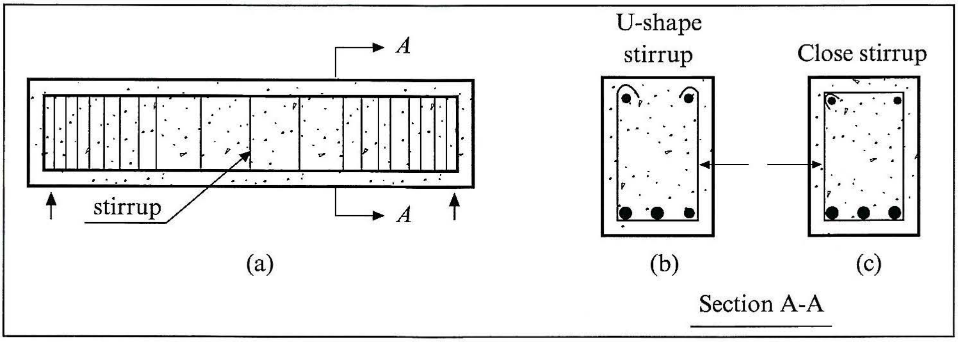


Figure 5.8 Types of stirrups.

Fig.5.8b and c show two types of stirrups. (1) U-shape is around tension bars and hooked with bars at compression zone. (2) Closed stirrup is around all bars and hooked at around one of the bar that located at compression zone.

ACI code specifies design shear $\phi\ V_n$ must be greater than or equal to shear force V_u that is:

$$\phi\ V_n \geq V_u \tag{5.20}$$

Where

V_n = nominal shear strength of the cross section.

ϕ = reduction factor 0.75

If the shear reinforcement is required, the nominal shear strength becomes:

$$V_n = V_c + V_s \tag{5.21}$$

Where

V_c = shear strength of the concrete

V_s = shear reinforcement

Substituted Eq. (5.21) into Eq. (5.20) to become:

$$\phi\ (V_c + V_s) \geq V_u \tag{5.22}$$

If the gravity load is used for shear strength V_u, the Eq. (5.21) is determined by:

$$V_u = 1.2\ V_d + 1.6\ V_l \tag{5.23}$$

5.6 INCLINED AND VERTICAL STIRRUPS

Inclined stirrups

The inclined stirrups are assumed that the diagonal crack passes through the vertical stirrups from the tension zone to the top of compression zone in the 45° (Fig.5.9). As a result, the diagonal crack is passed through two legs of stirrups. That means, the area of the stirrups A_v includes two leg for U-shaped or closed stirrup.

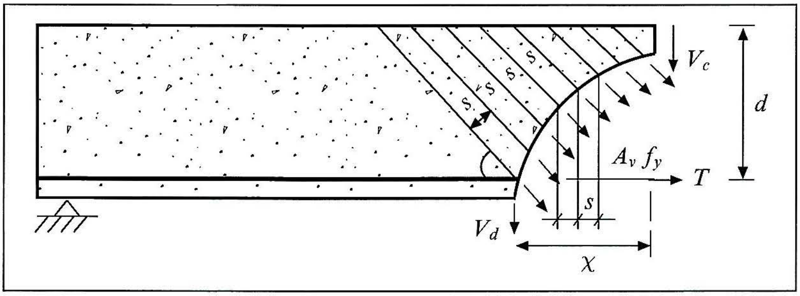


Figure 5.9 diagonal crack with inclined stirrups.

For inclined stirrups, the Eq. (5.21) is computed by:

$$\begin{aligned} V_s = A_v\, f_y\ \sin\theta &\leq 3\ \sqrt{f'_c}\ b_w d \\ &\leq 0.249\ \sqrt{f'_c}\ b_w d \qquad \text{SI} \end{aligned} \tag{5.24}$$

Thus

$$V_s = n_t\ A_v\, f_y\ \sin\theta \tag{5.25}$$

Where n_t, the total number of inclined stirrups, shear reinforcement is crossing with an angle θ and inclined crack is crossing with an angle 45°. The distance of d includes, the number n_t of stirrups through this distance are:

$$n = \frac{\chi}{s}(1 + \cot\theta) \tag{5.26}$$

$$\chi = d$$

$$V_s = \frac{d\,(1 + \cot 45 \;\tan\theta)}{s}\,(A_v\, f_y\; \sin\theta)$$

$$V_s = \frac{d\, A_v\, f_y\, (\sin\theta + \cos\theta)}{s} \tag{5.27}$$

Vertical stirrups

The vertical stirrups are perpendicular to the length of the beam or member and an angle θ is equal to 90° (Fig.5.10).

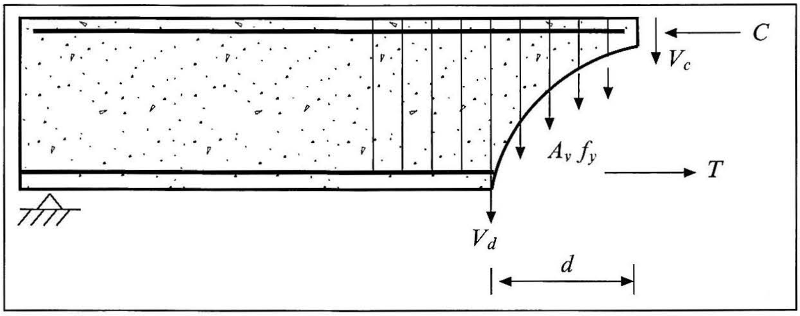


Figure 5.10 Diagonal crack with vertical stirrups.

When the stirrups are vertical to an angle $\theta =$ 90° the shear reinforcement V_s is computed by:

$$V_s = A_v\, f_y\, n \tag{5.28}$$

and the number of the stirrups equal to

$$n = \frac{d}{s} \tag{5.29}$$

Substituted Eq. (5.29) into Eq. (5.28) the V_s is written

$$V_s = \frac{d\, A_v\, f_y}{s} \tag{5.30}$$

From Eq. (5.30) the spacing between the stirrups is:

$$s = \frac{d\, A_v\, f_y}{V_s} \tag{5.31}$$

required $\phi\, V_s = V_u - \phi\, V_c$ (5.32)

5.7 LIMITATIONS FOR STIRRUP SPACING

ACI code required for maximum spacing of stirrups, should not be greater than $d/2$ or equal to 24 in. The shear reinforcement V_s is:

$$V_s \leq 4\sqrt{f'_c}\, b_w d \qquad \text{inch-pound} \qquad (5.33)$$

$$V_s \leq \frac{\sqrt{f'_c}}{3}\, b_w d \qquad \text{SI} \qquad (5.34)$$

If the shear reinforcement is between $4\sqrt{f'_c}\, b_w d$ and $8\sqrt{f'_c}\, b_w d$, the maximum spacing is decreased to $d/4$, or not exceeds 12 in.

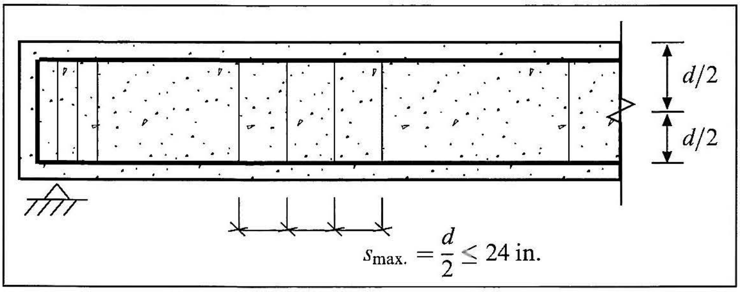


Figure 5.11 Maximum spacing.

5.8 REQUIREMENTS FOR MINIMUM SHEAR REINFORCEMENT

In order to ensure a required area of a minimum shear reinforcement, A_v at spacing s is:

$$A_{v,\min} = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \geq 50\,\frac{b_w s}{f_y} \qquad \text{inch-pound} \qquad (5.35)$$

$$A_{v,\min} = \frac{1}{16}\sqrt{f'_c}\,\frac{b_w s}{f_y} \geq \frac{1}{3}\,\frac{b_w s}{f_y} \qquad \text{SI} \qquad (5.36)$$

Where $A_{v,\min.}$ in Eq. (5.36) is in mm^2 and f_y in MPa

Substituted Eq. (5.35) into Eq. (5.30), the minimum shear reinforcement V_s is equal to

$$V_s = \frac{d\, f_y\, A_v}{s} = 0.75\sqrt{f'_c}\, b_w d \geq 50\, b_w d \qquad (5.37)$$

5.9 CRITICAL SECTIONS

The critical section is located at the distance d from the interior face of the support. At that section, the nominal shear strength is located at the diagonal crack. In this case, the shear strength V_u reached its maximum at the interior face of the beam support (Fig.5.12). The code permits for the section located between the face of support and the critical section must be designed for shear force V_u.

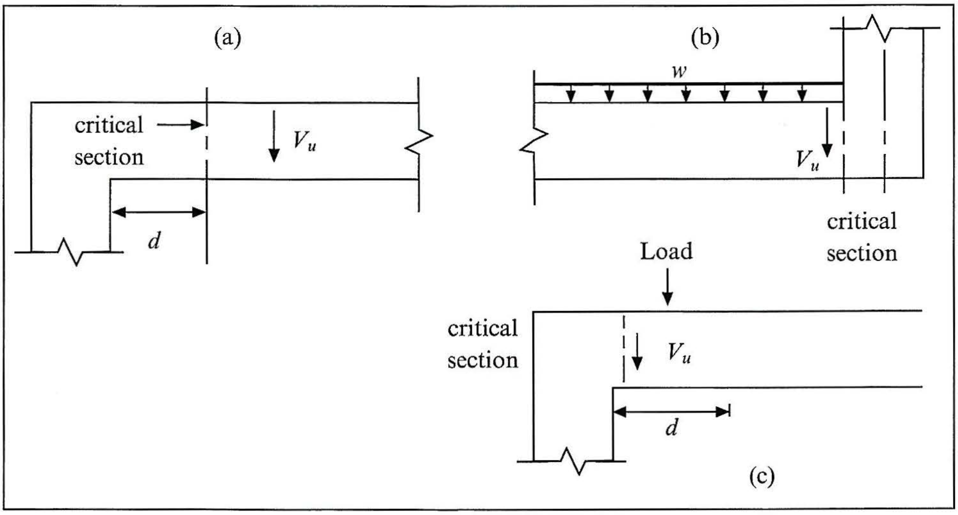


Figure 5.12 Critical section.

5.10 REQUIREMENTS FOR DESIGN PROCEDURE

The shear force V_u values at the center of the beam and at the end of the beam is calculated by:

$$V_u = 1.2\ V_d + 1.6\ V_l \tag{5.23}$$

and

$$V_u \leq \phi\ V_n \qquad \text{(no stirrups required)} \tag{5.20}$$

Where V_n is shear strength and also equal to:

$$V_n = V_c + V_s \tag{5.21}$$

V_c = shear strength in concrete

V_s = shear strength in steel

To obtain value of V_c

$$V_c = 2\sqrt{f'_c}\, b_w d \tag{5.7}$$

or

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u}\right) b_w d$$

If $\quad V_u \geq \phi V_n \quad$ and $\quad V_s \leq 8\sqrt{f'_c}\, b_w d$

$$V_s = V_u/\phi - V_c \tag{5.32}$$

For vertical stirrups V_s equal to:

$$V_s = \frac{d\, A_v\, f_y}{s} \tag{5.30}$$

$$s = \frac{d\, A_v\, f_y}{V_s} \tag{5.31}$$

If $\quad V_s \leq 4\sqrt{f'_c}\, b_w d \qquad (5.33)$

$$s = d/2 \quad \text{or} \quad s < 24 \text{ in. (610 mm)}$$

If $\quad V_s \geq 4\sqrt{f'_c}\, b_w d \qquad \left(\frac{\sqrt{f'_c}}{3}\, b_w d\right) \quad$ SI

$$s = d/4 \quad \text{or} \quad s < 12 \text{ in. (305 mm)}$$

Where $V_s > 8\sqrt{f'_c}\, b_w d$ or $(0.66\sqrt{f'_c}\, b_w d)$, for SI units, the cross-section of beam needs to increase when $V_u < \frac{1}{2}\phi V_c$. If V_u is greater than ϕV_c, the depth of the cross section should be increased.

$$A_v = \frac{V_s\, s}{f_y\, d} > A_{v,\min}$$

Where

$$A_{v,\min} = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \geq 50\,\frac{b_w s}{f_y}$$

$$= \frac{1}{16}\sqrt{f'_c}\,\frac{b_w s}{f_y} \geq \left(\frac{b_w s}{3 f_y}\right) \qquad \text{SI} \tag{5.35}$$

Example 5.3

Compute the spacing of stirrups for #4 bars, ($A_v = 0.4$ in^2 for two legs). If $f_y = 50$ ksi (344 MPa), $f'_c = 3$ ksi (21.7 MPa) and shear force V_u is 60 kips (178 KN).

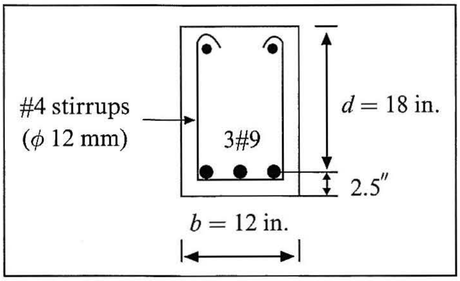


Figure 5.13

Solution.

Form Eq. (5.7) the V_c is:

$$V_c = 2\sqrt{f'_c}\, b_w d = 2\sqrt{3000}\,(12 \times 18)\,\frac{1}{1000} = 23.66 \text{ k (105 KN)}$$

If $\phi\ V_c/2$ is less than V_u the stirrups are important, where $\phi = 0.75$

$$\frac{0.75\ (23.66)}{2} = 8.87 \text{ kips}$$

When $V_u = 60$ kips $> \dfrac{\phi\ V_c}{2}$, the stirrups are needed.

$$V_s = \frac{V_u}{\phi} - V_c = \frac{60}{0.75} - 23.66 = 56.34 \text{ kips}$$

$$s = \frac{A_v f_y\ d}{V_s} = \frac{0.4\ (50)\ 18}{56.34} = 6.4 \text{ in.} \qquad \text{use 6 in.}$$

$$V_s \le 4\sqrt{f'_c}\, b_w d$$

$$4\sqrt{3000}\,(12 \times 18)\,\frac{1}{1000} = 47.32$$

$$V_s = 56.34 > 47.32 \text{ k}$$

From above value of V_s, the s is not greater than $d/4$

$$s = \frac{18}{4} = 4.5 \text{ in.} \qquad \text{(controls)}$$

Check for $A_{v,\min}$, from Eq. (5.35)

$$A_{v,\min} = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y}$$

$$= 0.75\sqrt{3000}\,\frac{(12 \times 4.5)}{50000} = 0.044$$

$$= 50\,\frac{b_w s}{f_y} = 50\,\frac{(12 \times 4.5)}{50000} = 0.054\ \text{in}^2$$

$A_s = 0.4\ \text{in}^2 > A_{v,\min} = 0.054\ \text{in}^2$ **O.K**

Example 5.4

Compute the spacings to be used for ϕ 8 mm stirrups. If $f_y = 345$ MPa, $f'_c = 27.5$ MPa and shear force $V_u = 157$ KN. Check for the minimum shear reinforcement $A_{v,\min}$.

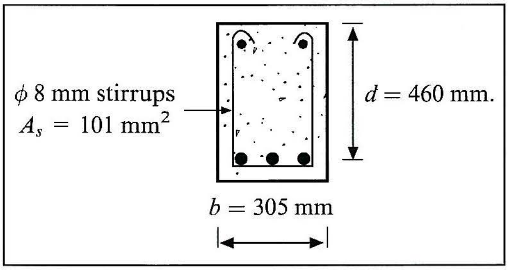


Figure 5.14

Solution.

From Eq. (5.8) SI units, the V_u is:

$$V_c = 0.166\sqrt{f'_c}\,b_w d$$

$$= 0.166\sqrt{27.5}\,(305 \times 460)\,\frac{1}{1000} = 122\ \text{KN (27.5 kips)}$$

$$V_u = 157\ \text{KN} > \frac{\phi V_c}{2} \qquad \text{stirrups are need}$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= \frac{157}{0.75} - 122 = 87.3\ \text{KN}$$

$$s = \frac{A_v\, f_y\, d}{V_s} = \frac{101\ (345)\ 460}{87300} = 183.6\ \text{mm} \approx 183\ \text{mm}$$

$$V_s \leq \frac{\sqrt{f'_c}}{3}\ b_w d \qquad \text{(for SI unit)}$$

$$\frac{\sqrt{27.5}}{3}\ (305 \times 460)\ \frac{1}{1000} = 254.2\ \text{KN}$$

$$V_s = 87.3\ \text{KN} \leq 254.2\ \text{KN}$$ **O.K**

The spacing s is less than $d/2$

$$d/2 = \frac{460}{2} = 230\ \text{mm}$$

use $s = 183$ mm

Check for $A_{v,\min.}$ from Eq. (5.35)

$$A_s = 101\ \text{mm}^2\ (0.22\ \text{in}^2)$$

$$A_{v,\min} = \frac{1}{16}\ \sqrt{f'_c}\ \frac{b_w s}{f_y} = \frac{1}{16}\ \sqrt{27.5}\ \frac{460\ (183)}{345} = 80\ \text{mm}^2$$

$$A_{v,\min} = \frac{1}{3}\ \frac{b_w s}{f_y} = \frac{1}{3}\ \frac{460\ (183)}{345} = 81.3\ \text{mm}^2$$

$$A_s = 101\ \text{mm}^2 > A_{v,\min} = 81.3\ \text{mm}^2$$ **O.K**

Example 5.5

Design the required spacing of U-shape stirrups in the simply supported beam as shown in Fig. 5.15. The beam has dead load of 3 kips/ft (43.8 KN/m) and live load of 5.7 kips/ft (83.2 KN/m). Use $f_y = 60$ kips/in^2 (413 MPa), $f'_c = 4.5$ kips/in^2 (31 MPa) and neglect weight of the beam.

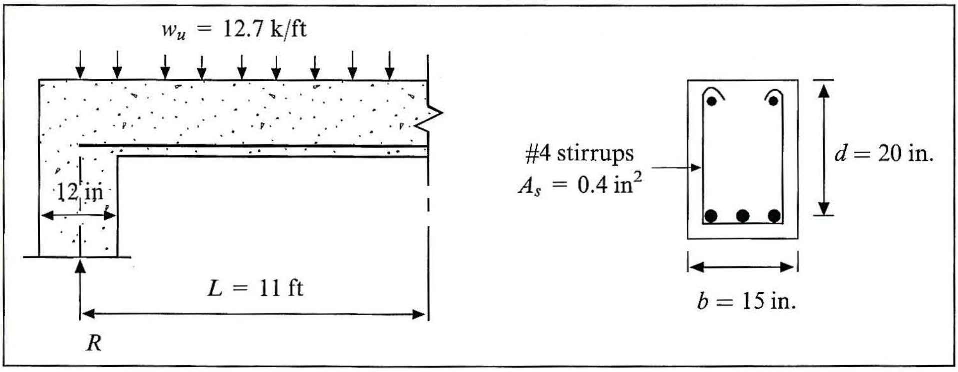


Figure 5.15

Solution.

a - Compute factored shear V_u

$$w_u = 1.2\ (3) + 1.6\ (5.7) = 12.7\ \text{k/ft}$$

Reaction of support

$$R = \frac{12.7 \times 22}{2} = 139.7\ \text{kips}$$

Shear force V_u at distance d from the face of support.

$$V_u = 139.7 - 12.7\left(\frac{26}{12}\right) = 112.2\ \text{kips}$$

b - Shear strength of concrete:

$$V_c = 2\sqrt{f'_c}\, b_w d$$

$$= 2\sqrt{4500}\ (15 \times 20)\ \frac{1}{1000} = 40.2\ \text{k}$$

$$\frac{\phi\, V_c}{2} = \frac{0.75\ (40.2)}{2} = 15\ \text{k}$$

$$V_u > \frac{\phi\, V_c}{2} \qquad \text{stirrups are needed}$$

c - Spacing of critical section:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{112.2}{0.75} - 40.2 = 109.4\ \text{k}$$

$$s_{\text{req.}} = \frac{A_v f_y\, d}{V_s} = \frac{0.4\ (60)\ 20}{109.4} = 4.4\ \text{in.} \qquad \text{use 4 in.}$$

d - Check for maximum spacing of stirrups:

$$s_{\text{max.}} \le \frac{d}{2} = \frac{20}{2} = 10\ \text{in} \quad \text{or} \le 24\ \text{in}$$

use $s_{\text{max.}} = 10$ in

e - Check minimum area of stirrups

$$A_{v,\min} = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} = \frac{0.75\,\sqrt{4500}\,(15)\,(4)}{60000} = 0.0503\ \text{in}^2$$

$$A_{v,\min} = \frac{50\,b_w s}{f_y} = \frac{50\,(15)\,4}{60000} = 0.05\ \text{in}^2$$

$A_s = 0.4\ \text{in}^2 > A_{v,\min} = 0.0503\ \text{in}^2$ **O.K**

f - Compute the distance χ where stirrup is not needed that located between center of the beam and value of $\frac{\phi\,V_c}{2}$

$$\frac{\phi\,V_c}{2} = \frac{0.75\,(40.2)}{2} = 15\ \text{kips}$$

$$\text{distance}\ \chi = \frac{15\,(132)}{139.7} = 14\ \text{in}$$

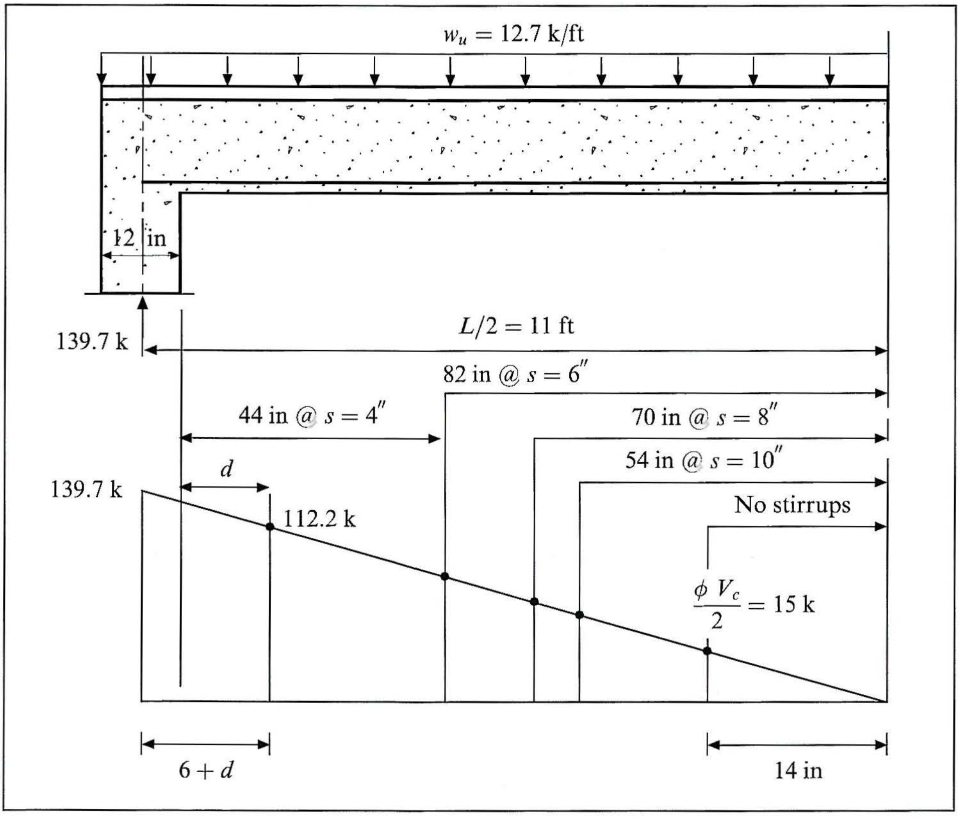


Figure 5.16

The maximum distance for $s_{max.} = 10$ in from the centerline of beam.

$$V_s = \frac{A_v f_y d}{10}$$

$$= \frac{0.4\ (60)\ 20}{10} = 48 \text{ kips}$$

$$V_u = \phi\ (V_c + V_s)$$

$$= 0.75\ (40.2 + 48) = 66 \text{ kips}$$

$$X_{10} = \frac{66\ (132)}{139.7} = 62 \text{ in.}$$

$$62 - 14 = 48 \text{ in.}$$

$$\frac{48}{10} = 4.8 \text{ stirrups} \qquad \text{use 4 stirrups}$$

$$4\ (10) = 40 \text{ in.}$$

In this example, the distance between 4 - and 10 - in. That critical section and maximum spacing should choose other number as 6, 8 and 10.

Try 8 in from the centerline of beam:

$$V_s = \frac{A_v f_y d}{s} = \frac{0.4\ (60)\ 20}{8} = 60 \text{ kips}$$

$$V_u = \phi\ (V_c + V_s) = 0.75\ (40.2 + 60) = 75.1 \text{ k}$$

$$X_8 = \frac{75.1\ (132)}{139.7} = 71 \text{ in.}$$

$$71 - 40 - 14 = 17 \text{ in.}$$

$$\frac{17}{8} = 2.125 \text{ stirrups} \qquad \text{(use 2 stirrups)}$$

$$2 \times 8 = 16 \text{ in.}$$

Try 6 in from the centerline of beam:

$$V_s = \frac{A_v f_y d}{s} = \frac{0.4\ (60)\ 20}{6} = 80 \text{ kips}$$

$$V_u = \phi\ (V_c + V_s) = 0.75\ (40.2 + 80) = 90.1 \text{ k}$$

$$\chi_6 = \frac{90.1\ (132)}{139.7} = 85.2 \text{ in.}$$

$$85.2 - 40 - 14 - 16 = 15.2 \text{ in.}$$

$$\frac{15.2}{6} = 2.5 \text{ stirrups} \qquad \text{(use 2 stirrups)}$$

$$2 \times 6 = 12 \text{ in.}$$

The critical section has $s = 4$ in. and the number of spacing is:

$$132 - 4\ (10) - 2\ (8) - 2 \times 6 - 14 - \left(\frac{1}{2} \text{ support } = 6''\right) = 44 \text{ in.}$$

$$\frac{44}{4} = 11 \text{ st.} \qquad \text{(use 11 stirrups)}$$

$$11 \times 4 = 44$$

Spaced the first stirrup 2 in. from the interior face of the support then run 11 stirrups at 4 in.

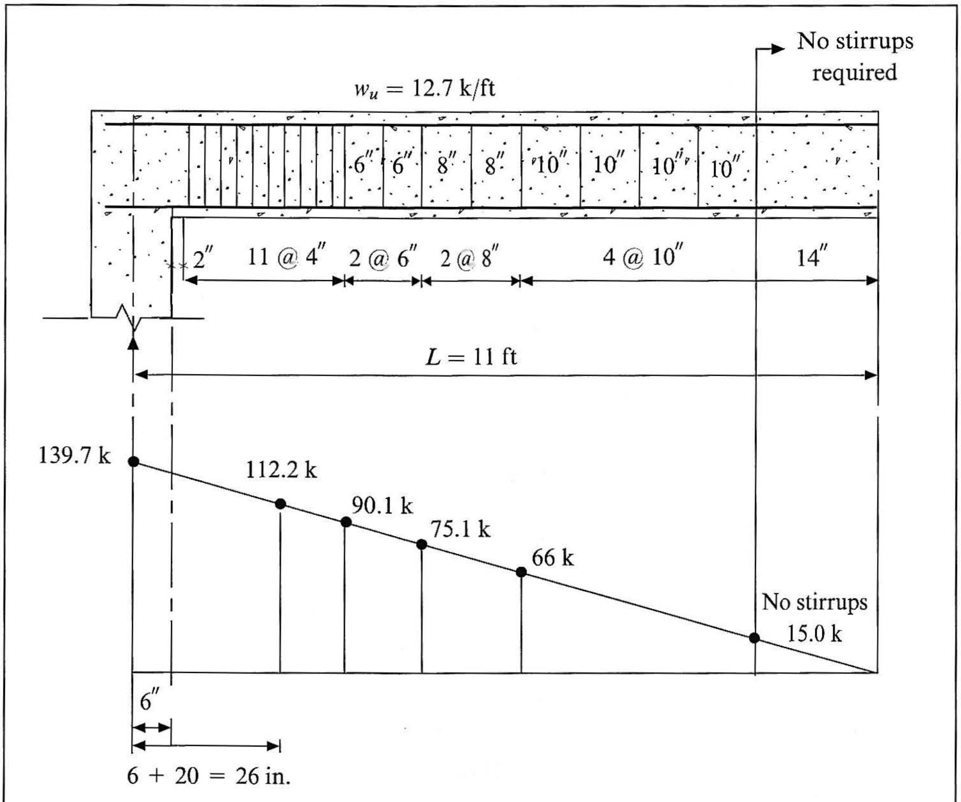


Figure 5.17

Example 5.6

Design the required spacing of U-stirrups for the beam of Fig.5.18. Using ϕ 10 mm bar, $f_y = 400$ MPa and $f'_c = 30$ MPa. The service dead load $D.L = 50$ KN/m (without own weight) and the service live load $L.L = 35$ KN/m. The area of steel $A_s = 462$ mm^2 (3 ϕ 14 mm), $A_v = 157$ mm^2, and support width = 250 mm.

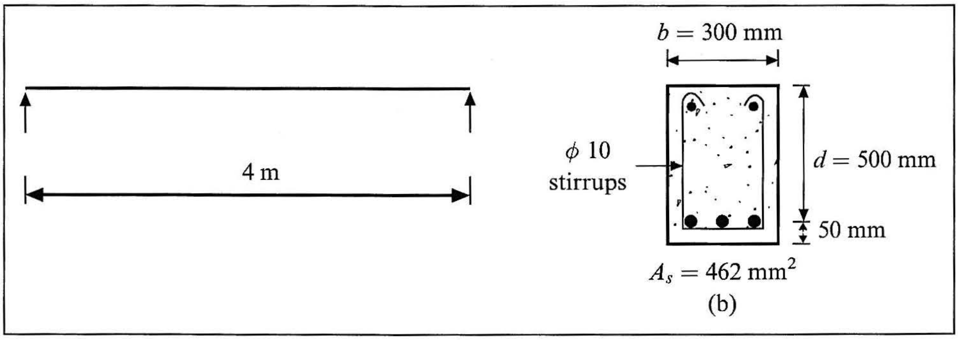


Figure 5.18

Solution.

a - Factored shear force V_u is

Own weight of beam $= (0.55 \times 0.3 \times 2500 \times 9.8)\ \dfrac{1}{1000} = 4$ KN/m

$$w_d = 1.2\,(50 + 4) = 64.8 \text{ KN/m}$$

$$w_l = 1.6\,(35) = 56 \text{ KN/m}$$

$$w_t = 64.8 + 56 = 120.8 \text{ KN/m}$$

$$V_u = \frac{120.8\ (4)}{2} = 241.6 \text{ KN}$$

Shear force V_u at distance d from support end

$$V_u = 241.6 - 120.8\ \frac{625}{1000} = 166.1 \text{ KN}$$

The V_u at midspan is

$$V_u = \frac{1}{8}\ (56)\ 4 = 28 \text{ KN}$$

b - Determine the spacing of stirrups and shear force of concrete:

$$V_c = 0.166\ \sqrt{f'_c}\ b_w d$$

$$= 0.166\ \sqrt{30}\ (300 \times 500)\ \frac{1}{1000} = 136.4 \text{ KN}$$

Shear force of steel:

$$V_s = \frac{V_u}{\phi} - V_c - \frac{166.1}{0.75} - 136.4 = 85.1 \text{ KN}$$

Check for V_s, if less than or equal to $\frac{1}{3}\sqrt{f'_c}\, b_w d$

$$\frac{1}{3}\sqrt{30}\,(300 \times 500)\,\frac{1}{1000} = 273.86$$

$$V_s = 85.1 \text{ KN} < 273.86 \text{ KN}$$

$$\text{req.} \quad s = \frac{A_v f_y d}{V_s} = \frac{157\ (400 \text{ MPa})\ 500}{85100\ N} = 369 \text{ mm}$$

Since V_s is smaller than $\frac{1}{3}\sqrt{f'_c}\, b_w d$, the maximum spacing equal to $d/2$ or less than 600 mm.

$$s_{\text{max.}} \geq d/2 = \frac{500}{2} = 250 \text{ mm}$$

Use s equal to 250 mm from interior face of support to place of $\frac{\phi V_c}{2}$

$$\frac{0.75\ (136.4)}{2} = 51.1 \text{ KN}$$

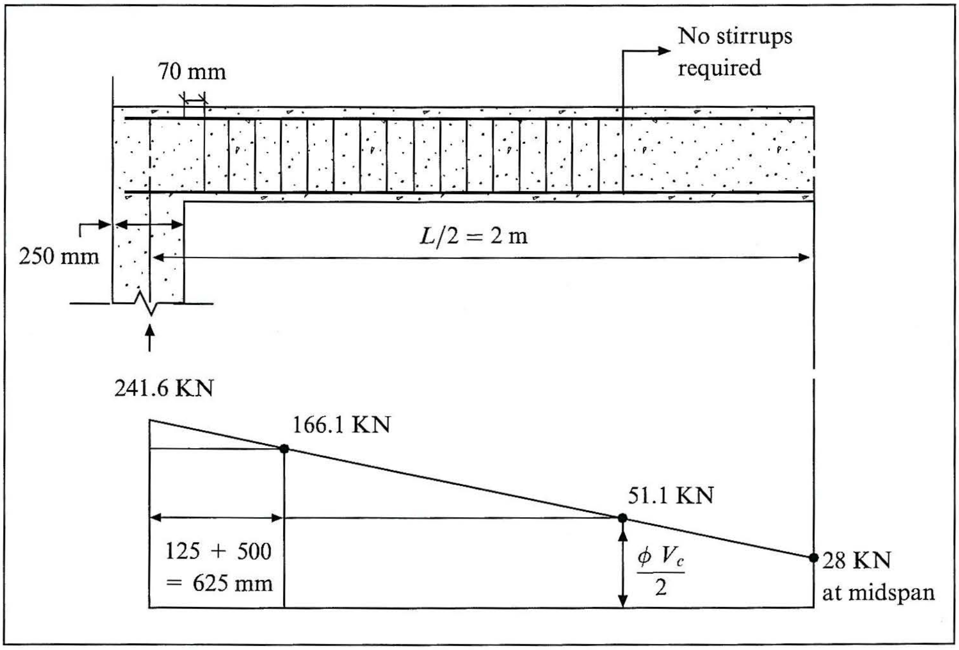


Figure 5.19

Example 5.7

A simply supported rectangular beam has 18 ft span, $f_y = 60$ ksi, $f'_c = 3$ ksi, $b = 14$ in and $d = 24$ in. Compute the maximum distributed load w_u and design the spacing of vertical stirrups. The reinforcement ratio ρ is 0.0128.

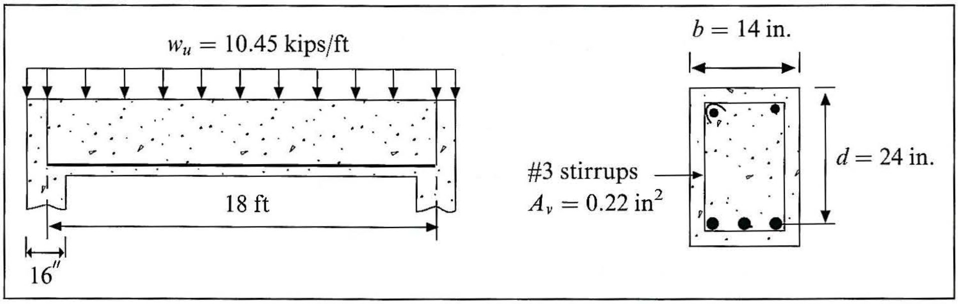


Figure 5.20

Solution.

a - maximum distributed load w_u

The maximum moment for Fig. 5.20 is

$$M_u = \frac{w_u\, l^2}{8}$$

Solve for moment from equilibrium equation.

$$T = C$$

$$T = A_s f_y = 0.85 f'_c\, ba$$

$$A_s = \rho bd = 0.0128\ (24 \times 14) = 4.3 \text{ in}^2$$

Use 3# 11 bars, $A_s = 4.68$ in^2

$$T = 4.68\ (60) = 281 \text{ kips}$$

$$a = \frac{281}{0.85\ (3)\ 14} = 7.87 \text{ in.}$$

$$M_n = T \text{ or } C\left(d - \frac{a}{2}\right)$$

$$= 281\ \left(24 - \frac{7.87}{2}\right) = 470 \text{ ft-kips}$$

$$\phi\, M_n = M_u$$

$$M_u = 0.9\ (470) = 423 \text{ ft-kips}$$

$$M_u = \frac{w_u\, l^2}{8}$$

$$w_u = \frac{8\ (423)}{(18)^2} = 10.45 \text{ kips/ft}$$

$$R = \frac{w_u l}{2} = \frac{10.45\ (18)}{2} = 94 \text{ k}$$

b - Design the spacing of vertical stirrups.

Determine the shear force V_u at distance d from support end

$$d = 24 \text{ in.}$$

$$V_u = 94 - 10.45\left(\frac{32}{12}\right) = 66.1 \text{ kips}$$

Determine the V_c

$$V_c = \left[1.9\ \sqrt{f_c} + 2500\ \rho_w\ \frac{V_u d}{M_u}\right] b_w d \leq 3.5\ \sqrt{f_c}\ b_w d$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{4.68}{14 \times 24} = 0.0139$$

Determine the moment M_u at $d = 24$ in. from the end of support

$$M_u = 94\ \frac{32}{12} - 10.45\left(\frac{32}{12}\right)\frac{1}{2}\left(\frac{32}{12}\right) = 213.5 \text{ ft-kips}$$

$$\frac{V_u d}{M_u} = \frac{66.1\left(\frac{24}{12}\right)}{213.5} = 0.62 < 1.0 \qquad \textbf{O.K}$$

$$V_c = \left[1.9\ \sqrt{3000} + 2500\ (0.0139)\ (0.62)\right]\ (14 \times 24)\ \frac{1}{1000} = 42.2 \text{ kips}$$

$$3.5\ \sqrt{3000}\ (14 \times 24)\ \frac{1}{1000} = 64.4 \text{ kips}$$

$$V_c = 42.2 \text{ kips} \leq 64.4 \text{ kips} \qquad \textbf{O.K}$$

Required

$$V_s = \frac{V_u}{\phi} - V_c = \frac{66.1}{0.75} - 42.2 = 45.9 \text{ kips}$$

$$s = \frac{A_v\, f_y\, d}{V_s} = \frac{0.22\ (60)\ 24}{45.9} = 6.9 \text{ in.}$$

(at critical section)
use 6.0 in.

Since $s = 6$ in at critical section.

$$s_{max.} = \frac{d}{2} = \frac{24}{2} = 12 \text{ in.} \qquad \text{or} = 24 \text{ in.}$$

use $s_{max.} = 12$ in.

At distance $\frac{\phi V_c}{2}$, stirrups are not required

$$\frac{0.75 \times 42.2 \text{ kips}}{2} = 15.8 \text{ k}$$

distance χ from centerline of beam:

$$\chi = \frac{15.8\ (9)}{94} = 1.50 \text{ ft} = 18.0 \text{ in.}$$

$$V_u = 66.1 > \frac{\phi V_c}{2} \qquad \text{(stirrups are required)}$$

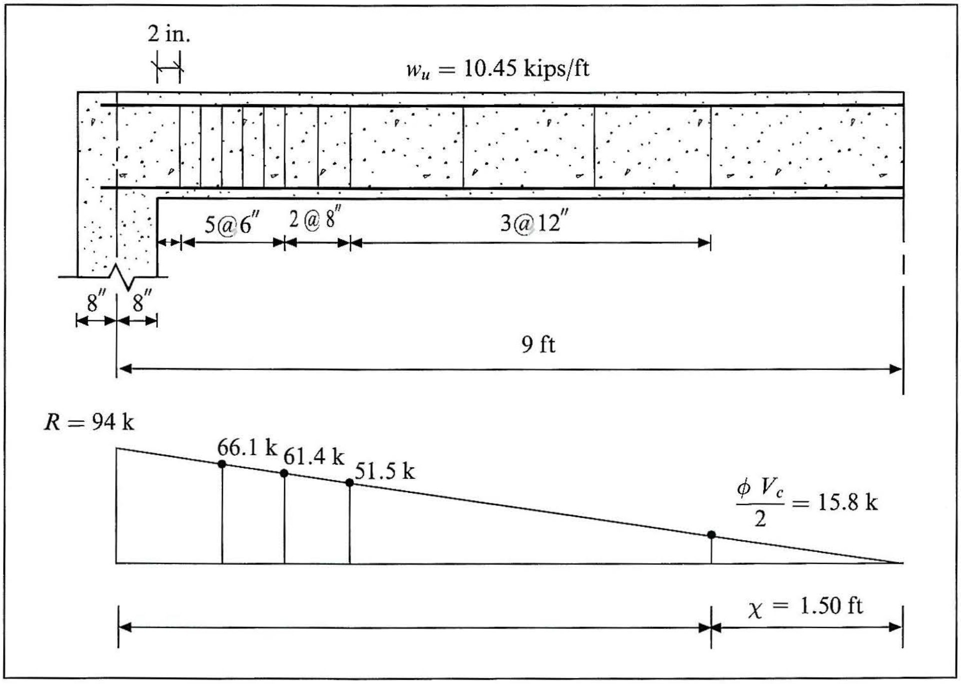


Figure 5.21

Since s is between 6 in. to 12 in. the code is limited; the maximum s is not greater than $d/2$. In this example, the $d/2 = 24/2 = 12$ in. is the maximum chosen, that related with V_u as mentioned early. To compute the actual spacing between 6 in. to 12 in. are 12, 10, 8 and 6 in.

1 - $s_{max.} = 12$ in

$$V_s = \frac{A_v f_y d}{s} = \frac{0.22\ (60)\ 24}{s} = \frac{316.8}{12} = 26.4 \text{ kips}$$

$$V_u = \phi\ (V_c + V_s) = 0.75\ (42.2 + 26.4) = 51.5 \text{ kips}$$

$$X_{12} = \frac{51.5\ (9 \times 12)}{94} = 59.2 \text{ in.}$$

From the center of beam:

$$59.2 - 1.50\ (12) = 41.2 \text{ in.}$$

$\frac{41.2}{12} = 3.4$ stirrups　　　　use 3 stirrups

$3 \times 12 = 36$ in.

2 - $s = 10$ in.

$$V_s = \frac{316.8}{10} = 31.7 \text{ kips}$$

$$V_u = 0.75\ (42.2 + 31.7) = 55.4 \text{ kips}$$

$$X_{10} = \frac{55.4\ (108)}{94} = 63.7 \text{ in.}$$

$63.7 - 36 - 18.0 = 9.6$ in.

$\frac{9.6}{10} = 0.96$ stirrups　　　　(no stirrups are required)

3 - $s = 8$ in.

$$V_s = \frac{316.8}{8} = 39.6 \text{ kips}$$

$$V_u = 0.75\ (42.2 + 39.6) = 61.4 \text{ kips}$$

$$X_8 = \frac{61.4\ (108)}{94} = 70.5 \text{ in.}$$

$70.5 - 36 - 18.0 = 16.5$ in.

$\frac{16.5}{8} = 2.06$ stir.　　　　(use 2 stirrups)

$2 \times 8 = 16$ in.

4 - $s = 6$ in. The remaining distance

$$108 - 36 - 18.0 - 16 - (0.5 \text{ support} = 8 \text{ in.}) = 30 \text{ in.}$$

$$\frac{30}{6} = 5 \text{ stir.} \qquad \text{(use 5 stirrups)}$$

$$5 \times 6 = 30 \text{ in.}$$

5.11 SHEAR - FRICTION

The shear friction is concerned with direct shear that is useful for precast composite material and the diagonal tension crack may be occurred in the composite construction without vertical steel reinforcement on the diagonal crack to prevent shear failure.

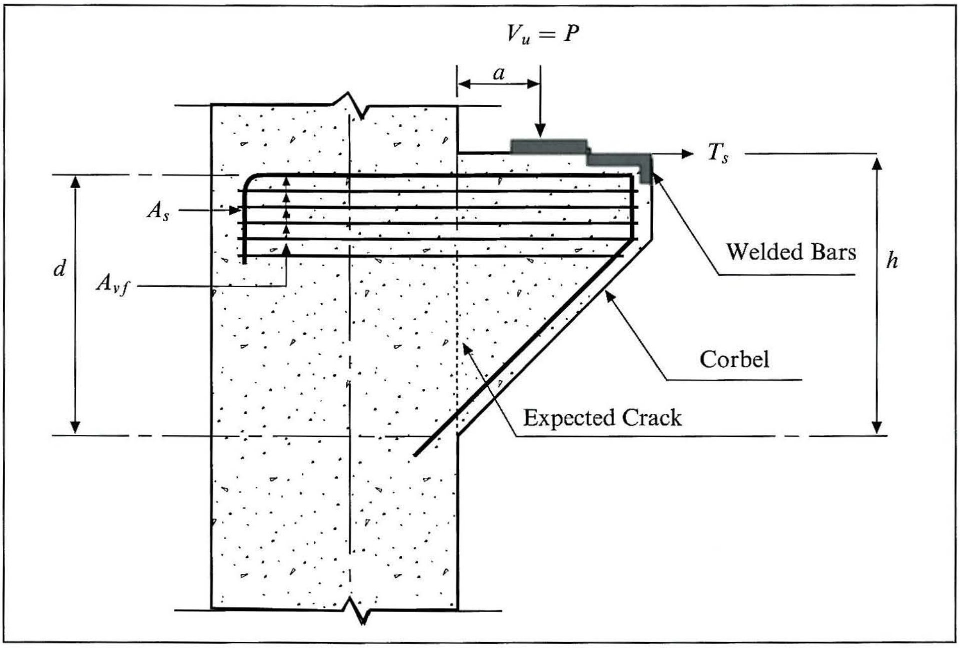


Figure 5.22 Shear - Friction in corbel or bracket.

Figure 5.22 shows concentrated load acting on cantilever of reinforced concrete and the expected crack started from adhere cantilever to main construction. The area of shear friction reinforcement A_{vf} is placed across on assumed crack to prevent shear-friction failure.

Figure 5.23 shows the concrete block without reinforcement shear. Failure plane occurs at the center of concrete block, but the friction reinforcement A_{vf} should be placed in the dashing line to prevent shear failure plane.

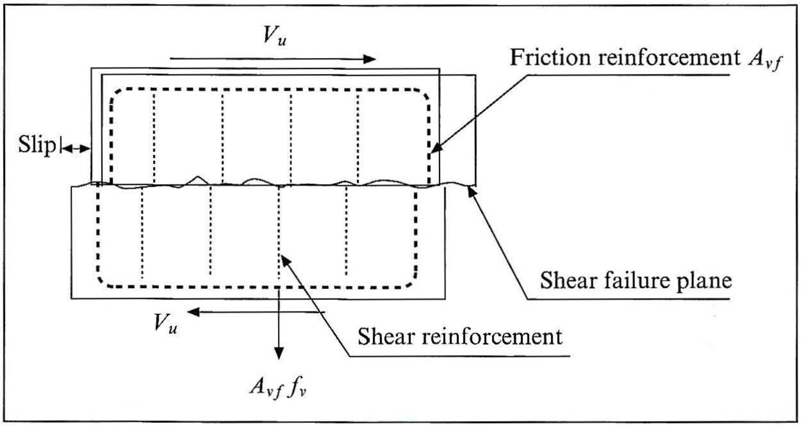


Figure 5.23 Shear - Friction in block of concrete.

The coefficient of friction μ is related with expected crack and composite material; the ACI code limited μ as following.

Smooth and hardened concrete	$0.6\ \lambda$
Rolled structural steel by steel bars	$0.7\ \lambda$
Roughened and hardened concrete for surface clean	$1.0\ \lambda$
Cast concrete (monolithic)	$1.4\ \lambda$
The value of λ is equal to	
Normal - weight concrete	$\lambda = 1.0$
Sand - lightweight concrete	$\lambda = 0.85$
All lightweight concrete	$\lambda = 0.75$

The nominal shear friction strenght V_n is:

$$V_u = \phi\ V_n$$

$$V_n = A_{vf} f_y\ \mu < 0.2 f_c'\ A_c \qquad (5.38)$$

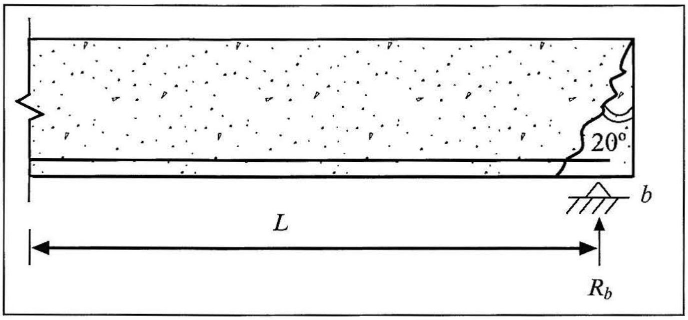
or $<$ (800 psi) A_c

$$V_n < 5.5\ A_c\ (N) \qquad \text{SI} \qquad (5.39)$$

Where A_c is the area of failure section of concrete, and A_{vf} is the area of shear friction reinforcement. N and P are factored loads and f_y should be less than or equal to 60 kips.

Example 5.8

Design precast beam for shear - friction across the crack at angle 20°, $f_y = 60$ ksi, $f'_c = 4.5$ ksi, use normal - weight concrete, temperature and shrinkage $T_s = 15$ kips. The dead load and live load are 60 and 50 kips and the depth at bearing 21 in. by 11 in. wide.



Figurre 5.24

Solution.

$$V_u = 1.2\ D.L + 1.6\ L.L$$

$$= 1.2\ (60) + 1.6\ (50) = 152 \text{ kips}$$

$$T_s = 1.2\ (15) = 18 \text{ kips}$$

To determine temperature and shrinkage, choose the ACI-02 factor load, which is 1.2 multiply by T_s effect.

$$V_n = \frac{V_u}{\phi} = \frac{152}{0.75} = 202.7 \text{ kips}$$

$$V_n = A_{vf} f_y \mu$$

Where $\mu = 1.0$ (normal - weight concrete)

$$A_{vf} = \frac{V_n}{f_y \mu} - \frac{202.7}{60\ (1)} = 3.38 \text{ in}^2$$

Temperature and shrinkage $T_s = 18$ kips, and the reinforcement to resist effective area of the concrete A_c equal to

$$T_{ts} = A_n f_y$$

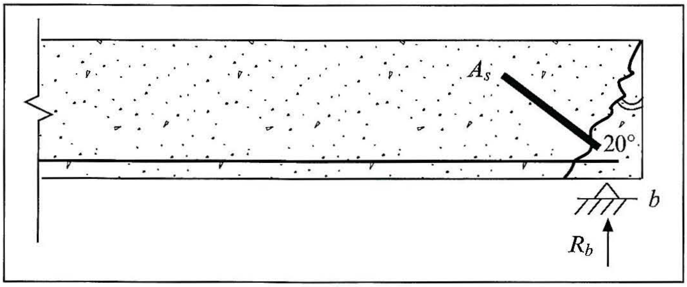
$$A_n = \frac{T_s}{f_y} = \frac{18}{60} = 0.3 \text{ in}^2$$

For uniform distribution along expected crack:

$$A_s = A_n + A_{vf} = 0.3 + 3.38 = 3.68 \text{ in}^2$$

use 5 # 8 (From Table 2.5)

$$A_s = 3.95 \text{ in}^2$$



Check for shear-friction in concrete:

$$V_n < 0.2 f'_c A_c$$

$$A_c = bd = 21 \times 11 = 231 \text{ in}^2$$

$$0.2\ (4500)\ (231)\ \frac{1}{1000} = 208 \text{ kips}$$

$$V_n = 202.7 \text{ k} < 208 \text{ k}$$ **O.K**

5.12 DESIGN PROCEDURE FOR CORBEL OR BRACKET

From equilibrium equation for vertical shear:

$$V_n = \mu\, T \tag{5.40}$$

$$T = A_{vf} f_y$$

$$V_n = \mu\, A_{vf} f_y \tag{5.38}$$

The ACI-11.9.1 limited the shear span to depth

$$a/d < 1.0 \tag{5.41}$$

Where a is depened on bearing strength if not increased d

$$V_u > N_{uc}$$

Where N_{uc} is tensile force and d take it from surface of support.

$$M_u = V_u a + N_{uc}\ (h - d) \tag{5.42}$$

From Eq. (5.38) the V_u equal to:

$$V_n < 0.2 f_c' A_c < 800 A_c$$

Where A_c is equal to b_w multiply by d, and the equation is computed by:

$$V_n \leq 0.2 f_c' b_w d \leq 800 b_w d$$

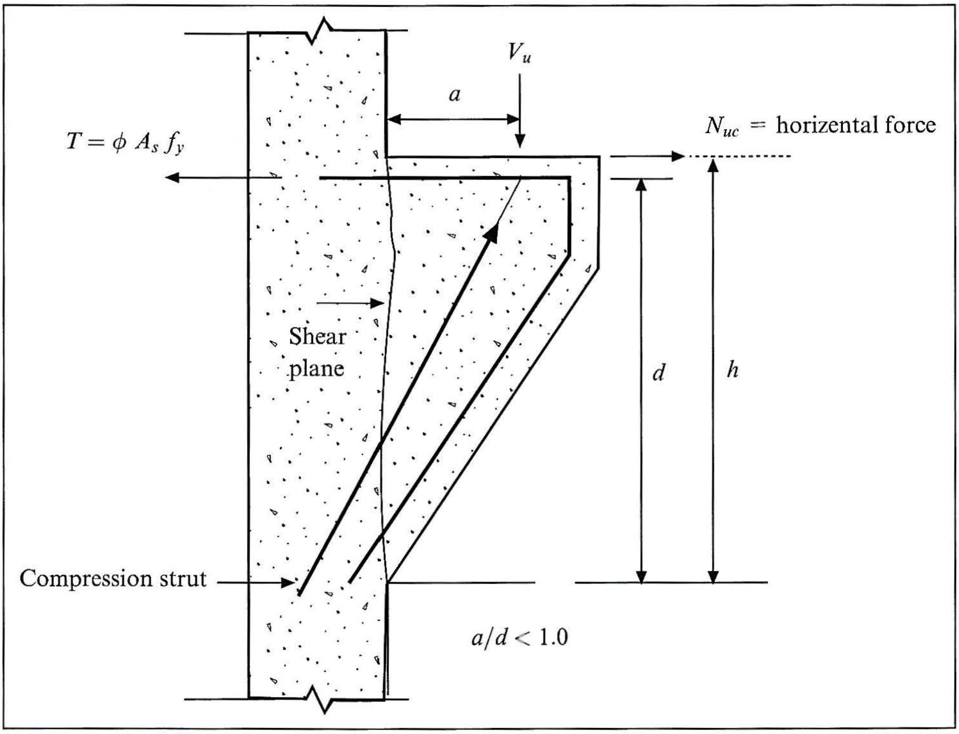


Figure 5.25 Forces on a Bracket.

The tensile force N_{uc} is less than $\phi A_s f_y$, but N_{uc} is computed to a live load and greater than $0.2 V_u$.

$$A_s = A_f + A_n \tag{5.43}$$

$$A_s = \frac{2}{3} A_{vf} + A_n \tag{5.44}$$

The area of tension reinforcement A_s, should be taken the greater of the Eq. (5.43) and (5.44).

The total area A_h of closed stirrups should be greater than

$$A_h \geq 0.5 (A_s - A_n) \tag{5.45}$$

Reinforcement ratio ρ is greater than $0.04 f'_c/f_y$

$$\rho = \frac{A_s}{bd} > 0.04 \frac{f'_c}{f_y} \tag{5.46}$$

Where A_s is main tension of bar and b is width of column

Example 5.9

Design a bracket shown in Fig. 5.26 that carries a dead load and live load of 25 kips and 35 kips. Compressive strength f'_c is 4.5 ksi and yield stress f_y is 60 ksi. Assume bearing plate 3 in and $N_{uc} = 15$ kips.

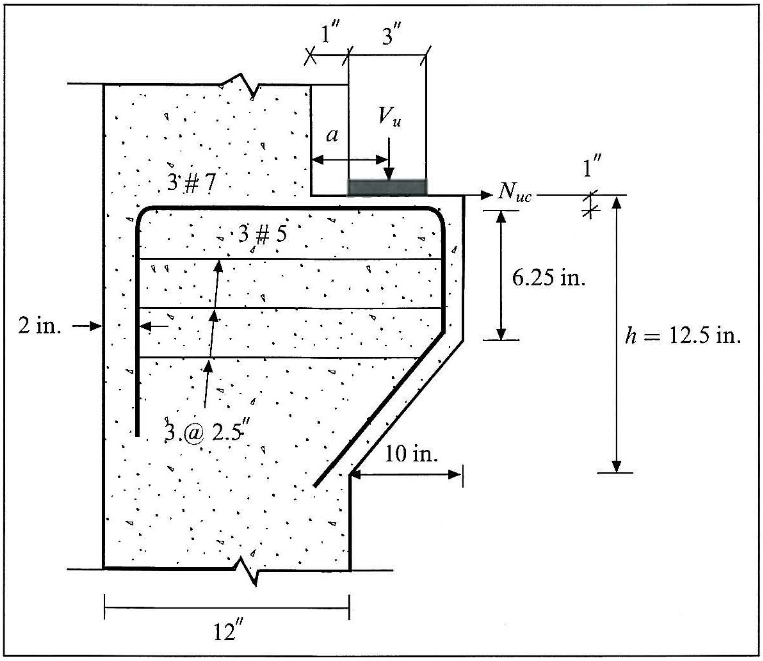


Figure 5.26

Solution.

a - Determine the total factor loads

$$V_u = 1.2\ D.L + 1.6\ L.L$$

$$= 1.2\ (25) + 1.6\ (35) = 86 \text{ kips}$$

$$N_{uc} = 1.6\ (15) = 24 \text{ kips}$$

$$V_n = \frac{V_u}{\phi} = \frac{86}{0.75} = 114.7 \text{ kips}$$

b - Compute shear-friction reinforcement:

$$A_{vf} = \frac{V_n}{f_y \, \mu} \qquad \mu = 1.0$$

$$= \frac{114.7}{60\ (1)} = 1.91 \text{ in}^2$$

$$a = \frac{1}{2}(3) + 1.0 = 2.5 \text{ in}^2$$

c - Determine the depth of bracket and assume 12 in square column

$$V_n = 0.2 f'_c \, b_w d$$

$$= 0.2\ (4500)\ b_w d = 900\ b_w d > 800\ b_w d$$

Use 800 $b_w d$ to determine depth of bracket:

$$V_n = 800\ (12)\ \text{d}$$

$$d = \frac{114700}{800\ (12)} = 11.9 \text{ in.} \qquad \text{use 11 in.}$$

$$\frac{a}{d} < 1.0$$

$$\frac{2.5}{11} = 0.23 < 1.0 \qquad \textbf{O.K}$$

d - Compute minimum reinforcement ratio $\rho_{\text{min.}}$

$$\rho_{\text{min.}} = 0.04 \frac{f'_c}{f_y} = 0.04 \frac{4.5}{60} = 0.003$$

The column has 12 in. × 12 in.:

$$A_f = 0.003\ (12 \times 11) = 0.4 \text{ in}^2$$

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{24}{0.75\ (60)} = 0.53 \text{ in}^2$$

$$A_s = A_f + A_n = 0.4 + 0.53 = 0.93 \text{ in}^2$$

$$A_s = \frac{2}{3} A_{vf} + A_n = \frac{2}{3}\ 1.91 + 0.53 = 1.8 \text{ in}^2$$

Take the greater of A_s

$$A_s = 1.8 \text{ in}^2$$

Use 3#7 bars, $A_s = 1.8 \text{ in}^2$

e - Determine A_h by Eq. (5.45) for closed Stirrups.

$$A_h \geq 0.5\ (A_s - A_n)$$

$$A_h = 0.5\ (1.8 - 0.53) = 0.64 \text{ in}^2$$

Use 3 # 5 bars, $A_h = 0.93 \text{ in}^2$

From ACI 11.9.4. Determine the spacing of stirrups

$$2/3\ \frac{11}{3} = 2.44 \text{ in} \approx 2.5 \text{ in}$$

try h = 11 + 1 (cover) + 0.5 (dim. bar)

= 11 + 1 + 0.875/2 = 12.4 in. use 12.5 in.

Use outer face of a bracket, is half of overall depth h

$$\text{Front face} = \frac{h}{2} = \frac{12.5}{2} = 6.25 \text{ in.}$$

Example 5.10

Design a bracket shown in Fig.5.27. If $f_y = 60$ ksi, $f'_c = 5$ ksi, live load = 30 kips and $V_u = 100$ kips. Use length of bearing 12 in. × 4 in. and $b_w = 13$ in.

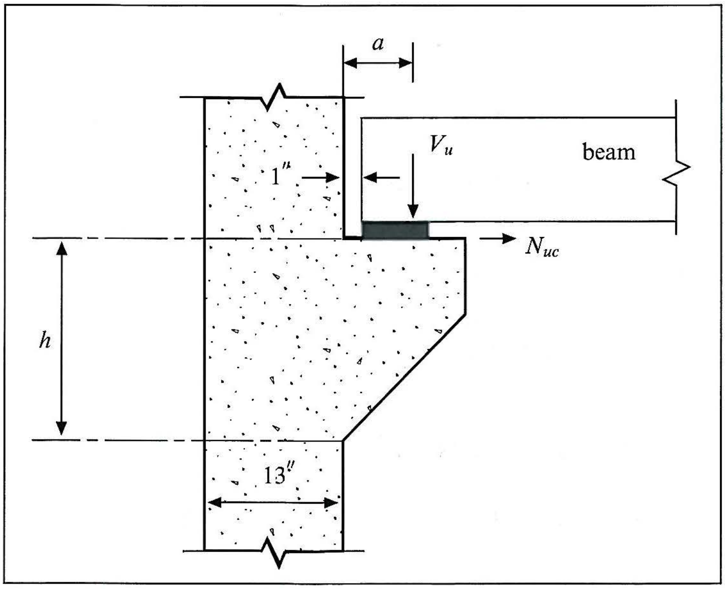


Figure 5.27

Solution.

$$V_u = 100 \text{ kips}$$

$$N_{uc} = 1.6\ (L.L) = 1.6\ (30) = 48 \text{ kips}$$

$$V_n = \frac{V_u}{\phi} = \frac{100}{0.75} = 133.3 \text{ kips}$$

Compute shear-friction reinforcement and minimum reinforcement, assume sand-lightweight concrete $\lambda = 0.85$

$$\mu = 1.0\ (0.85) = 0.85$$

$$A_{vf} = \frac{V_n}{f_y\ \mu} = \frac{133.3}{60\ (0.85)} = 2.61 \text{ in}^2$$

$$\rho_{\text{min.}} = 0.04 \frac{f_c'}{f_y} = 0.04\ \frac{5}{60} = 0.003$$

$$M_u = V_u a + N_{uc}\ (h - d)$$

$$a = \left(\frac{1}{2}\right)\ 4 + 1.0 = 3 \text{ in}$$

Try $h = 16$ in, d $= 14$ in

$$a/d = \frac{3}{14} = 0.21 < 1.0$$ **O.K**

$$M_u = 100\ (3) + 48\ (16 - 14) = 396 \text{ in-kips}$$

$$A_f = 0.003\ (13 \times 14) = 0.55 \text{ in}^2$$

where b is equal to 13 in. for column:

$$A_f = \frac{396}{0.85 \times 60 \times 14} = 0.55 \text{ in}^2$$

$$A_n = \frac{N_{uc}}{\phi\ f_y} = \frac{48}{0.75\ (60)} = 1.07 \text{ in}^2$$

$$A_s = A_f + A_n = 0.55 + 1.07 = 1.61 \text{ in}^2$$

$$A_s = 2/3\ A_{vf} + A_n = \frac{2}{3}\ (2.61) + 1.07 = 2.81 \text{ in}^2$$

Choose the greater of A_s:

$$A_s = 2.81 \text{ in}^2$$

Use 3 # 9 bars, $A_s = 3.00 \text{ in}^2$

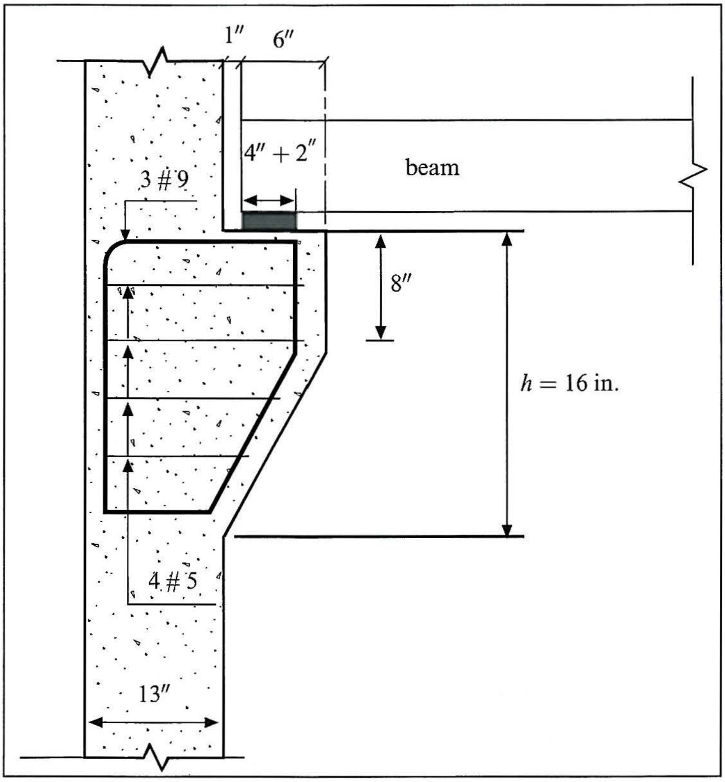
Compute for shear reinforcement A_h:

$$A_h \geq 0.5\,(A_s - A_n)$$

$$A_h = 0.5\,(3.00 - 1.07) = 0.97 \text{ in}^2$$

Use 4 # 5 bars, $A_h = 1.24 \text{ in}^2$

$$s_{\text{max.}} = \frac{2}{3}\left(\frac{14}{4}\right) = 2.34 \text{ in.}$$ use 2.5 in. stirrups



Choose 8 in. $= \dfrac{h}{2}$ for exterior bracket.

or

$$h = \text{embedded plate} + d + \text{bar radius}\left(\frac{1.128}{2}\right)$$

$$h = 1 + 14 + 0.564 = 15.564 \text{ in}$$

use $h = 16$ in. **O.K**

5.13 PUNCHING SHEAR

A heavy concentrated load has a special attention when the area carries that load is small. The punching shear takes place around the column to effect on the footing or slab, which causes a shear failure. As a result, the inclined cracks will be longer where the loads increased and the diagonal crack started from the top of the footing and extend to the bottom diagonally as shown in Figure 5.28. The ACI code limited punching -shear failure occurs at critical section $d/2$ from all exterior sides of the column joined with footing or a slab.

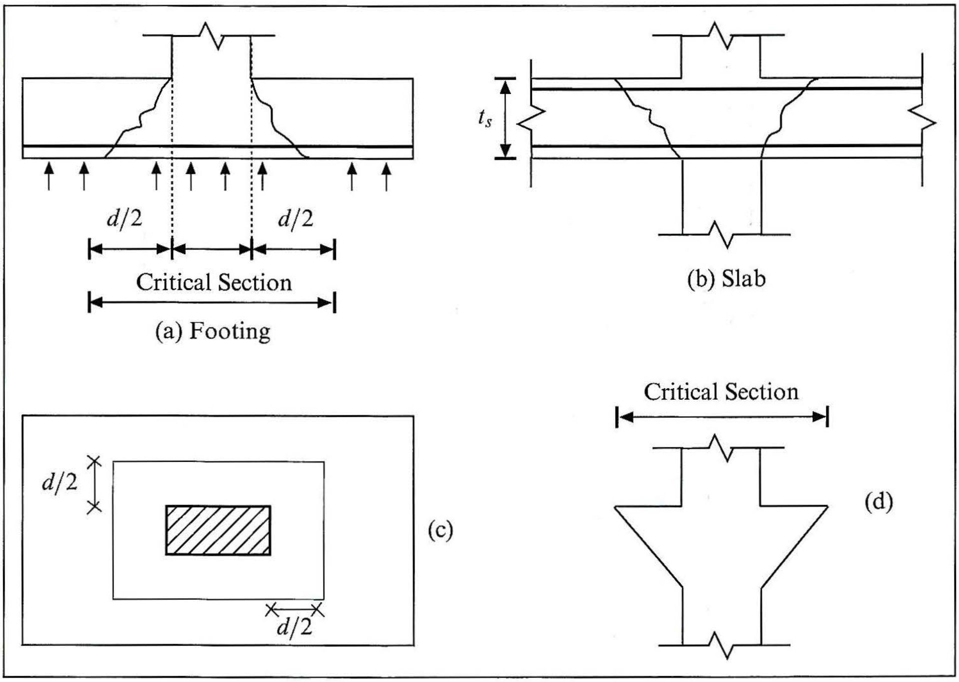


Figure 5.28 Punching shear.

ACI 11.12.2.1 determined the punching shear V_c for the smallest of:

(a) $$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d \quad (5.47)$$

(a₁) $$V_c = \left(1 + \frac{2}{\beta_c}\right) 0.166\sqrt{f'_c}\, b_o d \quad \text{SI} \quad (5.48)$$

(b) $$V_c = \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c}\, b_o d \qquad (5.49)$$

(b₁) $$V_c = \left(\frac{\alpha_s d}{b_0} + 2\right) 0.083 \sqrt{f'_c}\, b_o d \qquad \text{SI} \qquad (5.50)$$

(c) $$V_c = 4 \sqrt{f'_c}\, b_o d \qquad (5.51)$$

(c₁) $$V_c = 0.333 \sqrt{f'_c}\, b_o d \qquad \text{SI} \qquad (5.52)$$

Where

β_c = long side to short side ratio of the column or reaction area.

α_s = critical section for 40 interior column, 30 edge column and 20 corner. 4,3 and 2 sides.

b_o = distance from the exterior around faces of column.

d = footing depth.

V_c = punching shear.

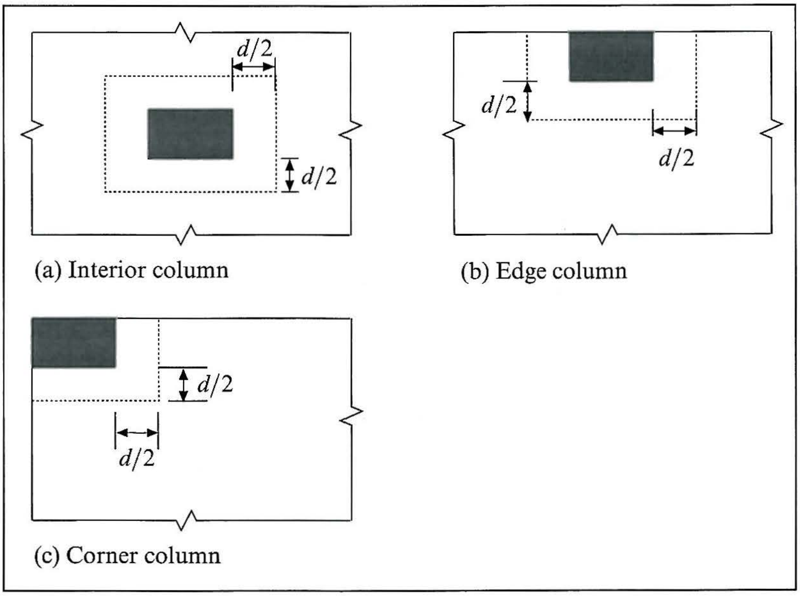


Figure 5.29 Column locations of slabs.

Example 5.11

Check punching-shear failure for footing and the compressive strength f'_c is 3 ksi (20.7 MPa). The dimensions are shown in Fig. 5.30.

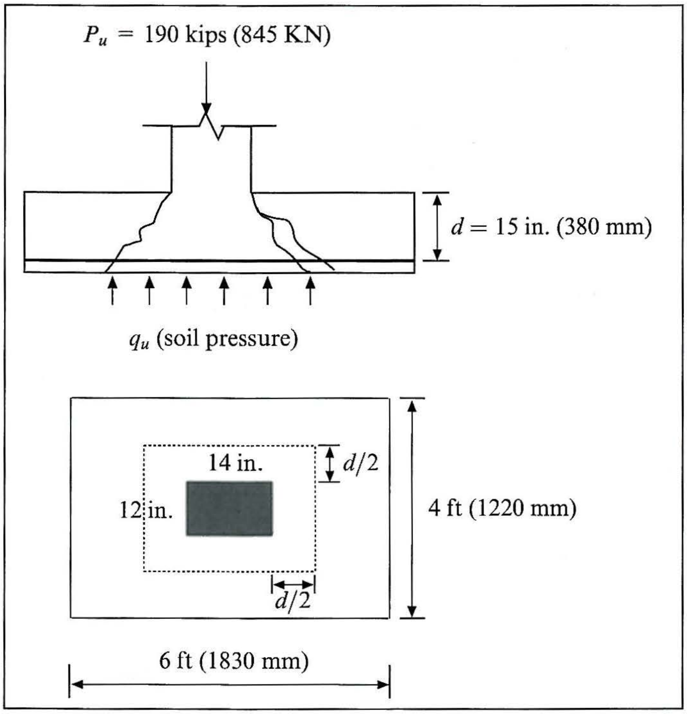


Figure 5.30

Solution.

$$b_o = 2\,(14 + 12 + 2\,d)$$

$$= 2\,(14 + 12 + 2\,(15)) = \frac{112}{12} = 9.34 \text{ ft}$$

$$q_u = \frac{190}{6 \times 4} = \frac{190}{24} = 7.92 \text{ kips/ft}^2$$

Where q_u is the soil pressure of the footing

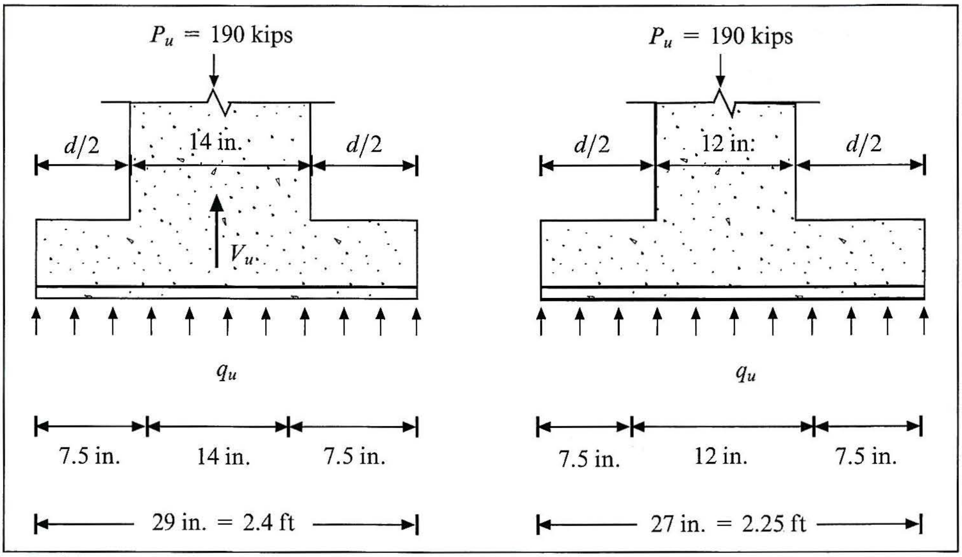
$$R = (2.25 \times 2.4)\ q_u = 5.4 \times 7.92 = 42.8 \text{ kips}$$

$$V_u = 190 - 42.8 = 147.2 \text{ kips}$$

The punching-shear failure is:

$$V_c = 4\sqrt{f'_c}\ b_o d = 4\sqrt{3000}\ (112 \times 15)\ \frac{1}{1000} = 368 \text{ kips}$$

$$\phi\, V_c = 0.75\,(368) = 276 \text{ kips} > V_u = 147.2 \text{ kips} \qquad \textbf{(safe)}$$



Example 5.12

Check punching-shear for solid slab with interior rectangular column 24 in. × 12 in. and $f'_c = 3.5$ ksi, the thickness of slab $t_s = 7$ in. and $d = 5.5$ in. The shear force V_u is 60 kips.

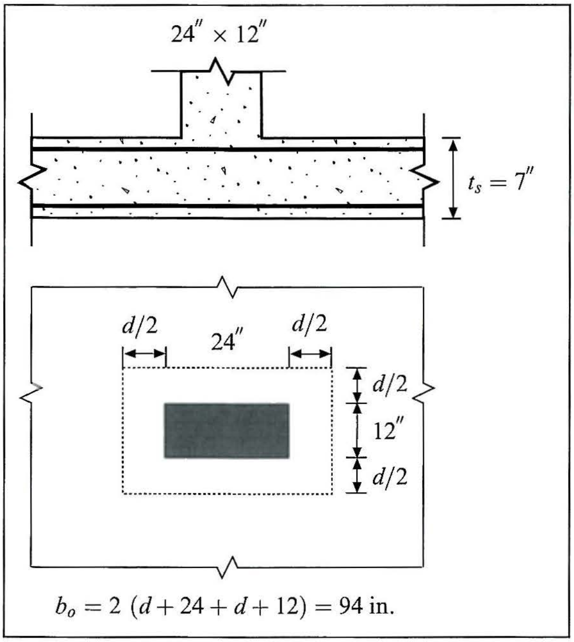


Figure 5.31

Solution.

a - Using Eq. 5.49

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right) \sqrt{f'_c}\, b_o d$$

$$\alpha_s = 40 \qquad \text{(for interior column)}$$

$$V_c = \left(\frac{40\ (5.5)}{94} + 2\right) \sqrt{3500}\ (94 \times 5.5)\ \frac{1}{1000} = 132.7 \text{ kips}$$

b - Using Eq. 5.51

$$V_c = 4 \sqrt{f'_c}\, b_o d$$

$$= 4 \sqrt{3500}\ (94 \times 5.5)\ \frac{1}{1000} = 122.3 \text{ kips}$$

The smallest of V_c is:

$$V_c = 122.3 \text{ kips}$$

$$\phi\ V_c = 0.75\ (122.3) = 91.7 \text{ kips}$$

$$\phi\ V_c = 91.7 \text{ kips} > V_u = 60 \text{ kips}$$

c - Determine the punching shear V_c for the smallest of the following by using Eq. (5.47).

$$V_c = \left(2 + \frac{4}{\beta_c}\right) \sqrt{f'_c}\, b_o d$$

β_c is long side to short side ratio of column

$$\beta_c = \frac{24}{12} = 2$$

$$d = 5.5 \text{ in.} \qquad d/2 = 2.75$$

$$b_o = 2\,(5.5 + 24 + 5.5 + 12) = 94 \text{ in.}$$

$$V_c = \left(2 + \frac{4}{2}\right) \sqrt{3500}\ (94 \times 5.5)\ \frac{1}{1000} = 122.3 \text{ kips}$$

5.14 DEEP BEAMS

Deep beams are defined in ACI-02, 10.7.1 as members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports. Deep beams should satisfy one of the following conditions:

a - Clear span to overall depth ratio l_n/h is not greater than 4; or

b - Regions loaded with concentrated loads within twice the member depth from the face of the support.

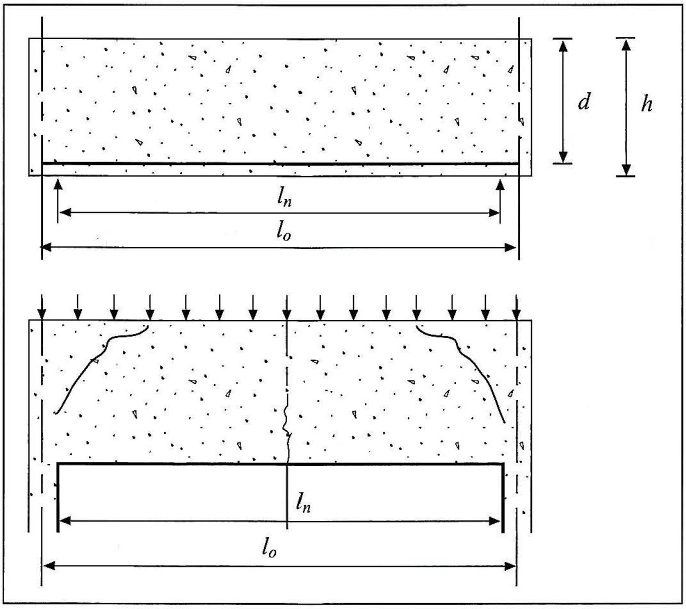


Figure 5.32 Deep beam with distributed load.

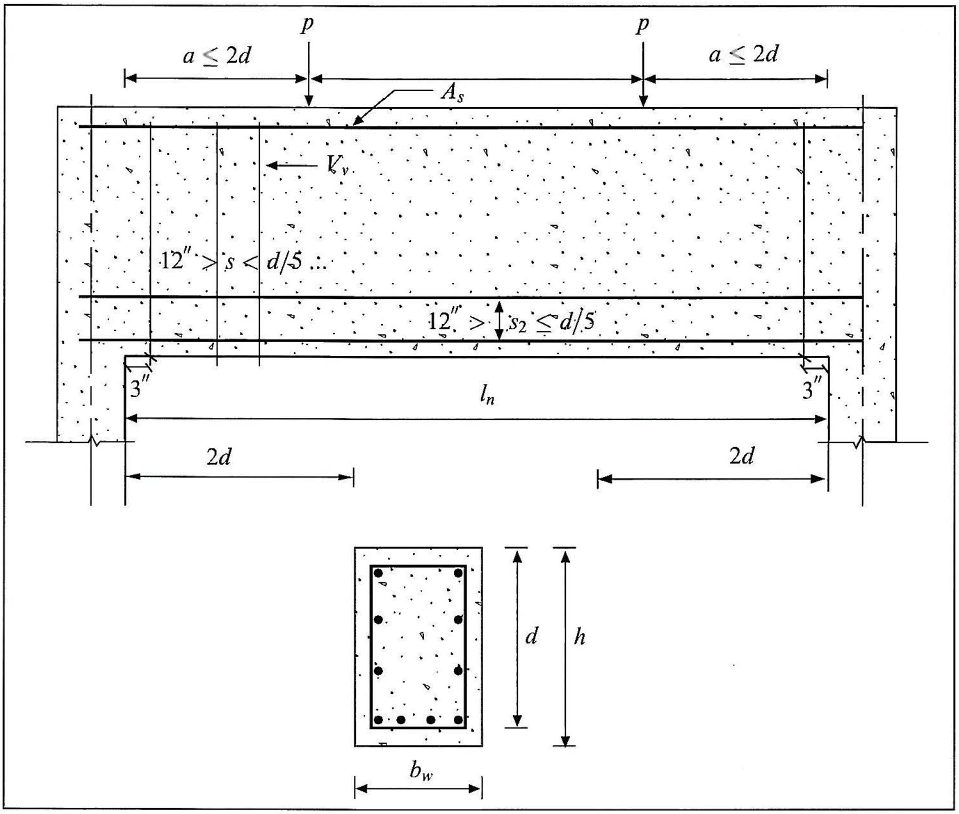


Figure 5.33 Simply supported beam with concentrated load.

Shear strength of Deep Beams

According to ACI-02, 11.8.1, deep beams shall be designed using either non-linear analysis or strut and tie model.

Shear strength V_n for deep beams shall not exceed the values given by the following equations:

$$V_n = 10\sqrt{f'_c}\, b_w d \qquad \text{inch} - \text{pound} \tag{5.53}$$

$$V_n = 0.83\sqrt{f'_c}\, b_w d \qquad \text{SI} \tag{5.54}$$

where

b_w = width of the beam web

d = depth of the beam (Fig. 5.32)

For simplicity earlier versions of the ACI code can be used to design the shear reinforcement of deep beams.

Simply Supported Deep Beams

The maximum shear force measurad for critical section at a distance χ from the interior face of the support.

$$\chi = 0.15\ l_n \leq d \qquad \text{(uniform loading beam)}$$

$$\chi = 0.5\ a \leq d \qquad \text{(concentrated load)}$$

$$V_u < \phi\ V_n$$

$$V_n = V_c + V_s$$

$$V_n = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f'_c}\ b_w d \quad \text{for} \quad 2 \leq \frac{l_n}{d} < 5 \tag{5.55}$$

Where

V_u = factored shear force

V_c = shear strength of concrete

V_s = shear strength of steel

d = depth of deep beam

l_n = clear span

a = distance of shear span from the interior face of support to the load.

For a simplified method ; thus

$$V_c = 2\sqrt{f'_c}\ b_w d \tag{5.56}$$

$$V_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right)\left(1.9\sqrt{f'_c} + 2500\ \rho_w \frac{V_u d}{M_u}\right) b_w d \tag{5.57}$$

The first part of Eq. 5.57

$$\left(3.5 - 2.5\frac{M_u}{V_u d}\right) \leq 2.5 \tag{5.58}$$

and V_c is not greater than Eq. (5.59)

$$V_c \leq 6\sqrt{f'_c}\ b_w d \tag{5.59}$$

Where M_u is the factored moment at the critical section
Design procedure for V_s

$$V_s = \left[\frac{A_v}{s}\left(\frac{1 + \frac{l_n}{d}}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11 - \frac{l_n}{d}}{12}\right)\right] f_y d \tag{5.60}$$

Where

A_v = area of vertical stirrup

s = distance between stirrups

l_n = distance between both interior face of supports

A_{vh} = area of shear reinforcement parallel to main reinforcement with a distance s_2

s_2 = vertical spacing between stirrups

d = depth of deep beam.

Continuous Deep Beams

For simplifed method: thus

$$V_c = 2\sqrt{f'_c}\, b_w d$$

and

$$V_u \le \phi V_c \text{ if not use the following equation}$$

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u}\right) b_w d \le 3.5\sqrt{f'_c}\, b_w d$$

$$V_s = \frac{A_v f_y d}{s}$$

Minimum Shear Reinforcement (ACI-02)

The area of vertical shear reinforcement A_v shall not be less than:

$$A_v \ge 0.0025\, b_w s \tag{5.61}$$

The area of horizontal shear reinforcement (parallel to the span) A_{vh} shall not be less than:

$$A_{vh} \ge 0.0015\, b_w\, s_2 \tag{5.62}$$

Where

s = spacing of vertical shear reinforcement

s_2 = spacing of horizontal shear reinforcement

The spacings s and s_2 should not exceed:

$$12 \text{ inch} \ge s \le d/5 \tag{5.63}$$

$$12 \text{ inch} \ge s_2 \le d/5 \tag{5.64}$$

Minimum Flexural Reinforcement

Minimum flexural tension reinforcement is given by ACI-02, 10.7.3 to be the same as for other flexural members as:

$$A_{s,\min} = \begin{cases} \dfrac{200\, b_w d}{f_y} \le \dfrac{3\sqrt{f'_c}}{f_y}\, b_w d & \text{inch-pound} \quad (5.65) \\[2ex] \dfrac{1.4\, b_w d}{f_y} \le \dfrac{\sqrt{f'_c}}{4 f_y}\, b_w d & \text{SI} \quad (5.66) \end{cases}$$

Example 5.13

A simply supported beam carries two columns at the spacing of 3 ft. from the both faces of support, and the columns have live loads of 90 kips. The clear span of 9 ft, depth d of 30 in. and 15 in. width. Use $f_y = 40$ ksi and $f'_c = 4$ ksi. Compute the shear reinforcement and determine the requiremed steel for both horizontal and vertical reinforcement. The beam has unit weight $\gamma_c = 150$ Ib/ft^3 (2400 kg/m^3) and $A_s = 4.68$ in^2 (3 # 11 bars).

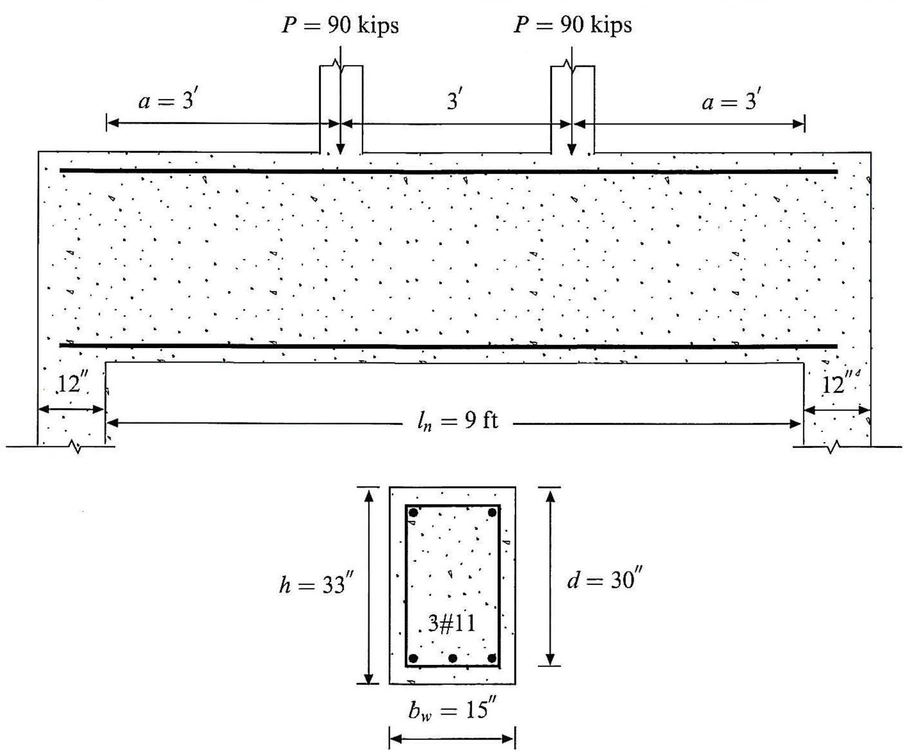


Figure 5.34

Solution

a - Own weight $= \dfrac{33 \times 15}{144}(150) = 515.6$ Ib/ft $= 0.515$ k/ft

$$\frac{l_n}{h} = \frac{9\ (12)}{33} = 3.27 < 4$$

Since $\dfrac{l_n}{h} < 4$ the simply support is deep beam.

$$\frac{1}{2}a = \frac{1}{2}(3 \times 12) = 18 \text{ in.} < \text{d} = 30 \text{ in.}$$

Critical section is 18 in from interior face of columns.

b - Compute shear force V_u

$$V_u = V_{L.L} + V_{D.L}$$

$$V_{L.L} = 1.6\ (90) = 144 \text{ kips}$$

$$\frac{l_n}{2} - \frac{a}{2} = \frac{9}{2} - 1.5 = 3 \text{ ft}$$

$$V_{D.L} = 1.2\ (0.515 \text{ k/ft})\ 3 \text{ ft} = 1.85 \text{ kips}$$

$$V_u = 144 + 1.85 = 145.85 \text{ kips}$$

$$M_u = 144\ (1.5) = 216 \text{ ft-kips}$$

$$\frac{M_u}{V_u d} = \frac{216}{145.85\left(\frac{30}{12}\right)} = 0.59$$

$$3.5 - 2.5\frac{M_u}{V_u d} = 3.5 - 2.5\ (0.59) = 2 < 2.5 \qquad \textbf{O.K}$$

$$V_c = 2\left(1.9\sqrt{f'_c} + 2500\,\rho_w\frac{V_u d}{M_u}\right) b_w d$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{4.68}{15\ (30)} = 0.0104$$

$$V_c = 2\left[1.9\sqrt{4000} + 2500\ (0.0104)\frac{1}{0.59}\right]\frac{(15'' \times 30'')}{1000} = 147.5 \text{ kips}$$

$$\text{max. allowed } V_n = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f'_c}\, b_w d$$

$$\text{max. } V_n = \frac{2}{3}\ (10 + 3.6)\sqrt{4000}\ (15 \times 30)\frac{1}{1000} = 258.2 \text{ kips}$$

$$\text{max. } V_n = 10\sqrt{f'_c}\, b_w d = 10\sqrt{4000}\ (15 \times 30)\frac{1}{1000} = 284.6 \text{ kips}$$

Shear force at critical section

Required $\quad V_n = \dfrac{145.85}{0.75} = 194.5$ kips c - Nominal shear strength

$$V_n = 258.2 \text{ kips} > V_n = 194.5 \text{ kips} \qquad \textbf{O.K}$$

and

$$V_n = 194.5 \text{ kips} > V_c = 147.5 \text{ kips}$$

(horizontal and vertical shear steel is required)

d - Compute for horizontal and vertical reiforcement

$$V_s = V_n - V_c = 194.5 - 147.5 = 47 \text{ kips}$$

Assume no. of bars for horizontal reiforcement then solve vertical bars.

Try #3 bars horizontally then check for minimum

$$\text{max. } s_2 \leq \frac{d}{5} = \frac{30''}{5} = 6 \text{ in.}$$

use spacing $s_2 = 6$ inch

min. $A_{vh} = 0.0015\ b\ \ s_2 = 0.0015\ (15 \text{ in.})\ 6 = 0.135 \text{ in}^2$

Use # 3 bars, $A_{vh} = 0.11\ (2) = 0.22 \text{ in}^2 > 0.135 \text{ in}^2$ **O.K**

e - Design shear

$$\left[\frac{A_v}{s}\left(\frac{1+\frac{l_n}{d}}{12}\right) + \left(\frac{A_{vh}}{s_2}\right)\left(\frac{11-\frac{l_n}{d}}{12}\right)\right] = \frac{V_s}{f_y d}$$

$V_s = 47$ kips, $\frac{l_n}{d} = 3.6$ in, $A_{vh} = 0.22$ in^2, $b = 15$ in and $f_y = 40000$ psi.

$$\left[\frac{A_v}{s}\left(\frac{1+3.6}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11-3.6}{12}\right)\right] = \frac{47}{40 \times 30} = 0.039$$

$$\left[\frac{A_v}{s}\left(\frac{4.6}{12}\right) + \frac{A_{vh}}{6}\ (0.616)\right] = 0.039$$

$$\frac{A_v}{s}\ (0.383) + (0.023) = 0.039$$

$$\frac{A_v}{s} = 0.043$$

Compute for vertical steel A_v

Use # 4 bars, $A_v = 2\ (0.2) = 0.4\ \text{in}^2$

$$s = \frac{0.4}{0.043} = 9.3 \text{ in} > \frac{d}{5} = 6 \text{ in.}$$

use 6 in.

Check minimum A_v

$$A_v = 0.0025\ (15)\ 6 = 0.225\ \text{in}^2$$

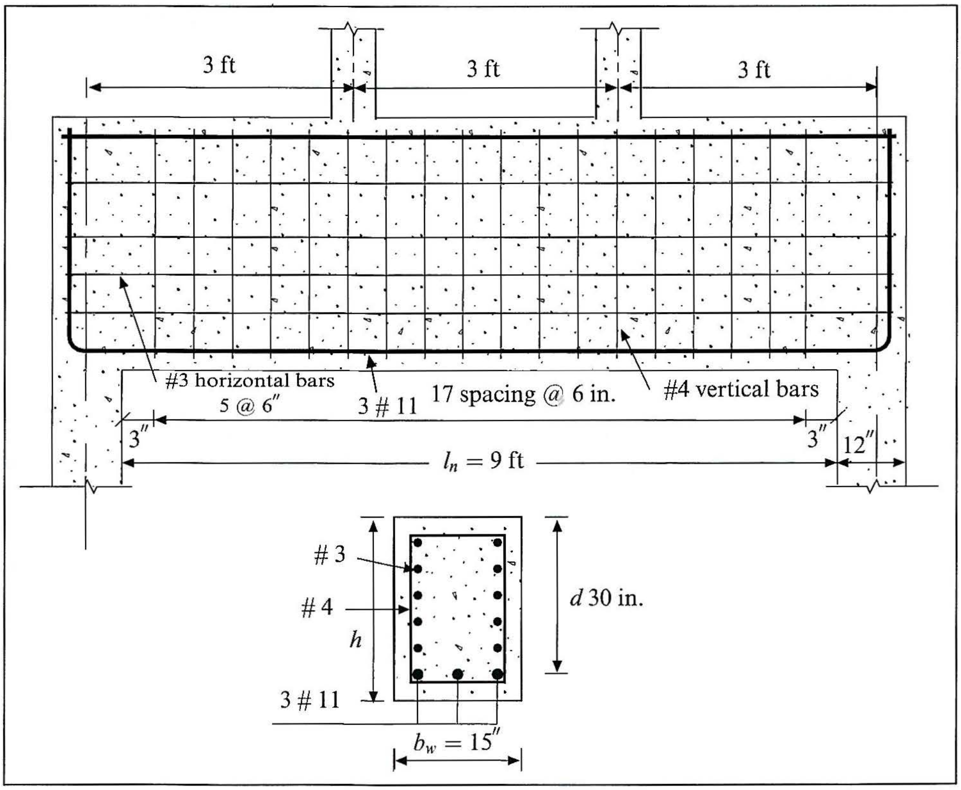
$$A_v = 0.4\ \text{in}^2 > 0.225\ \text{in}^2$$ **O.K**

For horizontal $l_n = 9$ ft

$$\frac{9 \text{ ft } (12) - 6}{6 \text{ in}} = 17 \text{ spaces}$$ from 3 in. of support

For vertical $d = 30$ in. and $s = 6$ in.

$$\frac{30}{6} = 5 \text{ spaces at bottom of no. 3 bar}$$



Example 5.14

A continuous beam is to carry distributed factored load $w_u = 25$ kips/ft. If $f'_c = 4$ ksi, $f_y = 40$ ksi and $A_s = 3$ in^2. Compute the shear reinforcement and determine the area of steel for both horizontal and vertical reinforcement.

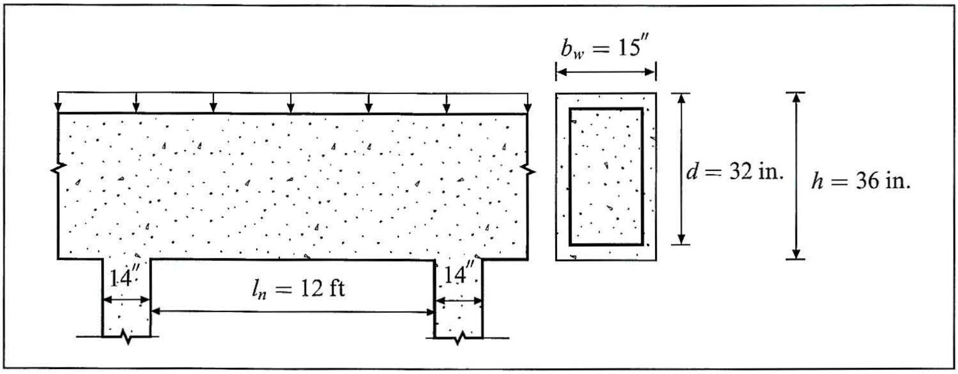


Figure 5.35

Solution.

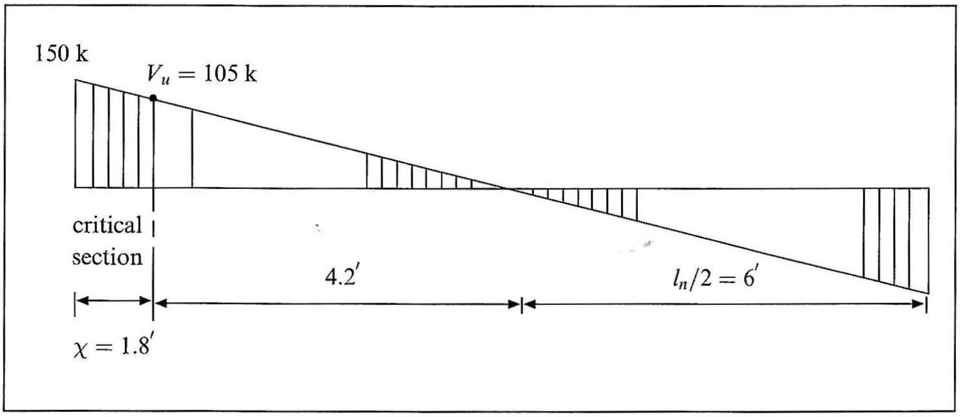
$$\frac{l_n}{h} = \frac{12\ (12)}{36} = 4$$

since $\frac{l_n}{h} \leq 4$ the continuous beam is deep beam

a - Determine the spacing between face of support and critical section χ.

$$\chi = 0.15\ l_n = 0.15\ (12) = 1.8 \text{ ft} \quad < d = 2.67 \text{ ft} \qquad \textbf{O.K}$$

and the shear factor at critical section V_u is:



$$R = \frac{w_u l}{2} = \frac{25\ (12)}{2} = 150 \text{ kips}$$

$$\frac{150}{6'} = \frac{V_u}{4.2'} \qquad V_u = 105 \text{ k}$$

Use simplified method

$$V_c = 2\sqrt{f'_c}\, b_w d = 2\sqrt{4000}\ (15 \times 32)\frac{1}{1000} = 60.7 \text{ kips} < V_u \qquad \textbf{n.g}$$

It is not enough to carry factored force V_u

$$V_c = \left(1.9\sqrt{f'_c} + 2500\ \rho_w \frac{V_u d}{M_u}\right) b_w d \le 3.5\sqrt{f'_c}\, b_w d$$

$$M_u = \frac{w_u\, l_n^2}{11} = \frac{25\ (12)^2}{11} = 327.3 \text{ ft-k (from Fig. 8.3b)}$$

$$\frac{V_u d}{M_u} = \frac{105\ (32)}{327.3\ (12)} = 0.85 < 1.0 \qquad \textbf{O.K}$$

Use 3#9 bars, $A_s = 3.0 \text{ in}^2$

$$\rho_w = \frac{3.0}{(15 \times 32)} = 0.006$$

$$3.5 - 2.5\ (0.85) = 1.37$$

$$V_c = 1.375\ [1.9\sqrt{4000} + 2500\ (0.006)\ 0.85]\frac{(15 \times 32)}{1000} = 87.7 \text{ kips}$$

$$3.5\sqrt{4000}\frac{(15 \times 32)}{1000} = 106.25 \text{ kips}$$

$$V_c = 87.7 \text{ k} < 106.25 \text{ k} \qquad \textbf{O.K}$$

b - Check minimum shear reinforcement

$$V_s = \frac{V_u}{\phi} - V_c = \frac{105}{0.75} - 87.7 = 52.3 \text{ kips}$$

$$V_n = V_c + V_s = 87.7 + 52.3 = 140 \text{ kips} > V_u = 105 \text{ kips} \qquad \textbf{O.K}$$

c - Check maximum shear reinforcement

$$\frac{l_n}{d} \ge 2$$

Use the following equation

$$V_n = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f'_c}\, b_w d$$

$$= \frac{2}{3}\ (10 + 4.5)\ \sqrt{4000}\ (15 \times 32)\ \frac{1}{1000} = 294 \text{ kips}$$

$$\phi\ V_n = 0.75\ (294) = 220.5 \text{ kips} > 87.7 \text{ kips} \qquad \textbf{O.K}$$

d - Horizontal shear reinforcement

Use #3 bars $A_{vh} = 2\ (0.11) = 0.22 \text{ in}^2$

$$A_{vh} = 0.0015\ b_w\ s_2$$

$$s_2 = \frac{0.22}{0.0015\ (15'')} = 9.8 \text{ in.}$$

$$s_2 \le \frac{d}{5} \text{ or } 12 \text{ in.}$$

max. $s_2 = d/5 = \frac{32}{5} = 6.4$ or 12 in.

use $s_2 = 6$ in.

min. $A_{vh} = 0.0015\ (15)\ 6 = 0.135 \text{ in}^2 < 0.22 \text{ in}^2$ **O.K**

use $A_{vh} = 0.22 \text{ in}^2$

$$d/s = \frac{32}{6} = 5.3 \qquad \text{use 5 spaces}$$

use #3 horizontal stirrups at 6 in.

e - Vertical shear reinforcement, use #3 bars, $A_v = 2\ (0.11) = 0.22 \text{ in}^2$

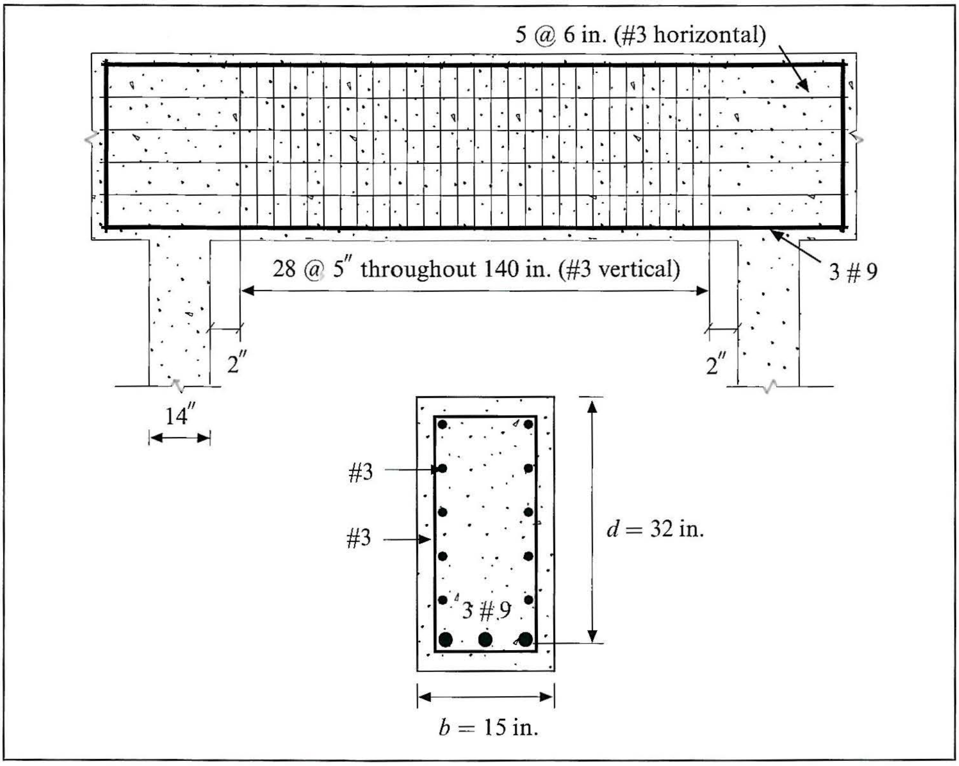
$$A_v = 0.0025\ b_w s$$

$$s = \frac{A_v}{0.0025\ (15)} = \frac{0.22}{0.0025\ (15)} = 5.87 \text{ in.}$$

max. $s = \frac{d}{5} = \frac{32}{5} = 6.4$ in. (use 5 in. spacing)

$$\frac{144 - 4}{5} = 28 \text{ spaces}$$

Use #3 bars at 5 in. throughout the span of beam.



PROBLEMS

5.1 Determine the shear strenght V_c, for the cross section of the beam as sketched in Fig. P5.1. Assume $DL = 3.5$ k/ft and $LL = 6$ k/ft. Use $f_y = 55$ ksi, $f'_c = 4.5$ ksi and $A_s = 3.81$ in^2.

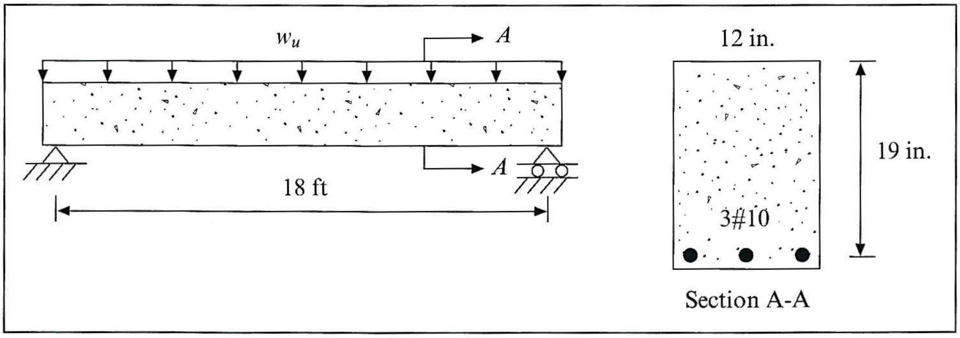


Figure P5.1

5.2 Recalculate the shear strength V_c for Prob. P5.1 by using SI units.

5.3 The beam of Fig. P5.1 is subjected to axial tension force with $N_u = -20$ kips and $f'_c = 3.5$ ksi. Determine the shear strength V_c.

5.4 What is the spacing of #4 stirrups where $A_v = 0.4$ in^2 (ϕ 12 mm, $A_v = 226$ mm^2) for two legs, the factored shear force $V_u = 47$ kips (209 KN). Use $f_y = 60$ ksi (420 MPa) $f'_c = 3$ ksi (20 MPa) and check for $A_{v,\text{min}}$.

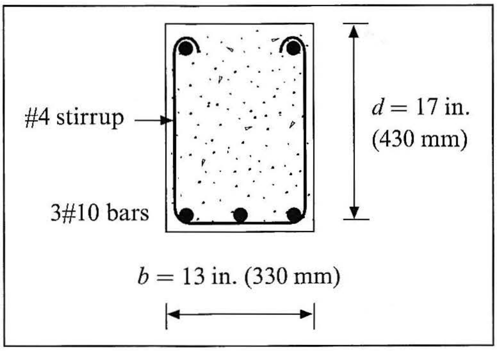


Figure P5.4

5.5 Determine the spacings to be used for #3 stirrups where $A_v = 0.22$ in^2. (ϕ 10 mm, $A_v = 157$ mm^2) as sketched in Fig P5.4. Use $V_u = 62$ kips (156 KN), $f_y = 50$ ksi, $f'_c = 4$ ksi and check for $A_{v,\text{min}}$.

5.6 Design the required spacing of stirrups for simply supported beam, shown in Fig. P5.6 to carry distributed live load of 2.5 k/ft (36.5KN/m) and distributed dead load of 2.0 k/ft (29.2 KN/m) neglected beam weight. If $f_y = 40$ ksi (345 MPa) and $f'_c = 4$ ksi (27.5 MPa).

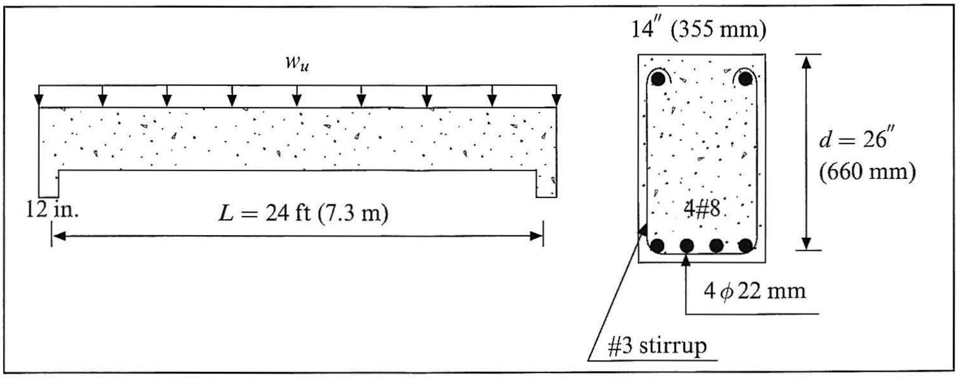


Figure P5.6

Case	f'_c	*DL* k/ft (KN/m)	LL k/ft (KN/m)	f_y ksi (MPa)
1	3.5 ksi (20 MPa)	2 k/ft (14.6 KN/m)	2.75 k/ft (25.5 KN/m)	40 ksi (280 MPa)
2	4 ksi (25 MPa)	2.5 k/ft (22 KN/m)	3.0 k/ft (29.2 KN/m)	45 ksi (310 MPa)
3	4.5 ksi (30 MPa)	2.75 k/ft (29.2 KN/m)	3.25 k/ft (40 KN/m)	45 ksi (310 MPa)
4	5 ksi (35 MPa)	3.0 k/ft (44 KN/m)	3.5 k/ft (58 KN/m)	60 ksi (420 MPa)

5.7 Design the required stirrups for the beam shown in Fig. P5.7. If $f_y = 50$ ksi (350 MPa) and $f'_c = 4.5$ ksi (30 MPa). Use #4 U-shape stirrups.

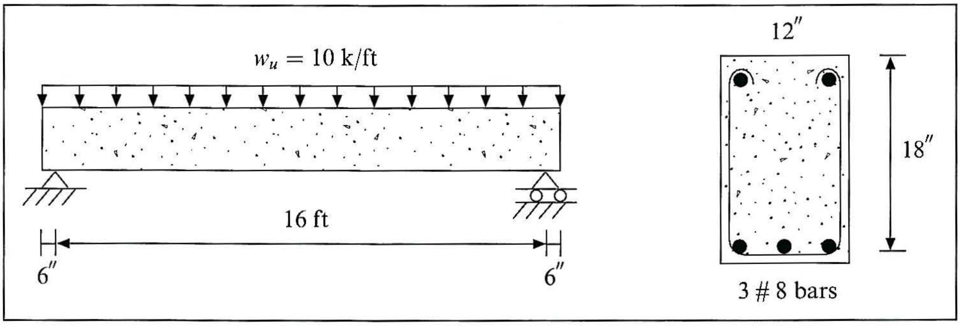


Figure P5.7

5.8 Determine the stirrups for T-beam shown in Fig. P5.8, if $f_y = 50$ ksi (350 MPa) and $f'_c = 4$ ksi (27.5 MPa). Use # 4 stirrups and span $L = 16$ ft.

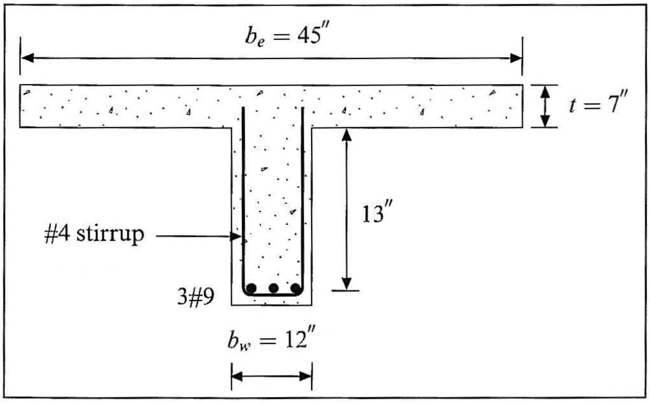


Figure P5.8

5.9 Redesign the stirrups in Prob. P5.8 by using SI units.

5.10 Design a bracket shows in Fig. P5.10 to support a dead load of 40 kips (178 KN) and live load of 60 kips (267 KN). Assume bearing plate 4 in (100 mm) and $N_{uc} = 25$ kips (111 KN). If $f_c' = 4$ ksi (27.5 MPa) and $f_y = 60$ ksi (420 MPa).

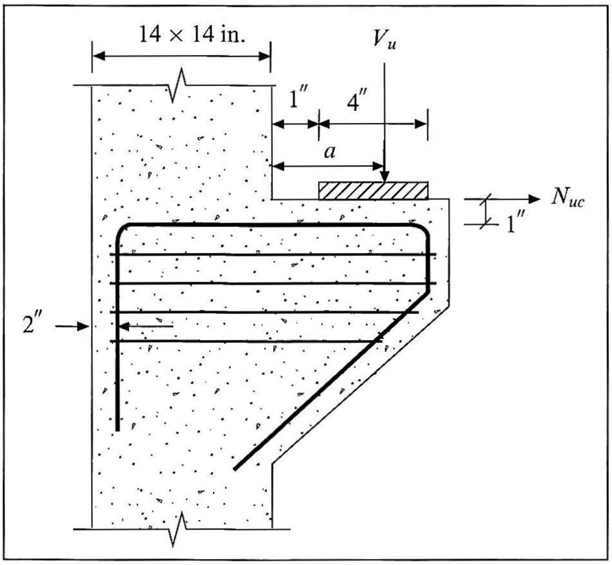


Figure P5.10

5.11 Redesign the bracket, shown in Fig. P5.10 to support $DL = 30$ k and $LL = 50$ kips. Assume $N_{uc} = 17$ kips.

5.12 Determine the shear reinforcement and the steel requirement to use in both vertical and horizontal reinforcement for a simply supported deep beam to carry dead load of 20 kips (89 KN) and live load of 50 kips (222 KN). Assume the unit weight of the concrete γ_c = 150 Ib/ft^3 (2400 kg/m^3), $f_c' = 4$ ksi (27.5 MPa) and $f_y = 50$ ksi (350 MPa).

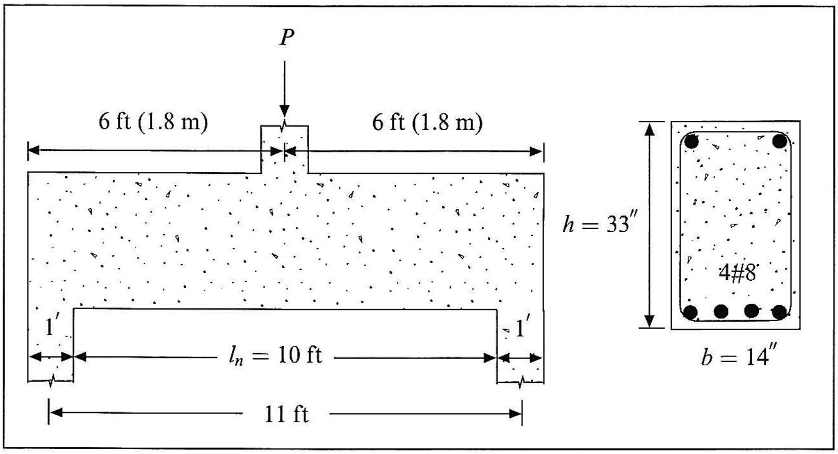


Figure P5.12

5.13 Redesign the vertical and horizontal reinforcement for a simply supported deep beam (Fig. P5.12) to carry distributed load of $w_u = 22$ k/ft (neglect concentrated load). If $f'_c = 3.5$ ksi and $f_y = 50$ ksi.