Lecture (1) - Basic Concepts

Topics

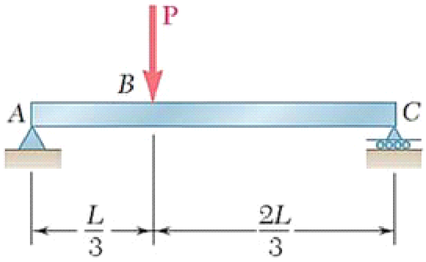
- Loading Types
- Support Types
- Beam Types
- Beam Reactions

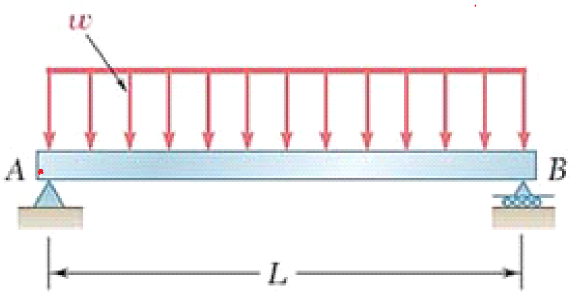
Introduction:

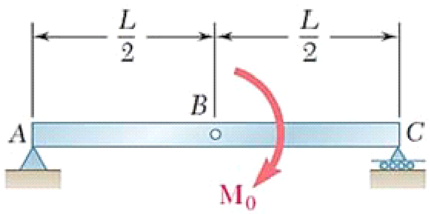
Loading Types:

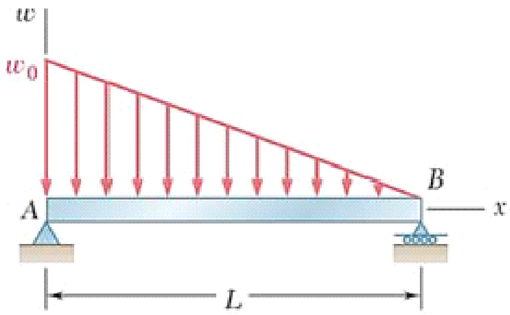
Concentrated Load
- Concentrated Force
- Concentrated Moment

Distributed Load
- Uniformly Distributed Load (UDL)
- Linearly Varying Distributed Load (LVDL)

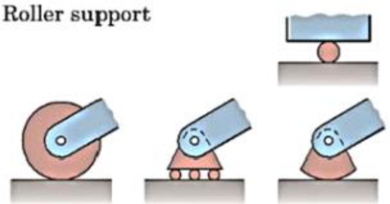
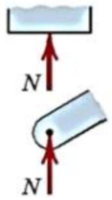
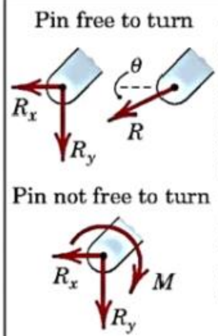
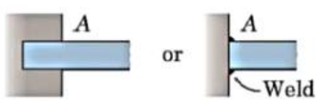
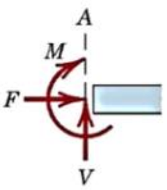








Support Types:

4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.	**Roller**
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.	**Hinge**
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.	**Fixed**

Supports

Roller

Hinge

Fixed

Introduction - Cont.

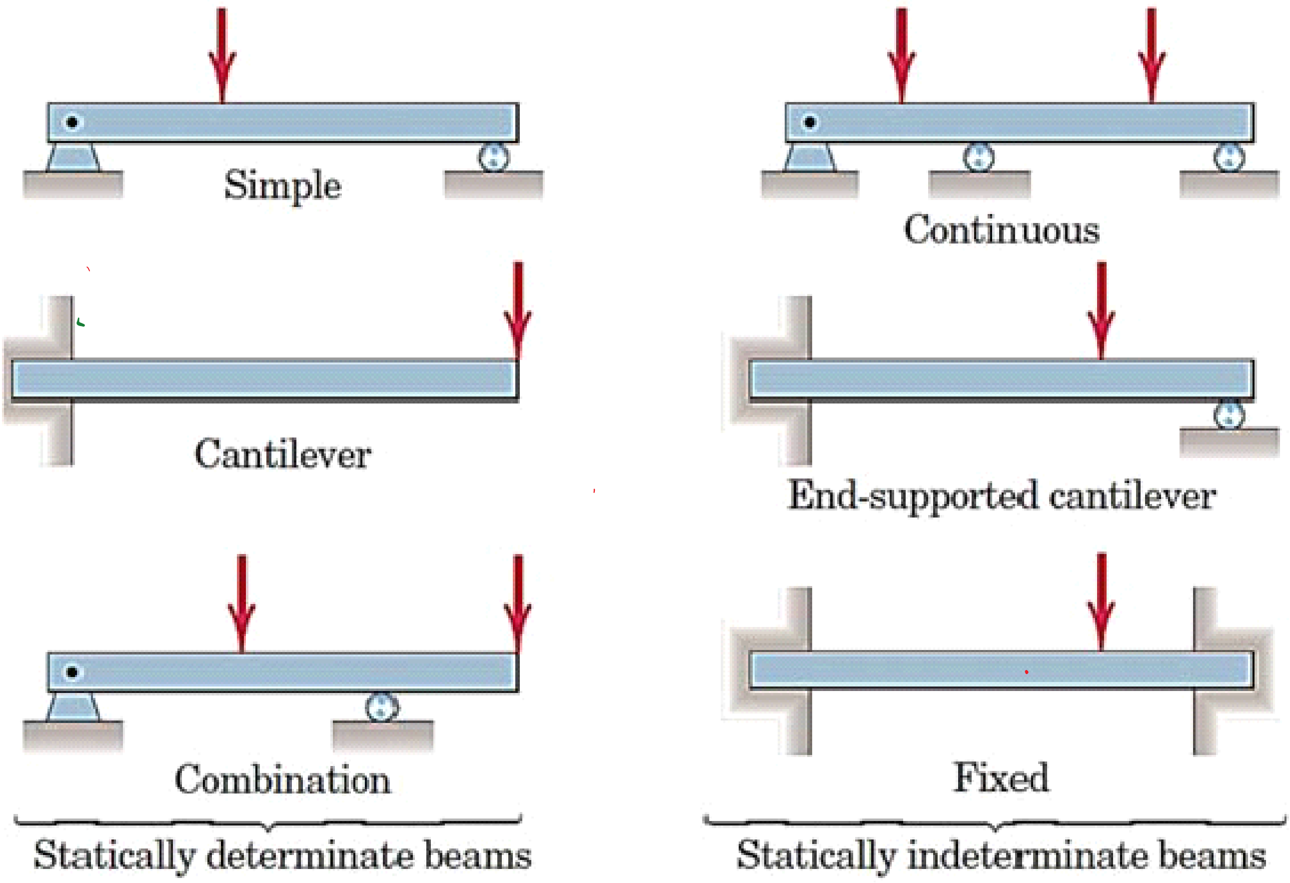
Beam Types:

Statically determinate beams:

- Simply supported beams
- One-sided over-hanging beam
- Two-sided over-hanging beam
- Cantilever beam

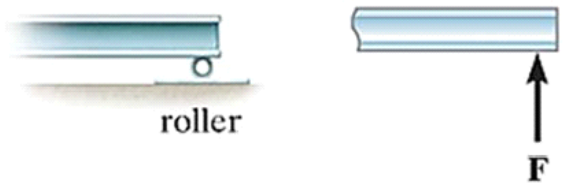
Statically indeterminate beams:

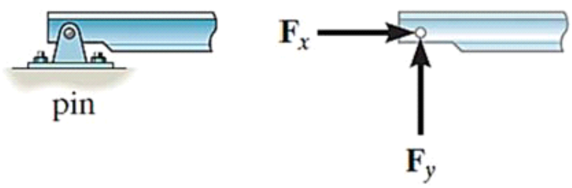
- Continuous beam
- End-supported cantilever
- Fixed at both ends

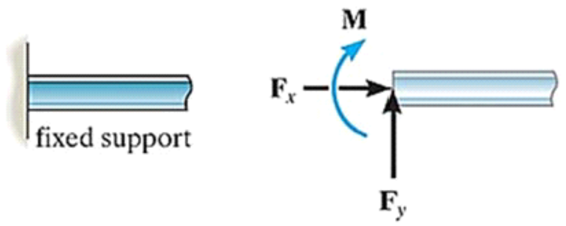


Support Reactions

Beam Reactions:





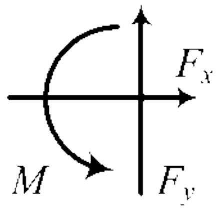


Sign Convention:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

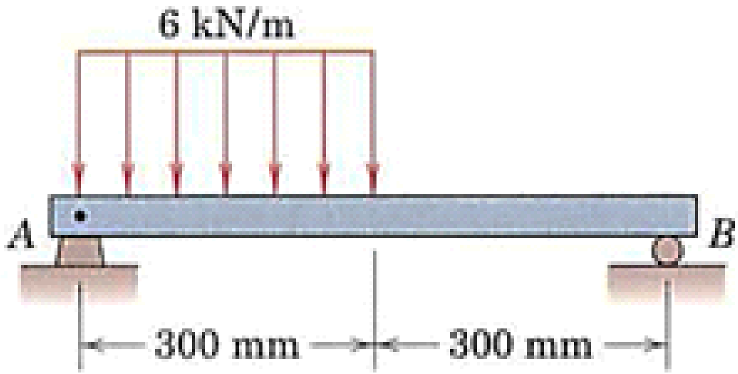


★ **HOW?**

Example (1)

Determine the reactions at A and B for the beam subjected to the uniform load distribution.

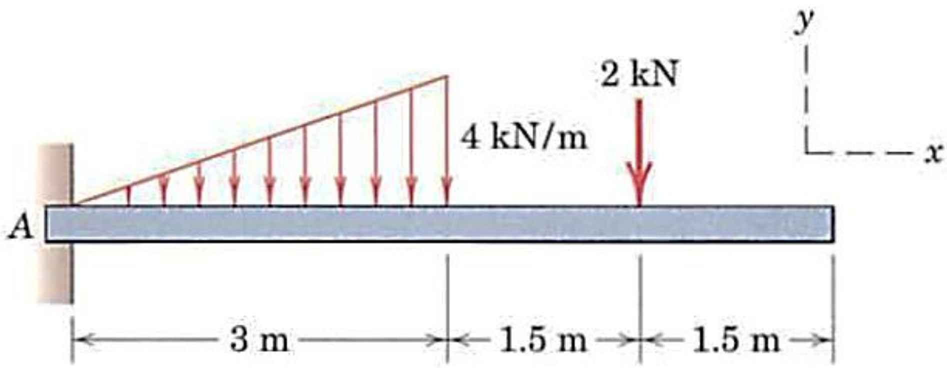
Ans. $R_A = 1.35$ kN, $R_B = 0.45$ kN



Example (2)

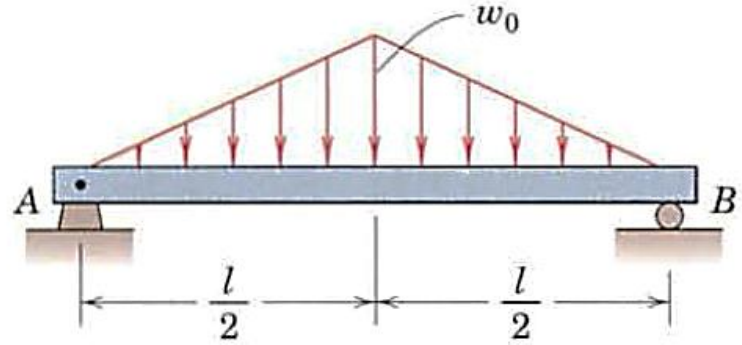
5/97 Determine the reactions at A for the cantilever beam subjected to the distributed and concentrated loads.

Ans. $A_x = 0$, $A_y = 8$ kN, $M_A = 21$ kN·m



Example (3)

5/94 Determine the reactions at the supports A and B for the beam loaded as shown.



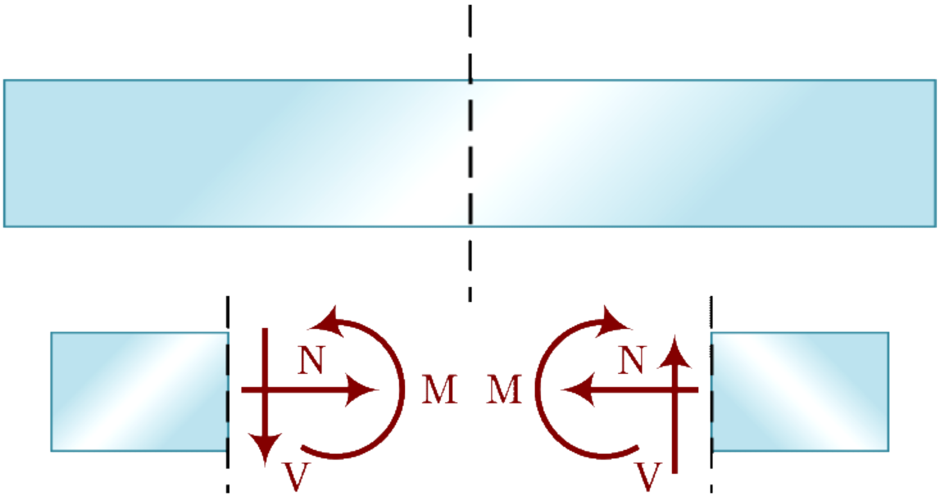
Lecture (2) - Internal Forces (Beams)

Topics

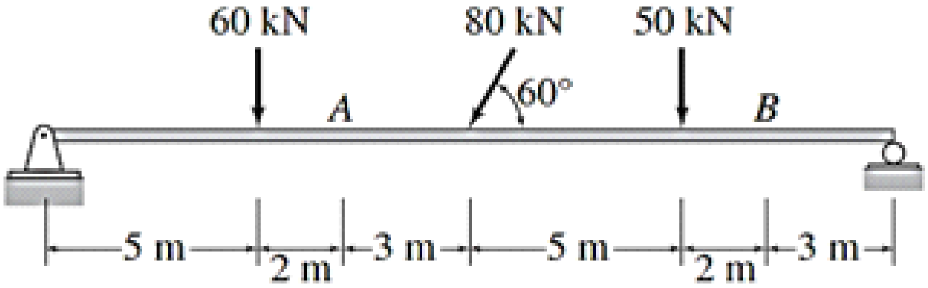
- **Internal Forces in Beams**
    - **Why?**
    - **Sign Convention**
    - **Procedure**

Internal Forces - Beams

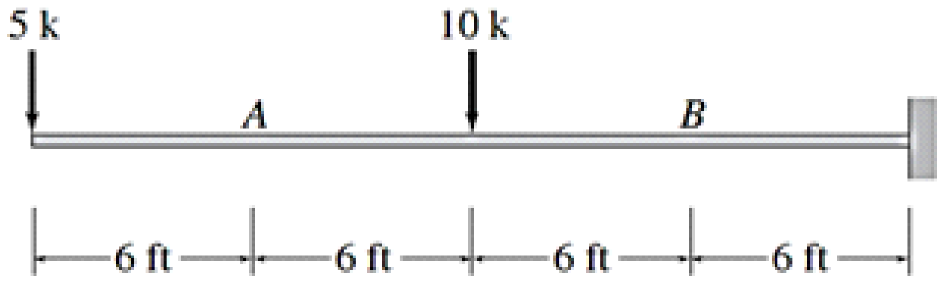
- Why?
- Sign convention
- Procedure



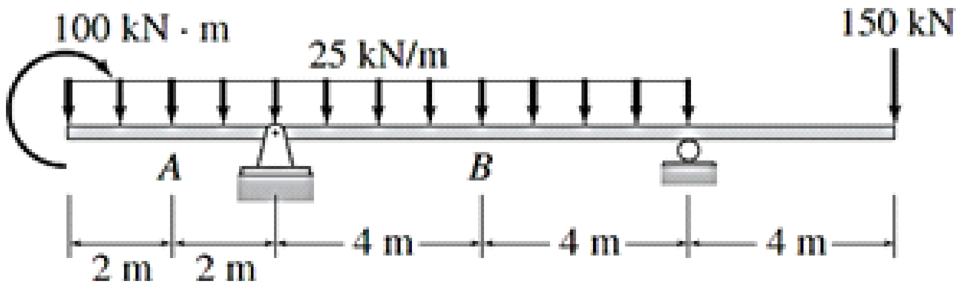
Example (1)



Example (2)



Example (3)



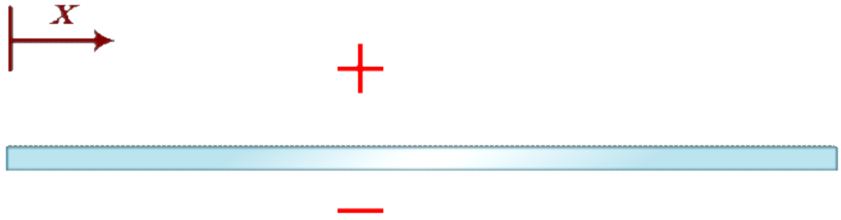
Lecture (3) - V and M Diagrams

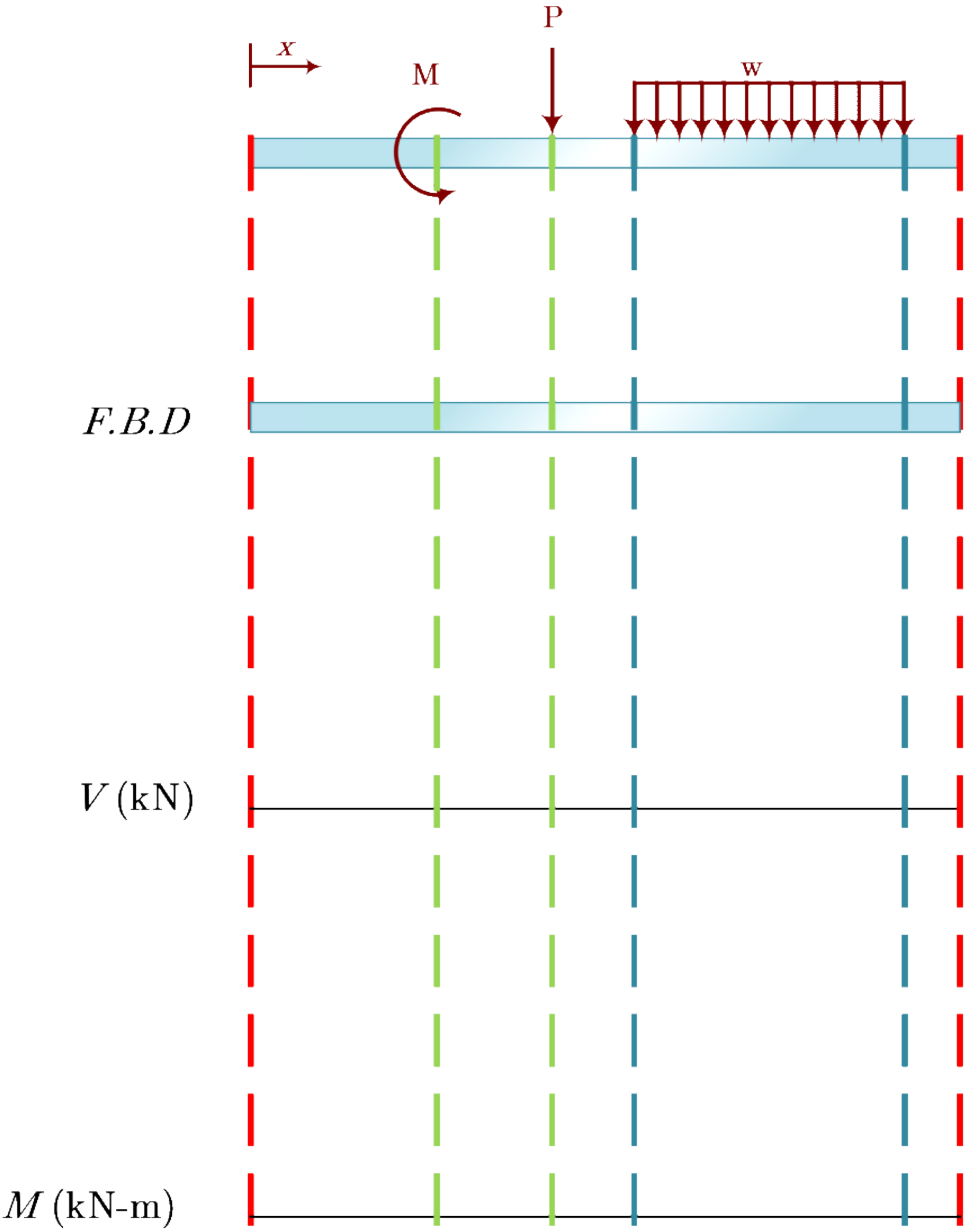
Topics

- Shear Force and Bending Moment Diagrams
  - Why?
  - Sign Convention
  - Procedure

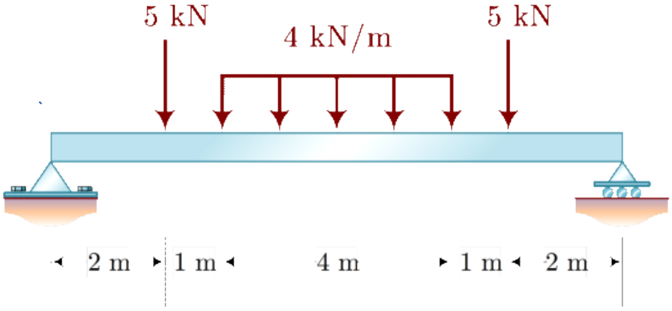
Shear Force and Bending Moment Diagrams

- Why?
- Sign convention
- Procedure

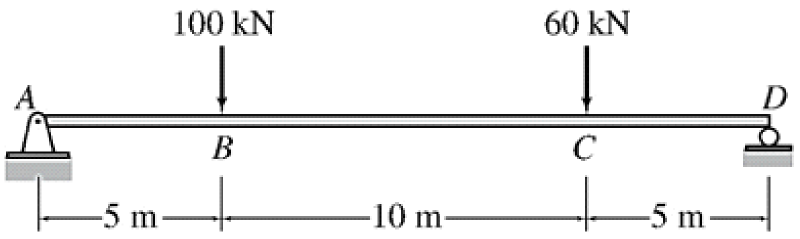


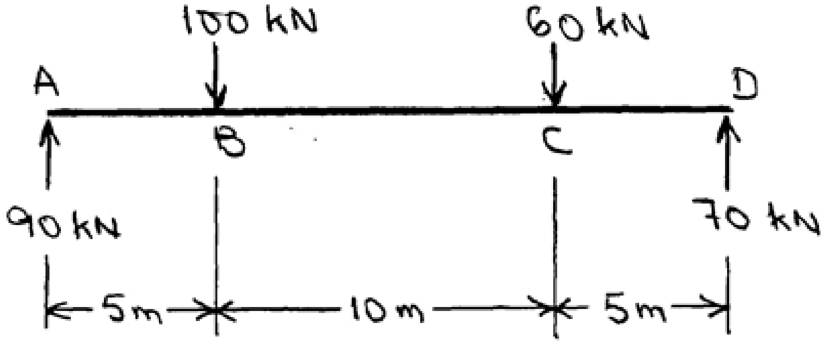


Procedure

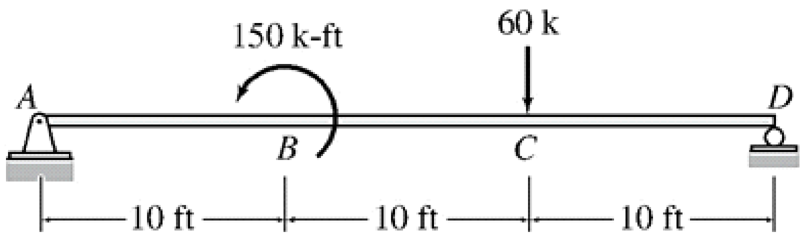


Example (1)

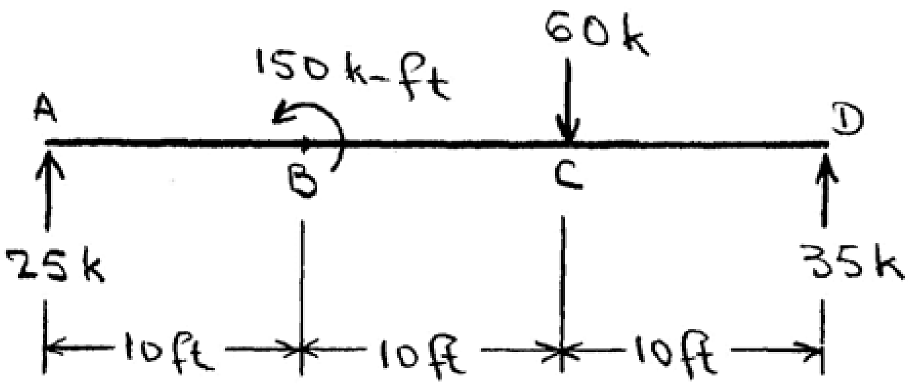




Example (2)

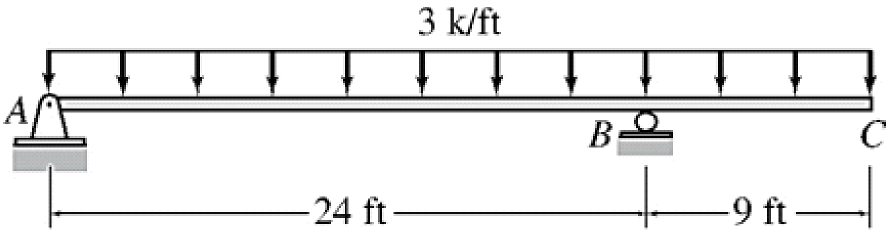


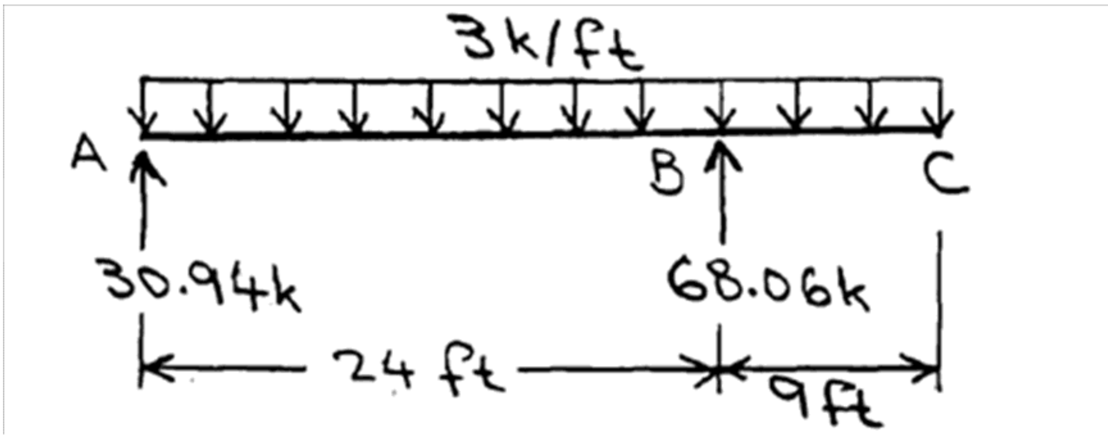
150 k-ft
60 k
A
B
C
D
10 ft
10 ft
10 ft



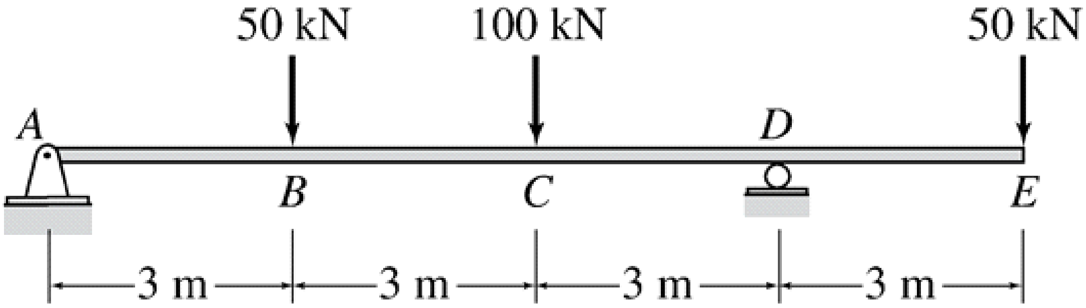
60k
150 k-ft
A
B
C
D
25k
35k
10ft
10ft
10ft

Example (3)

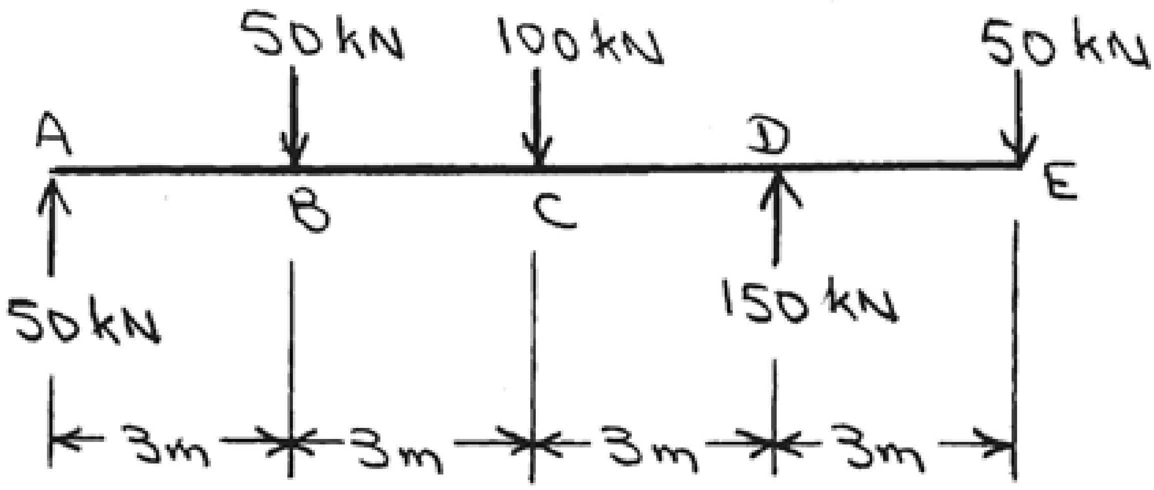




Example (4)

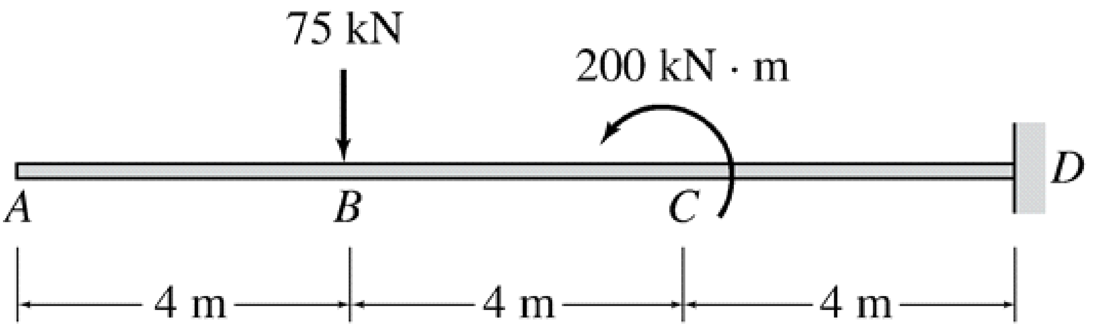


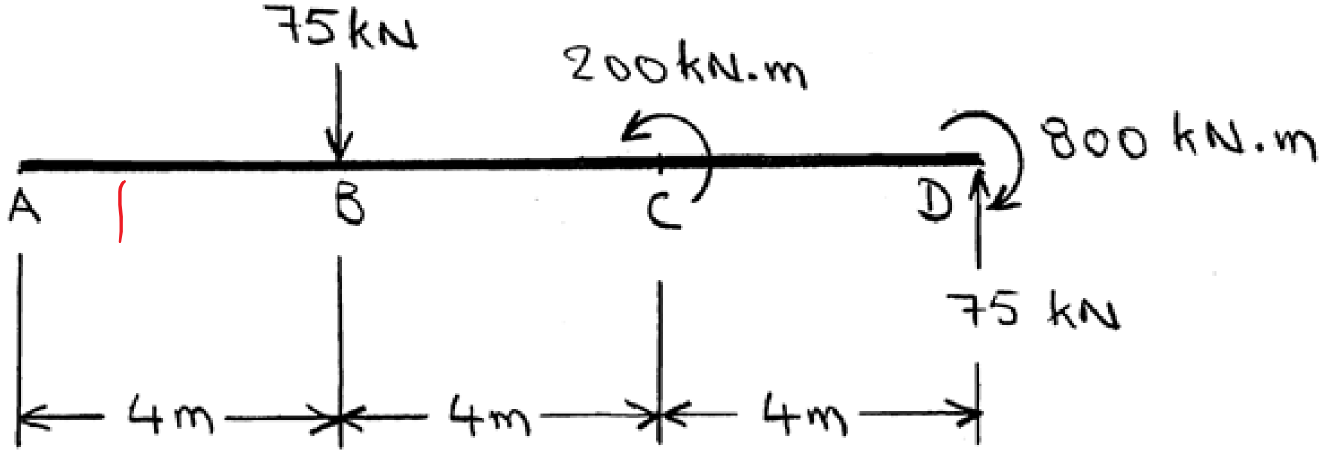
50 kN
100 kN
50 kN
A
D
B
C
E
3 m
3 m
3 m
3 m



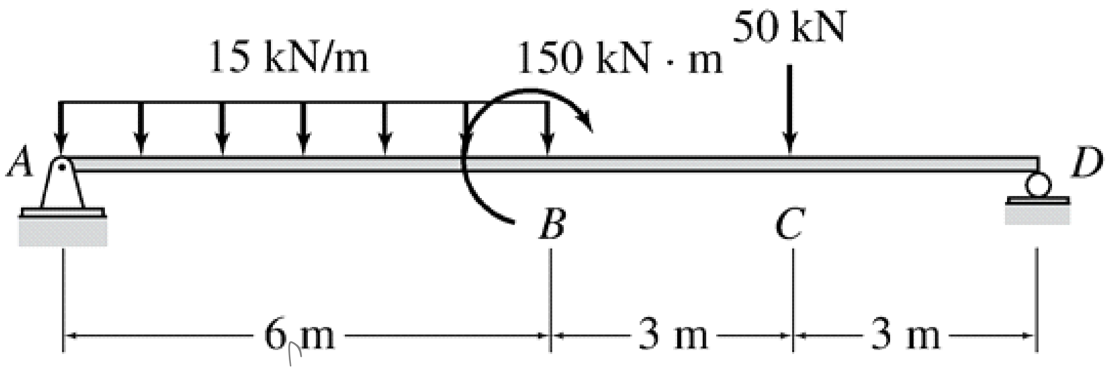
50 kN
100 kN
50 kN
A
D
E
B
C
50 kN
150 kN
3m
3m
3m
3m

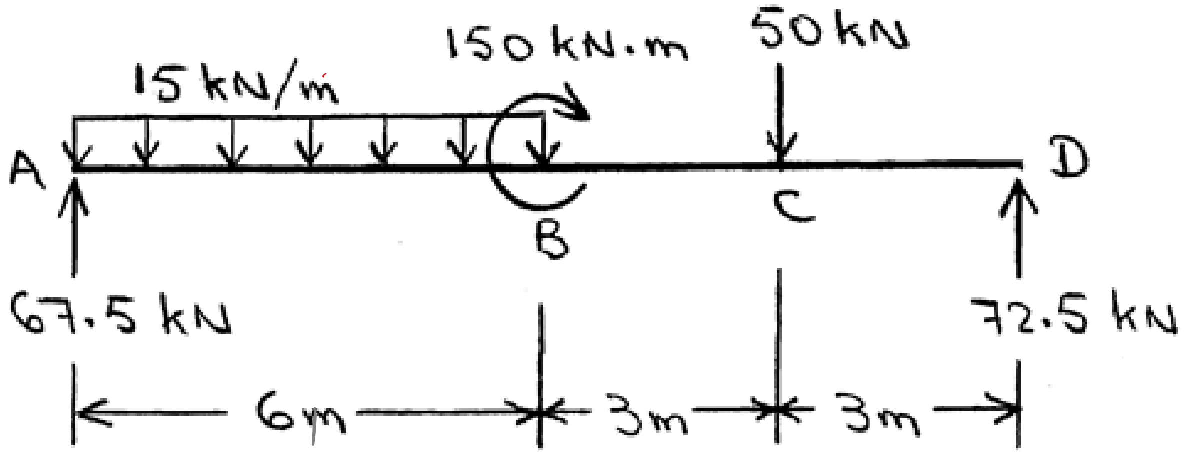
Example (5)


75 kN
200 kN · m
A
B
C
D
4 m
4 m
4 m


75kN
200kN.m
800 kN.m
A
B
C
D
75 kN
4m
4m
4m

Example (6)


15 kN/m
150 kN · m
50 kN
A
B
C
D
6 m
3 m
3 m

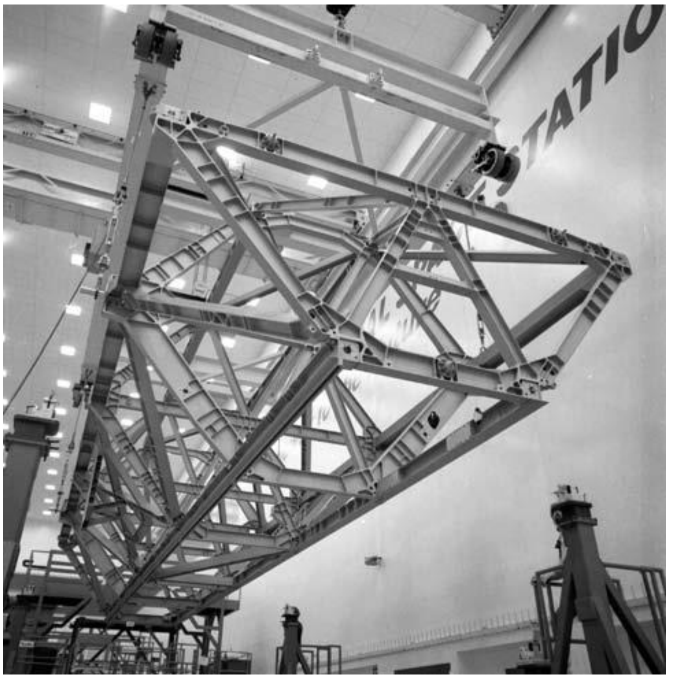

150 kN.m
50 kN
15 kN/m
A
B
C
D
67.5 kN
72.5 kN
6m
3m
3m

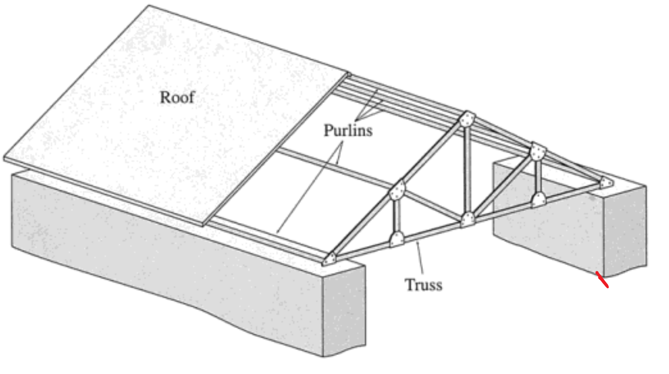
Lecture (4) - Trusses (MOJ)

Topics

- **What is a truss**
  - **Truss main parts (Terminology)**
  - **Assumptions**
  - **Procedure (MOJ , MOS) & when to use which**
  - **Sign convention (T or C)?**
  - **How to avoid confusion?**
  - **Table for final answer**

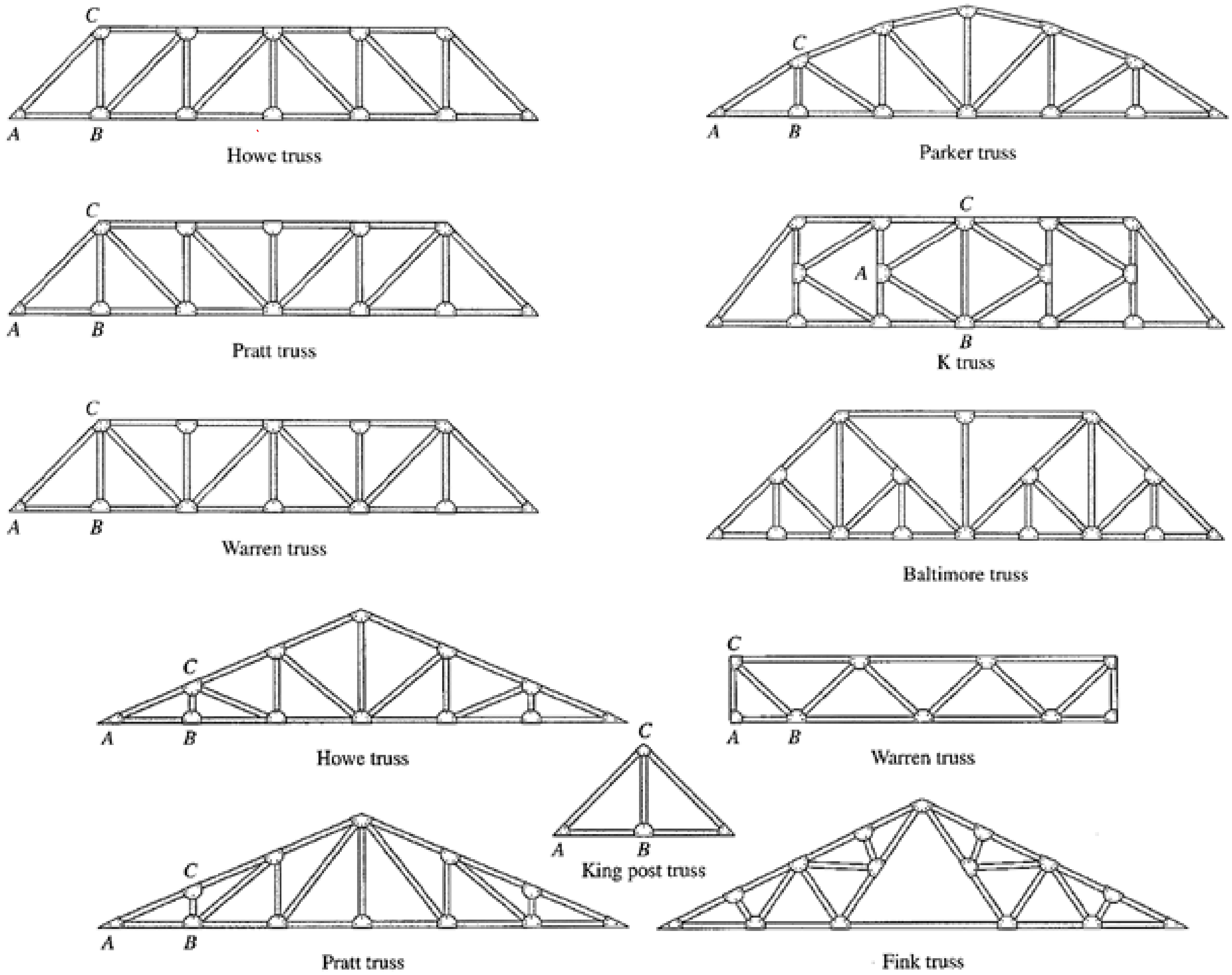
Examples





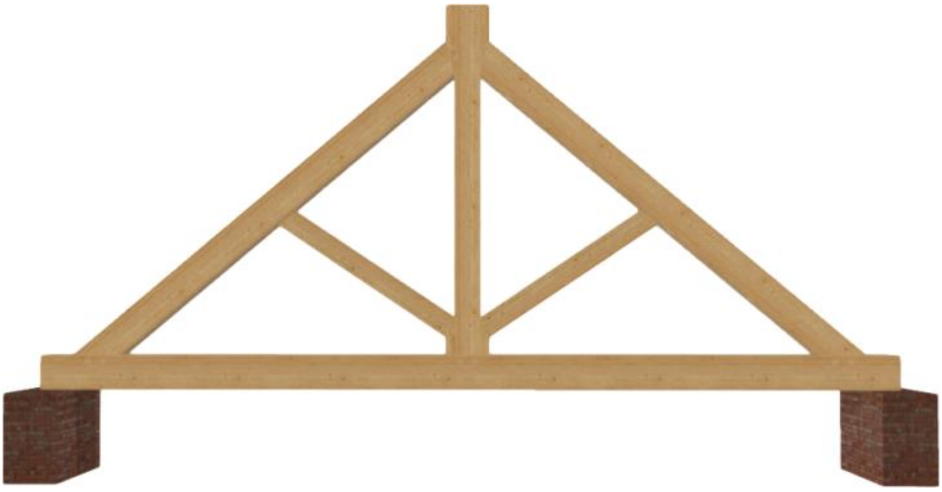
Trusses - Introduction

★ **What are they?**



Truss main parts (Terminology)

Truss main parts (Terminology)



Introduction Cont.

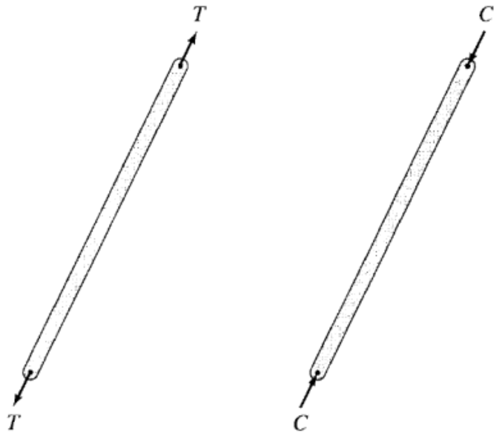
- Assumptions for Analysis of Trusses:

1- All members are connected only at their ends by frictionless hinges (No end moments)

2- All loads and support reactions are applied only at the joints.

3- The centroidal axis of each member coincides with the line connecting the centers of the adjacent joints.

- Two Force Members?!

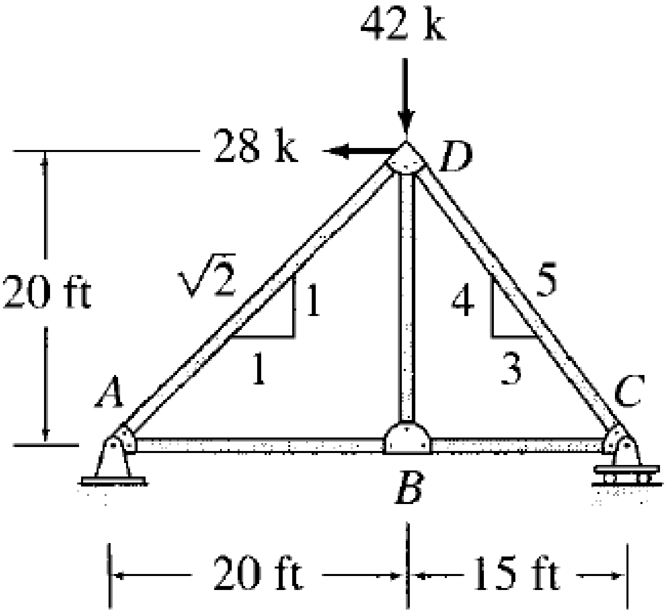


- Procedure (MOJ , MOS)
- When to use MOJ and MOS?
- Sign convention (T or C)?
- How to avoid confusion?
- Table for final answer

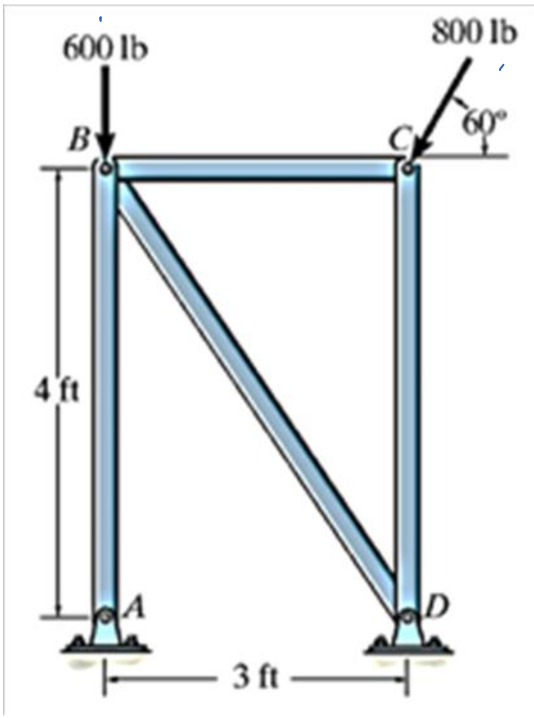
Method of Joints (MOJ)

- Procedure
- Sign convention (T or C)?
- How to avoid confusion?
- Table for final answer

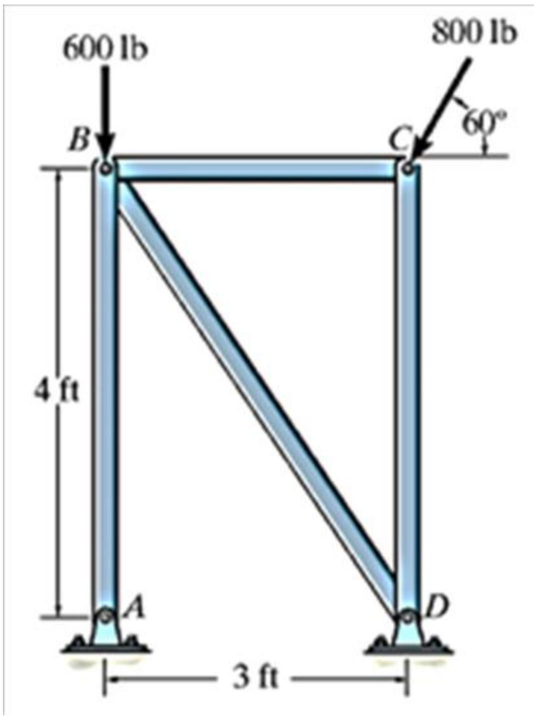
MOJ - Example



MOJ - Example (1)

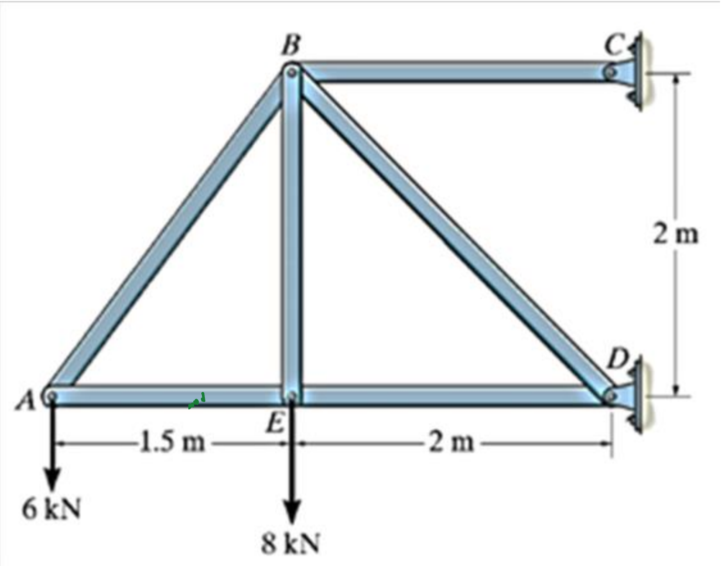


MOJ - Example (1) Cont

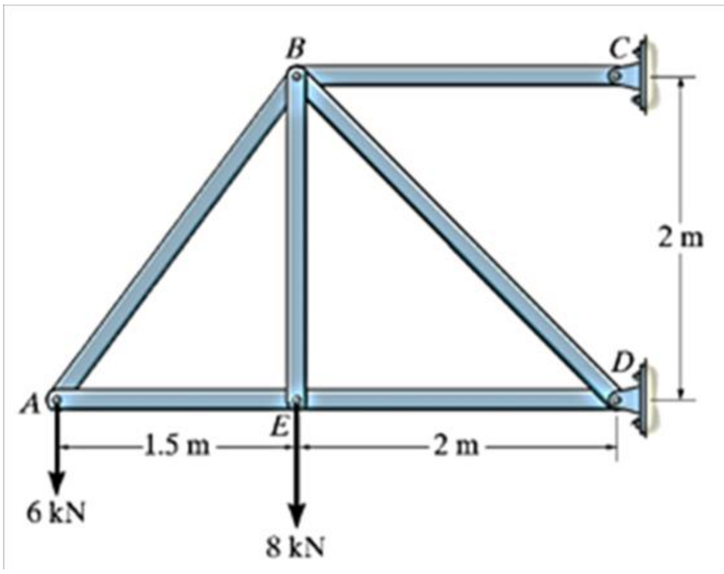


Member	Force (lb)	Type
AB		
BC		
CD		
DB		

MOJ - Example (2)

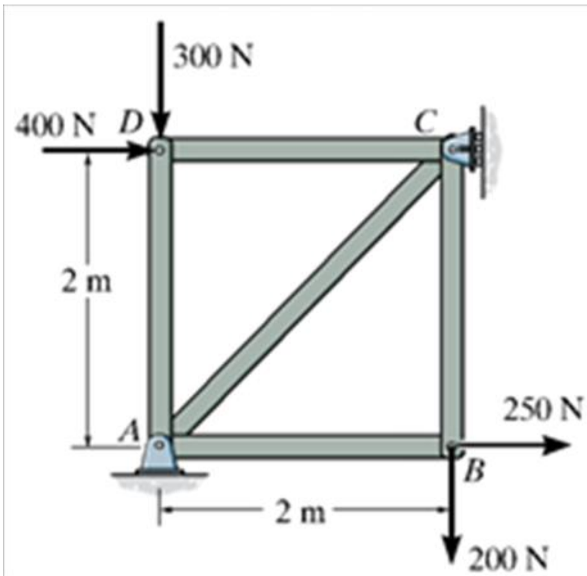


MOJ - Example (2) Cont

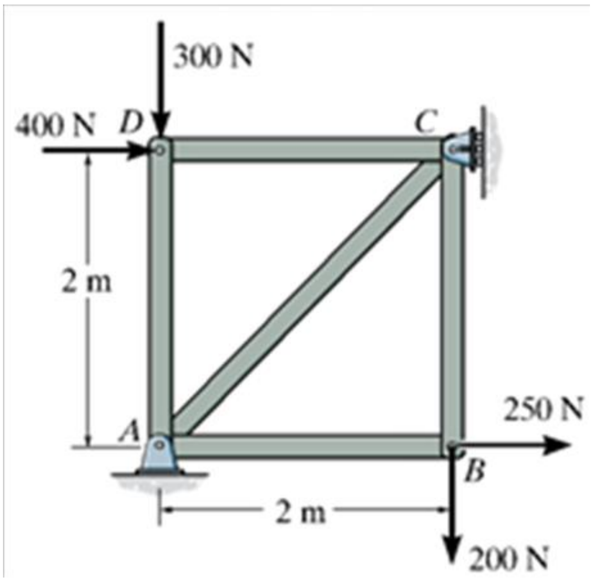


Member	Force (kN)	Type
AB		
BC		
BD		
AE		
ED		

MOJ - Example (3)



MOJ - Example (3) Cont



Member	Force (N)	Type
AB		
BC		
CB		
BA		
AC		

Lecture (5) - Trusses (MOS)

Topics

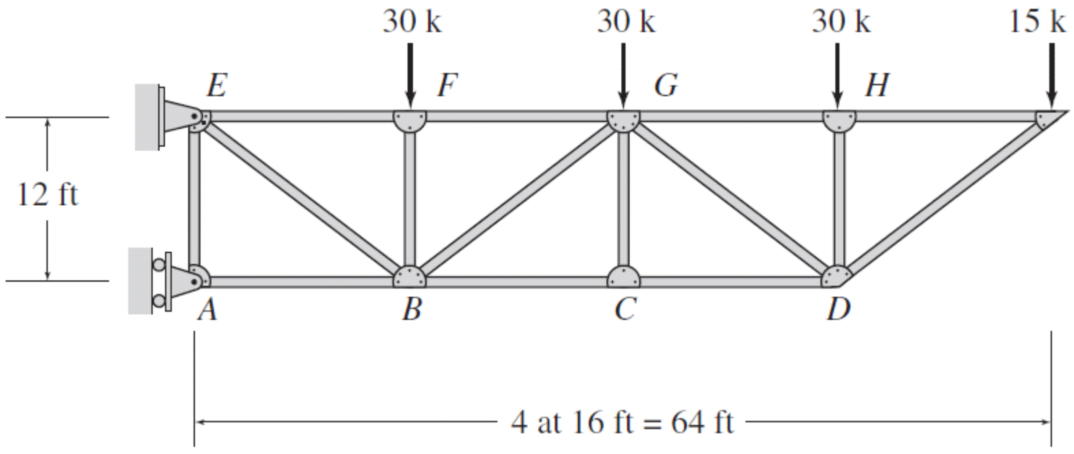
- Idea! & when to use MOS
- Procedure
- Sign convention (T or C)?
- How to avoid confusion?
- Table for final answer

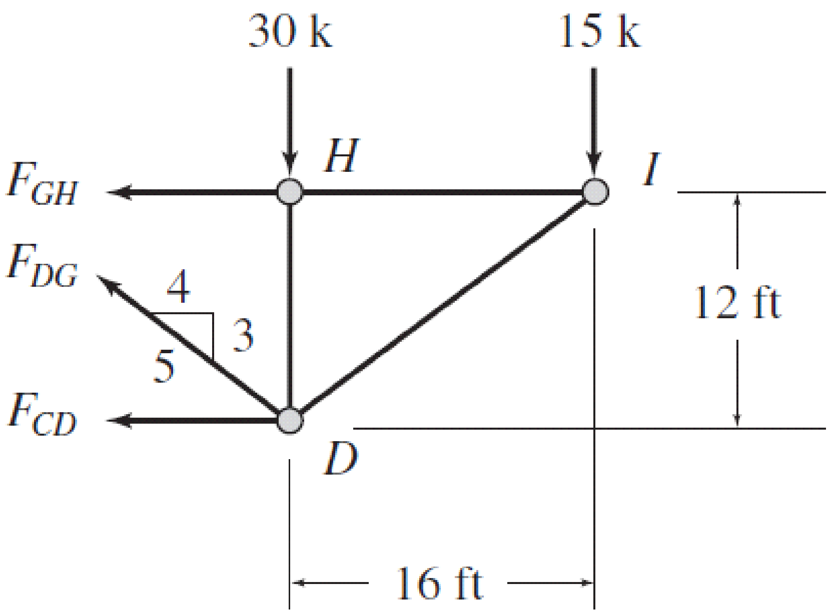
Method of Sections (MOS)

- Idea! And when to use MOS
- Procedure
- Sign convention (T or C)?
- How to avoid confusion?
- Table for final answer

MOS - Example (1)

- Need GH, GD, and CD



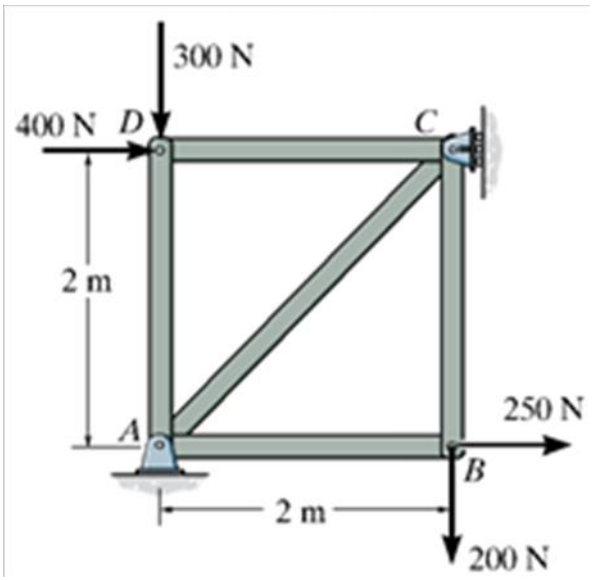


MOS - Example (1) - Cont.

Member	Force (k)	Type
GH		
GD		
CD		

MOS - Example (2)

- Need DC, AC, and AB

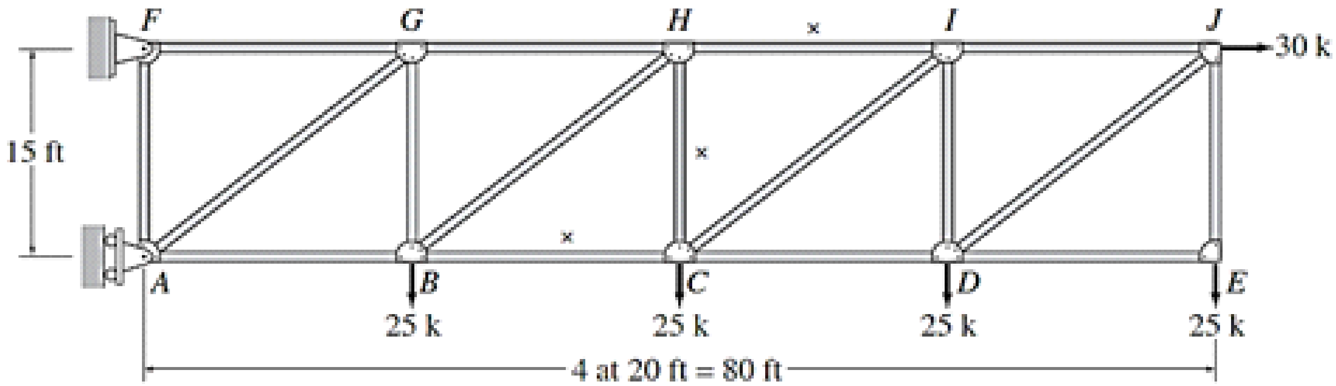


MOS - Example (2) - Cont.

Member	Force (N)	Type
DC		
AC		
AB		

MOS - Example (3)

- Need HI, HC, and BC

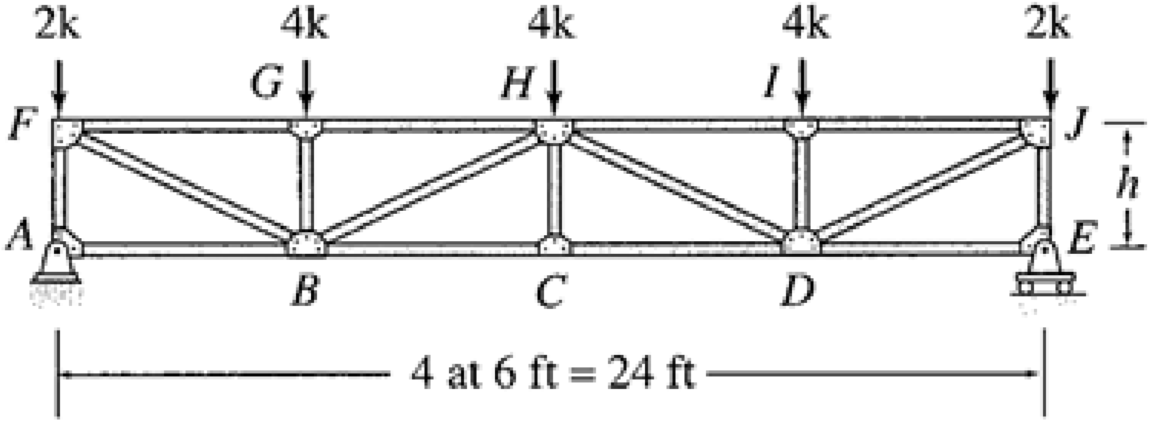


MOS - Example (3) - Cont.

Member	Force (k)	Type
HI		
HC		
BC		

MOS - Example (4)

- Need GH, BH, and BC



MOS - Example (4) - Cont.

Member	Force (k)	Type
GH		
BH		
BC		

Lecture (6) - Internal Forces (Frames)

Topics

- **What are Frames**
- **Frame reactions**
- **Frame internal forces**
- **Sign convention?**
- **How to avoid confusion?**

Frame Analysis - Introduction

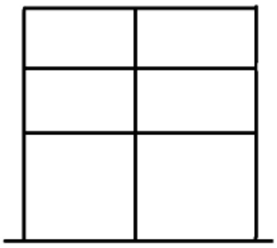
- What are they?

- Structures having the combination of beam, column and slab to resist the lateral and gravity loads.
- Usually used to overcome the large moments developing due to the applied loading.

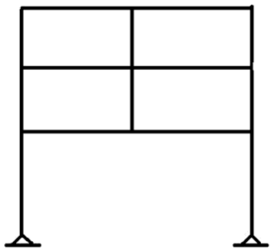
- Frames structures can be differentiated into: →
  - Rigid frame structure
  - Braced frame structure

Difference?!

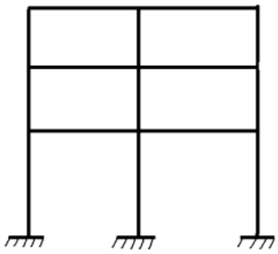
- Rigid frame structure:
  - Pin ended
  - Fixed ended
- Braced frame structure:
  - Gabled frames
  - Portal frames



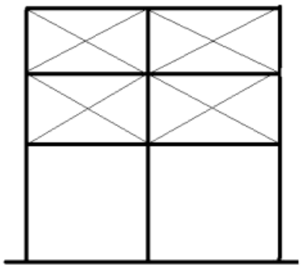
Rigid Structural Frame



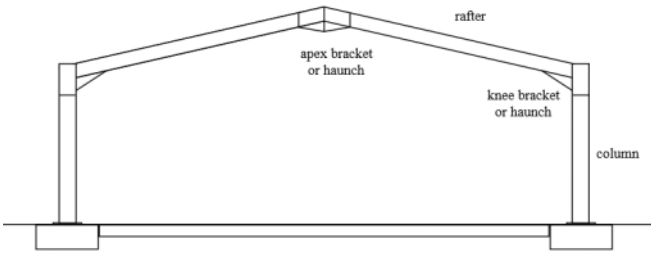
Pin Ended Rigid Structural Frame



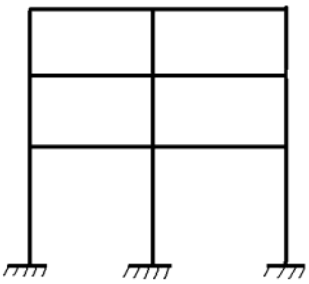
Fixed Ended Rigid Structural Frame



Braced Structural Frame



Gabled Structural Frame

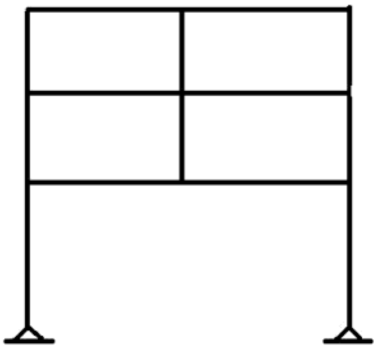


Portal Structural Frame

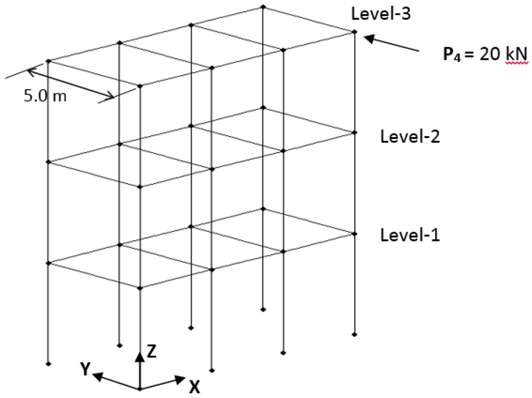
Frame Analysis - Cont.

Types:

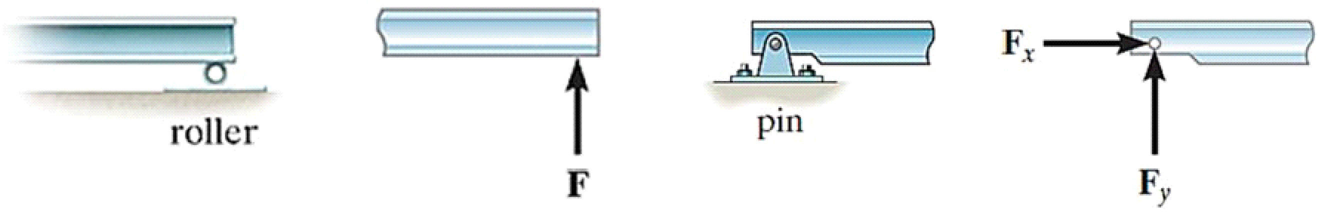
Plane Frame (2D)

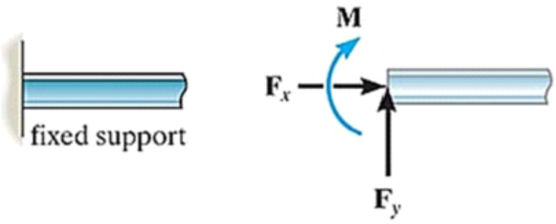


Space Frame (3D)



Reactions





$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

Internal Forces

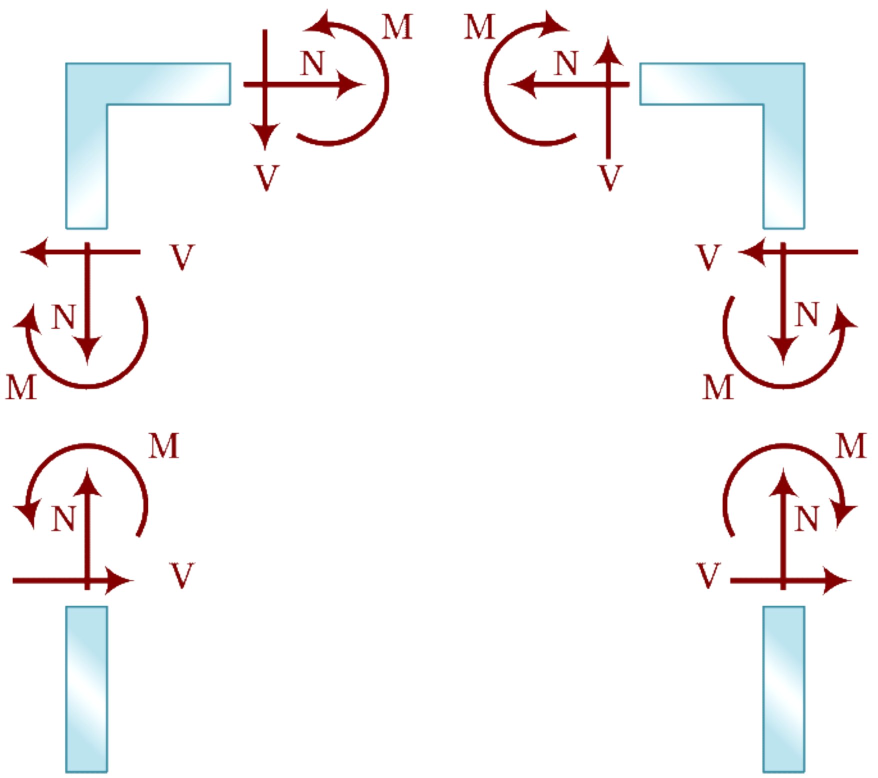
- Procedure
- Sign convention?
- How to avoid confusion?

- Procedure
  - Cut through desired point
  - Get support reactions (if needed)
  - Apply the 3 equilibrium equations

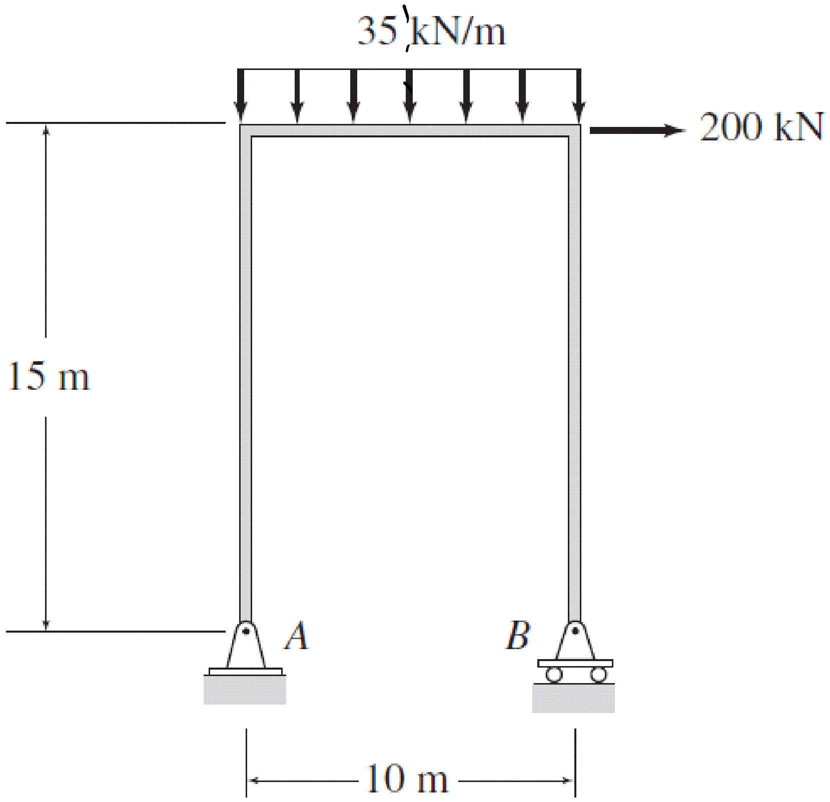
$$\sum F_x = 0$$

$$\sum F_y = 0$$

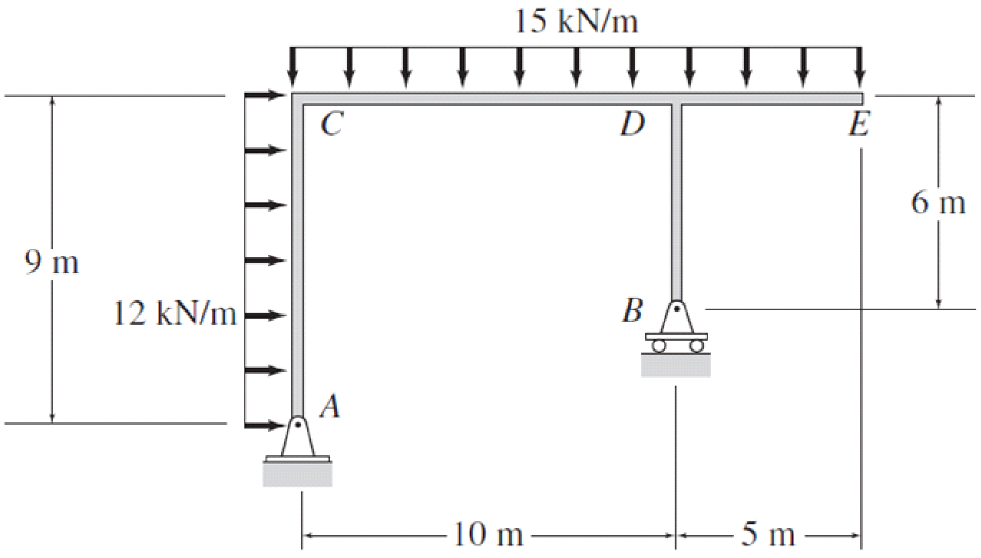
$$\sum M = 0$$



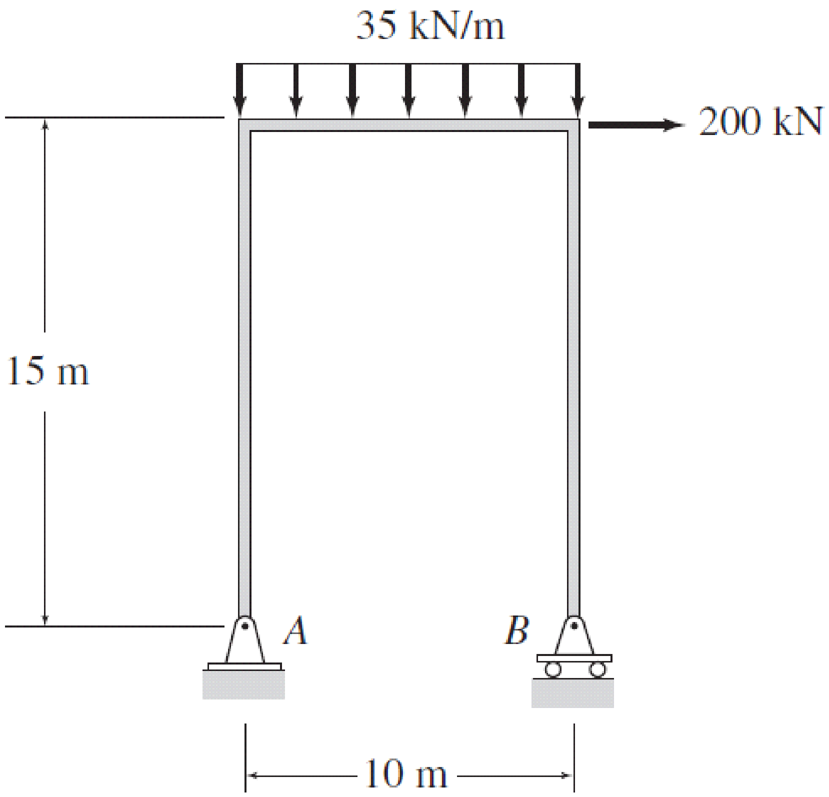
Frame Reactions + Internal Forces Example



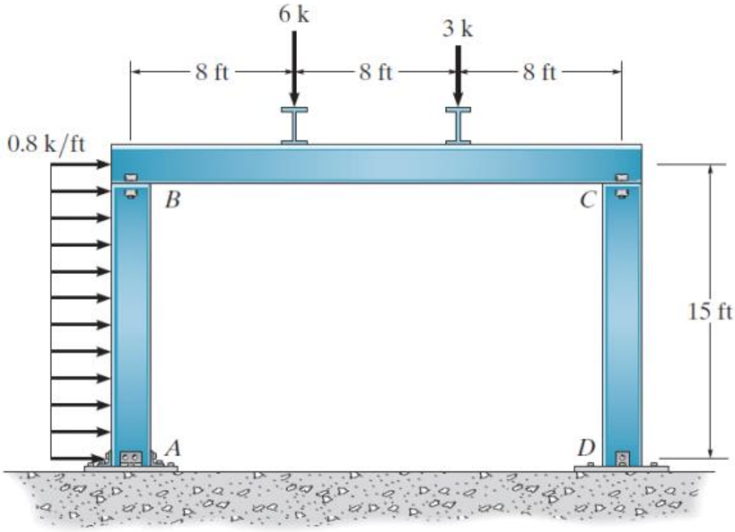
Frame Reactions (2)



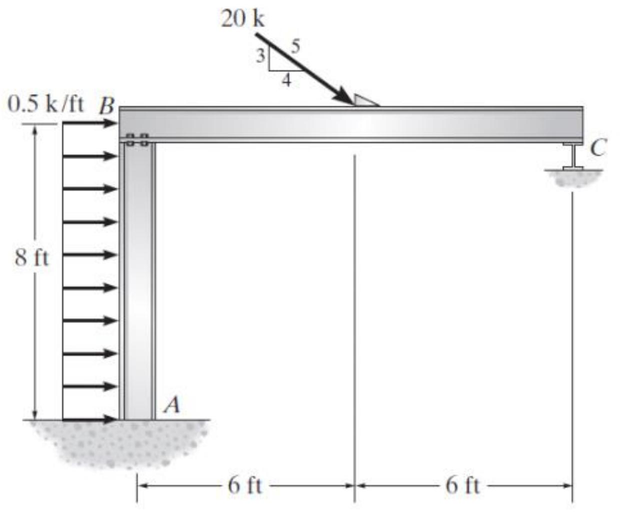
Internal Forces (1)



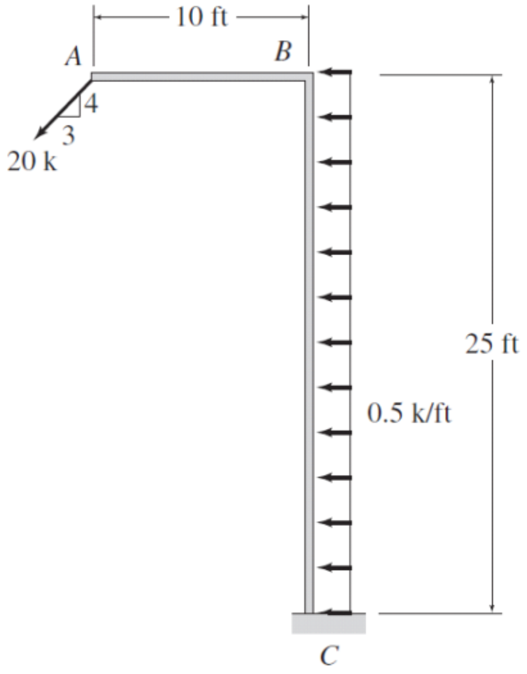
Internal Forces (2)



Internal Forces (3)



Internal Forces (4)



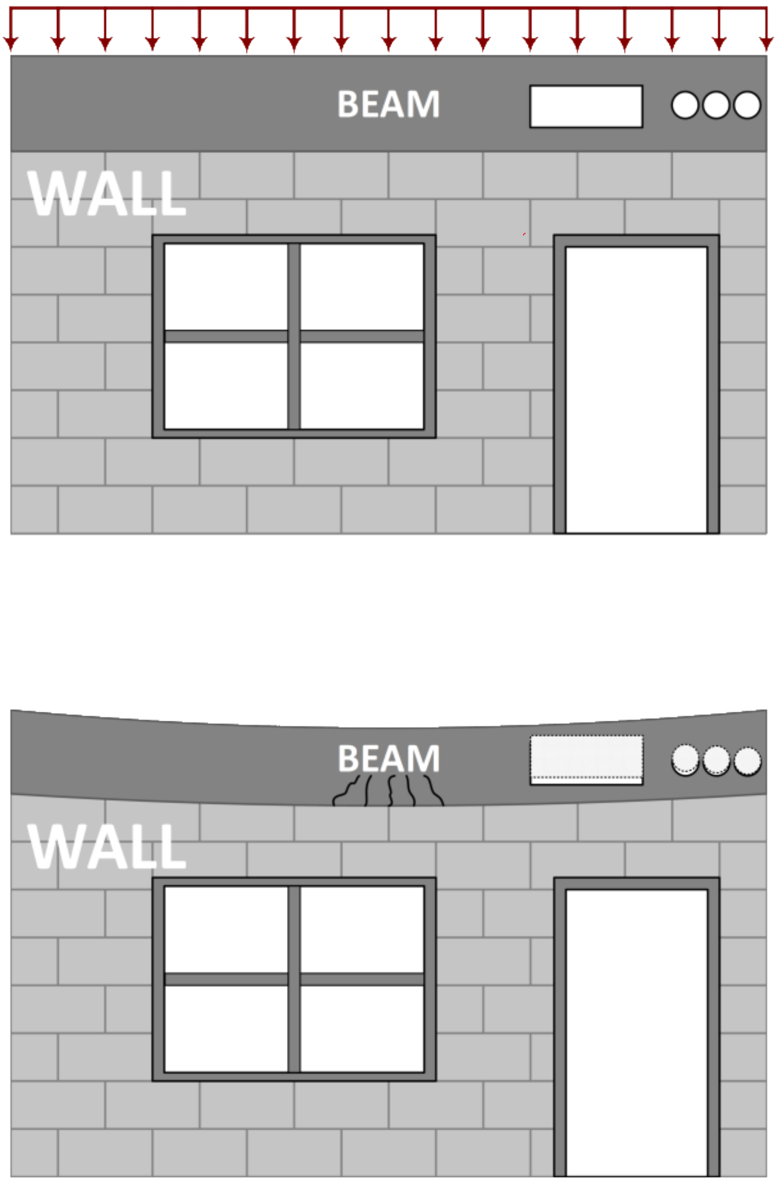
Lecture (7) - Beam Deflections

Topics

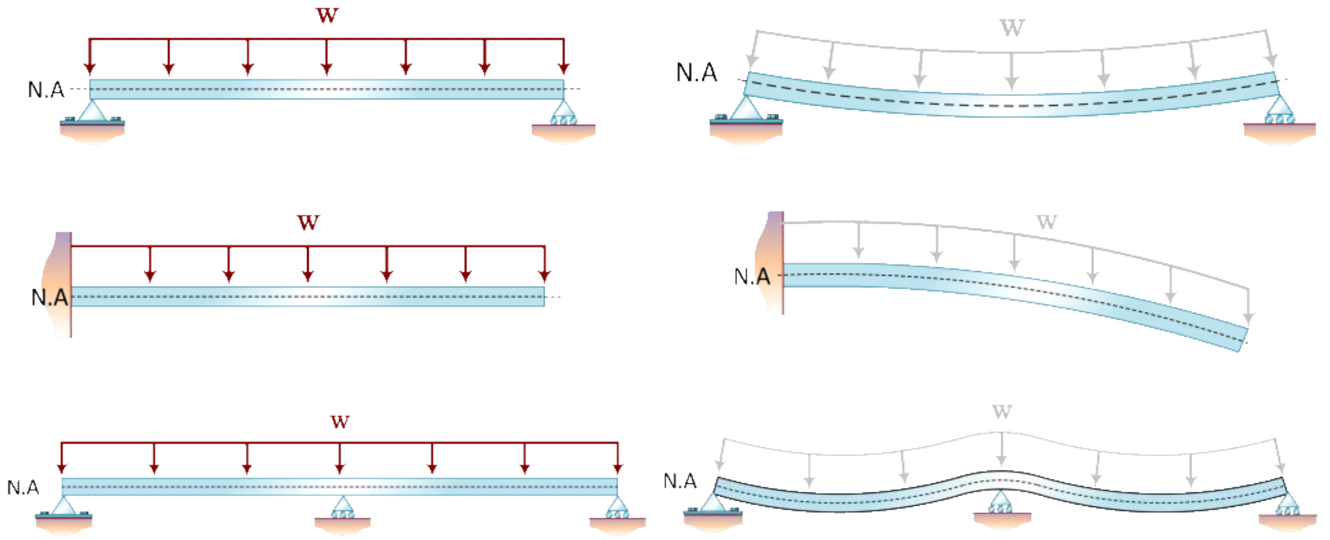
- **Why?**
- **Deflected beams shapes**
- **Factors?**
- **Sign convention**
- **Equations**

Why Study Beam Deflections?

- Forms the basis for analysis and design of indeterminate structures.
- To keep them within acceptable limits to avoid structural and none structural damages.



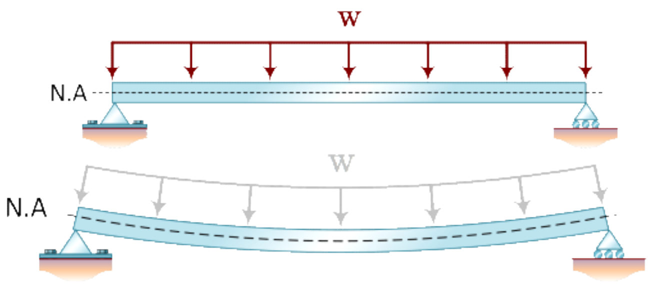
Examples of Beam Deflection



Factors Affecting Beam Deflections

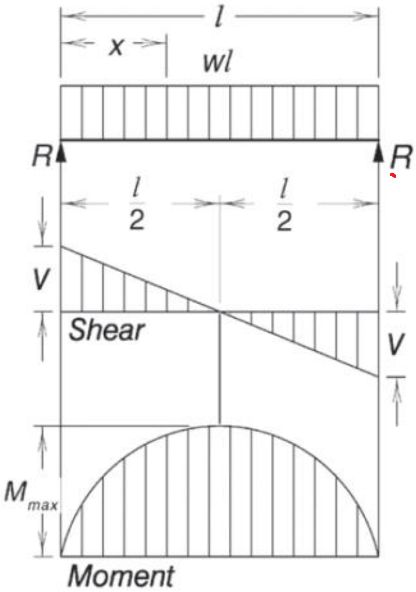
Factor	Symbol	Type
Applied load	*w*	Directly proportional
Span length	*L*	Directly proportional
Modulus of Elasticity	*E*	Inversely proportional
Moment of Inertia	*I*	Inversely proportional

$$\Delta_{max} = \frac{5\ w\ l^4}{384\ E\ I}$$



Tabulated Equations

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

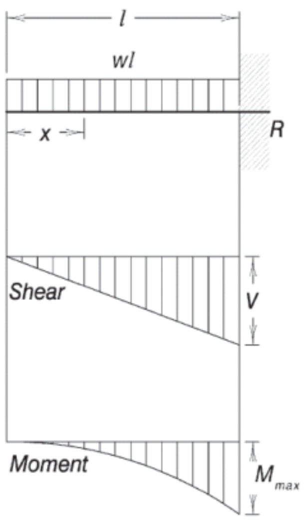
M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

19. CANTILEVERED BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= 4wl$

$R = V$ .... $= wl$

V_x .... $= wx$

M_{max} (at fixed end) .... $= \frac{wl^2}{2}$

M_x .... $= \frac{wx^2}{2}$

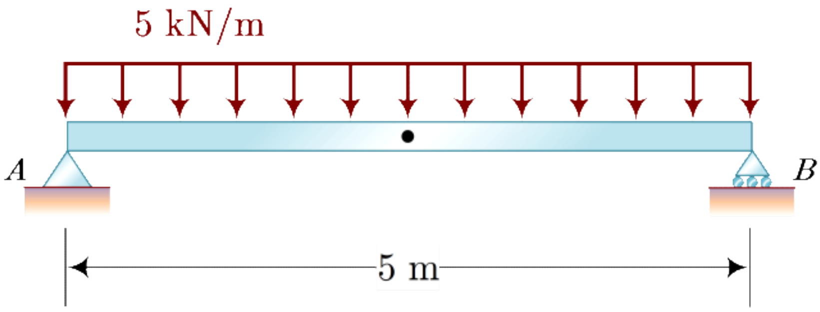
Δ_{max} (at free end) .... $= \frac{wl^4}{8EI}$

Δ_x .... $= \frac{w}{24EI}\left(x^4 - 4l^3x + 3l^4\right)$

Example

For the beam shown in the figure below, calculate the deflection mid-span of the beam shown in the figure. Given:

$$E = 200\,\text{GPa}, \quad I = 200 \times 10^{6}\ \text{mm}^{4}$$

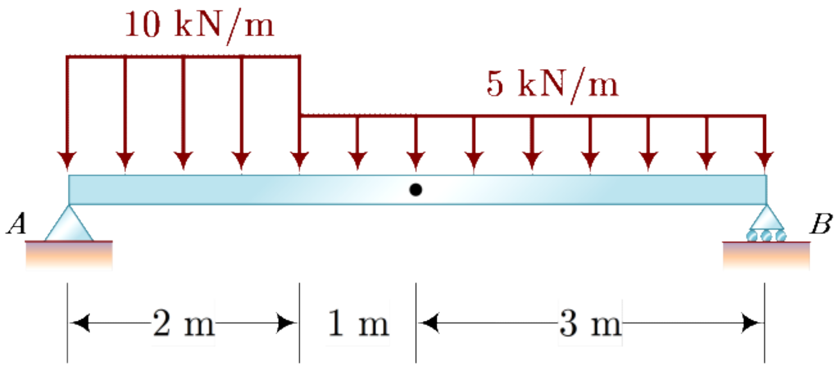


$$\Delta_{max} \quad \text{(at center)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad = \frac{5wl^4}{384\,EI}$$

Example (2)

For the beam shown in the figure below, calculate the deflection of the beam at the mid-span. Given:

$E = 95\,\text{GPa}, \quad I = 100 \times 10^6\ \text{mm}^4$



Lecture (8) - Loads on Structures

Topics

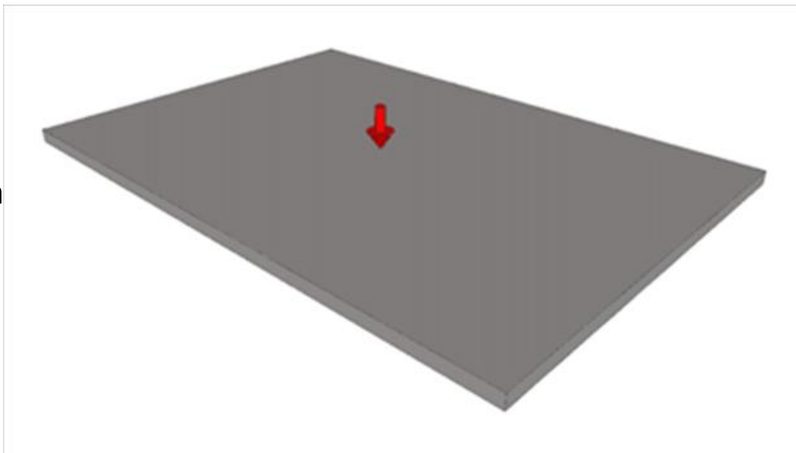
- **Load Types**
- **Load Categories**
- **Load Combinations**
- **D vs. L**
- **Load Paths - Tributary Areas (Columns)**
- **Load Paths - Tributary Areas (Beams)**

Loads on Structures - Introduction

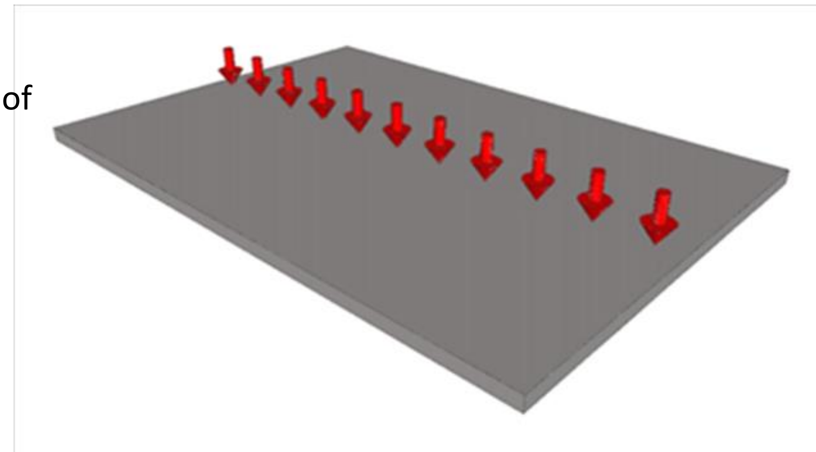
- Load Types
- Load Categories
- Load Combinations
- D vs. L

Load Types:

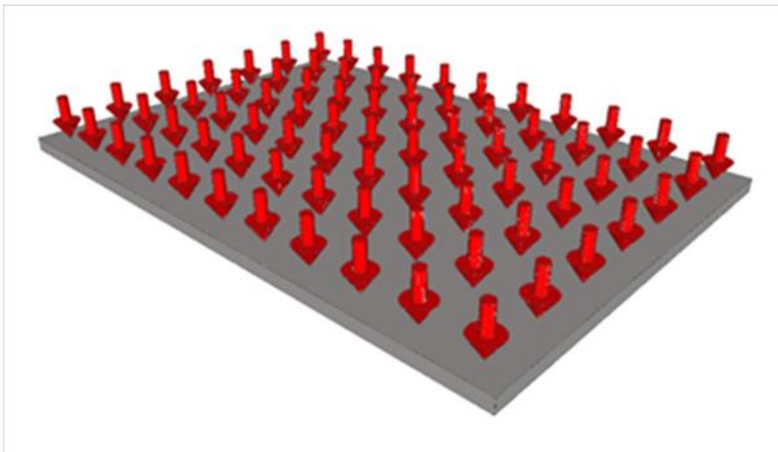
- **Concentrated loads:**
  - Applied over relatively small area
  - **Examples: Column loads, Vehicular wheel load**



- **Line Loads:**
  - Distributed along a narrow strip of the structure
  - **Examples: Beam self-weight, weight of wall or partition**



- **Surface Line Loads:**
  - Distributed over an area of the structure
  - **Examples: floor and roof loads**



Load Categories - Dead Load

- **Dead Loads:**
  - Various structural members & objects that are ***permanently*** attached.
  - Can be **calculated** knowing the **densities** and **dimensions** of the structural components.
  - Unit weights of typical **building materials (codes and standards)**
  - Unit weights of **service equipment (Manufactures)**
  - **Small** structures (**small** error) - can be **ignored**
  - **Multistory** structures (**high** error) - **cannot be ignored**.

✓	Roof Slab	✓	Walls
✓	Floor Slab	✓	Windows
✓	Beams	✓	Plumbing
✓	Girders	✓	Electrical Fixtures
✓	Columns	✓	Ducts

Construction Materials	Density (kg/m^3)*	Construction Materials	Density (kg/m^3)*
Water	1000	Cement mortar	2080
Sandy soil	1800	Concrete (P.C.C)	2400
Clay soil	1900	Concrete (R.C.C)	2500
Gravel soil	2000	Steel	7850
Sandstone	2000	Cast iron	7208
Silt	2100	Copper	8940
Asphalt	721	Iron	7850
Cement	1440	Glass	2580

***Note: to convert density from kg/m^3 to kN/m^3, multiply by (9.806×10^{-3})**

Load Categories - Live Load

- **Live Loads:**
  - **Vertical** loads due to **human occupancy, snow, rain ponding, furniture, partition walls and moveable equipment**.
  - **Horizontal (lateral)** loads due to **wind, earthquake, water pressure, blast/explosion, collision**, etc.
  - They can be caused by **weights of objects temporarily** placed on a structure, **moving vehicles, or natural forces**.

✓ Building Loads	✓ Snow Load
✓ Highway Bridge Loads	✓ Earthquake Loads
✓ Railroad Bridge Loads	✓ Hydrostatic Pressure
✓ Impact Loads	✓ Soil Pressure
✓ Wind Loads	✓ Other Environmental Loads

- **Floors** are assumed to be under **uniform live loads** which **depend on the purpose for which the building is designed**.
- These **loads are usually tabulated** in adapted code.
- These **values include some protection against overloading**, e**mergency situations, construction loads, and serviceability requirements due to vibration**.

TABLE 1.4 Minimum Live Loads*

Occupancy or Use	Live Load psf	Live Load kN/m²	Occupancy or Use	Live Load psf	Live Load kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	40	1.92	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			First-floor corridors	100	4.79
Light	125	6.00	Corridors above first floor	80	3.83
Heavy	250	11.97			

*Minimum Live Loads, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, American Society of Civil Engineers.

Load Combinations

- Why?
- How?

$$\sum(\text{Load} \times \text{Load factor}) \leq \text{Resistance} \times \text{resistance factor}$$

$$\sum \gamma_i Q_i \leq \phi R_n$$

γ_i : a load factor

Q_i : applied load

ϕ : resistance factor

R_n : the nominal resistance or strength

ϕR_n : the design strength

$$1.4(D+F) \quad (1)$$

$$1.2(D+F)+1.6(L+H)+0.5(L_r \text{ or } S \text{ or } R) \quad (2)$$

$$1.2(D+F)+1.6(L_r \text{ or } S \text{ or } R)+1.6H+(f_1L \text{ or } 0.5W) \quad (3)$$

$$1.2(D+F)+1.0W+f_1L+1.6H+0.5(L_r \text{ or } S \text{ or } R) \quad (4)$$

$$1.2(D+F)+1.0E+f_1L+1.6H+f_2S \quad (5)$$

$$0.9D+1.0W+1.6H \quad (6)$$

$$0.9(D+F)+1.0E+1.6H \quad (7)$$

Source: International Building Code (2015)

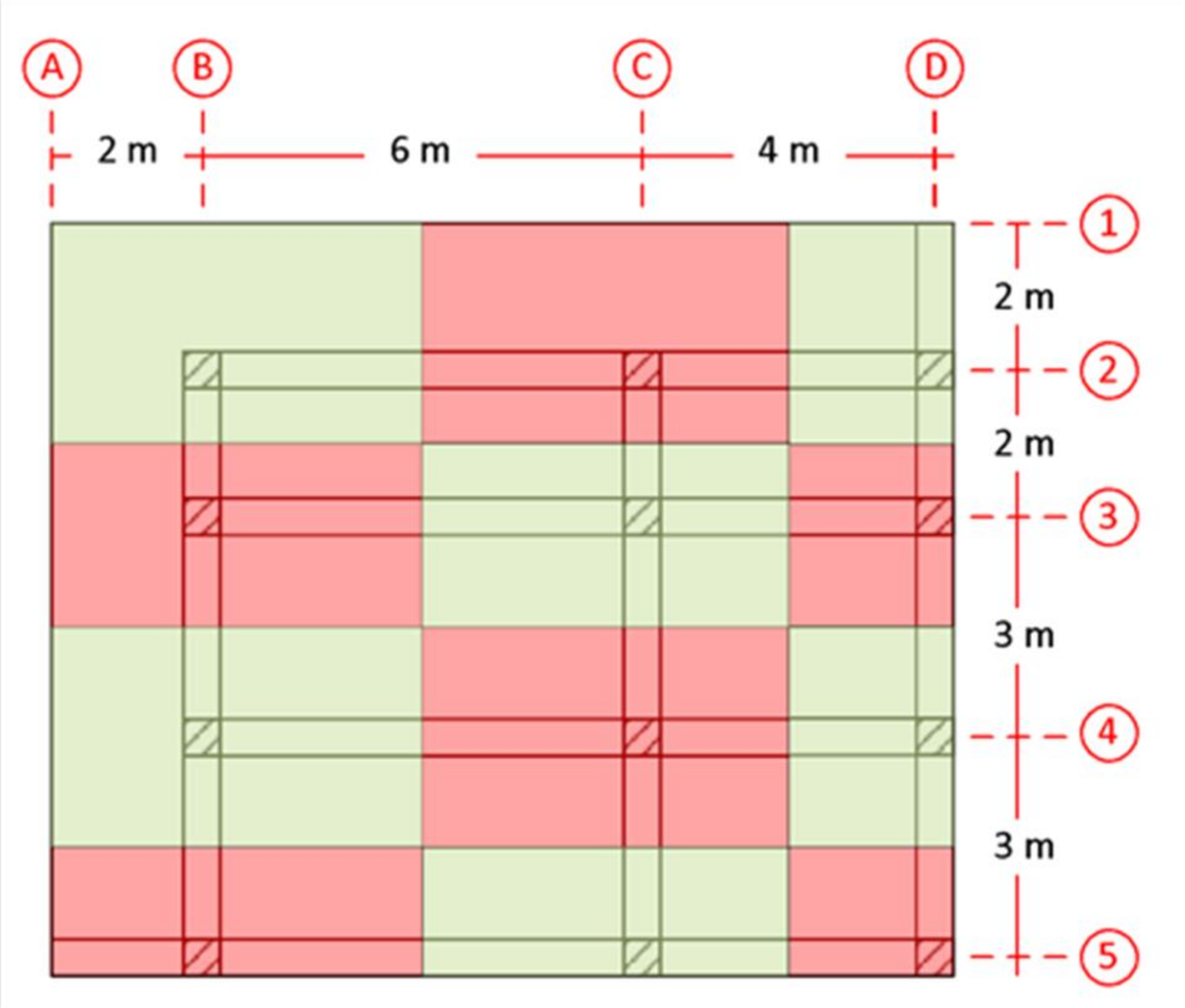
Where:

- D = Dead load
- E = Combined effect of horizontal and vertical earthquake induced forces
- F = Load due to fluids with well-defined pressures and maximum heights
- H = Load due to lateral earth pressures, ground water pressure or pressure of bulk materials.
- L = Roof live load greater than 20 psf (0.96 kN/m^2) and floor live load
- L_r = Roof live load of 20 psf (0.96 kN/m^2) or less
- R = rain load
- S = snow load
- W = wind load

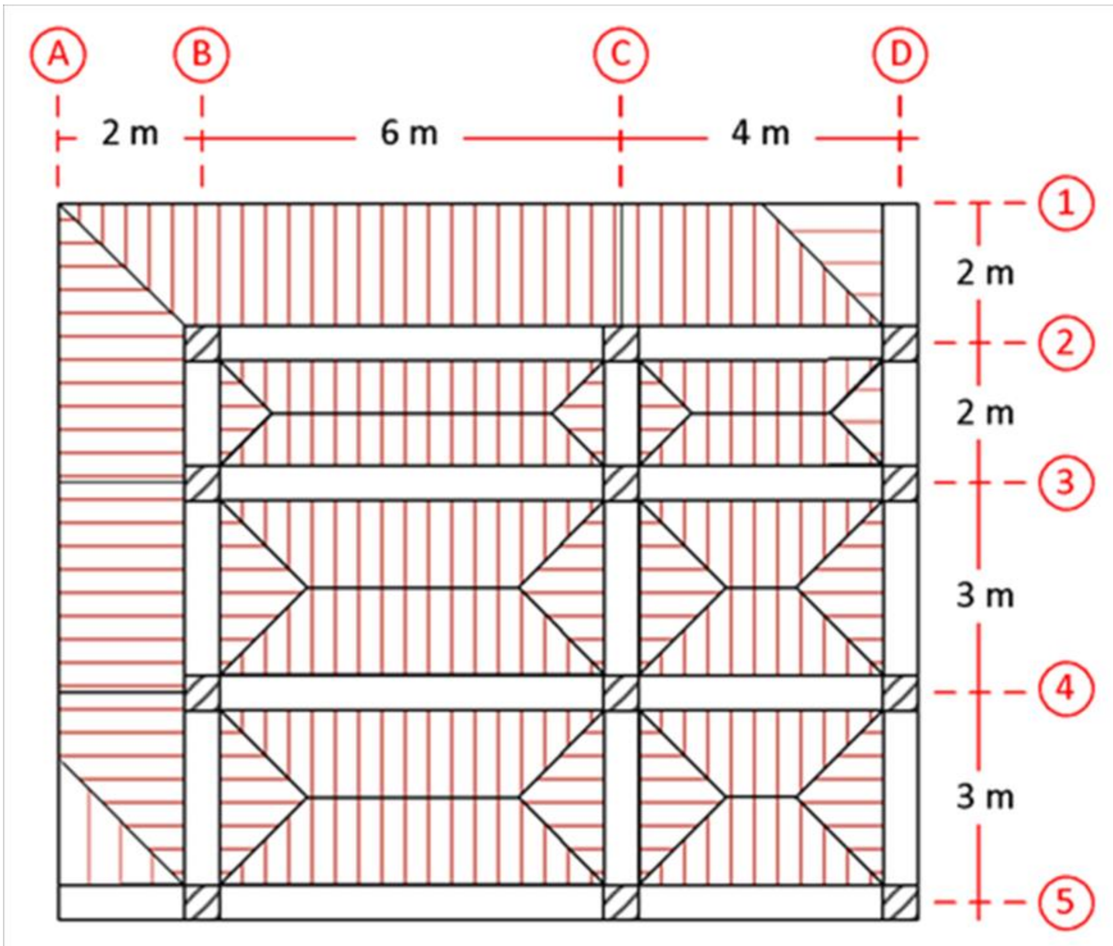
Load Paths - Tributary Areas (Columns)

Tributary Areas for beams and columns:

- Definition:
  - Beams: The area of slab that is supported by a particular beam is termed the beam's tributary area.
  - Columns: the area surrounding the column that is bounded by the panel centerlines



Load Paths - Tributary Areas (Beams)



- **Notes:**
  - Tributary area for **interior columns** is four time **(4x)** the tributary area **typical corner column.**
  - Tributary area for **beams surrounding a "square" slab share equal portion** of the load applied to that slab.
  - For **rectangular slabs**, the load shared by the beams in the **short direction** is **triangular** whereas the load shared by beams in the **long direction is trapezoidal.**

Approximate Methods:

- In short, the trapezoidal loads can be assumed as uniformly distributed over the beam span with some approximation techniques.

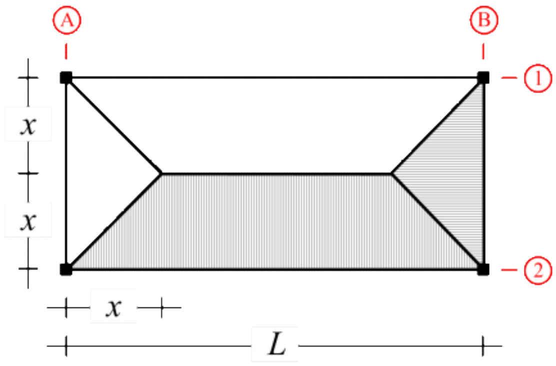
w: Uniformly distributed load per unit area

L: Span of beams

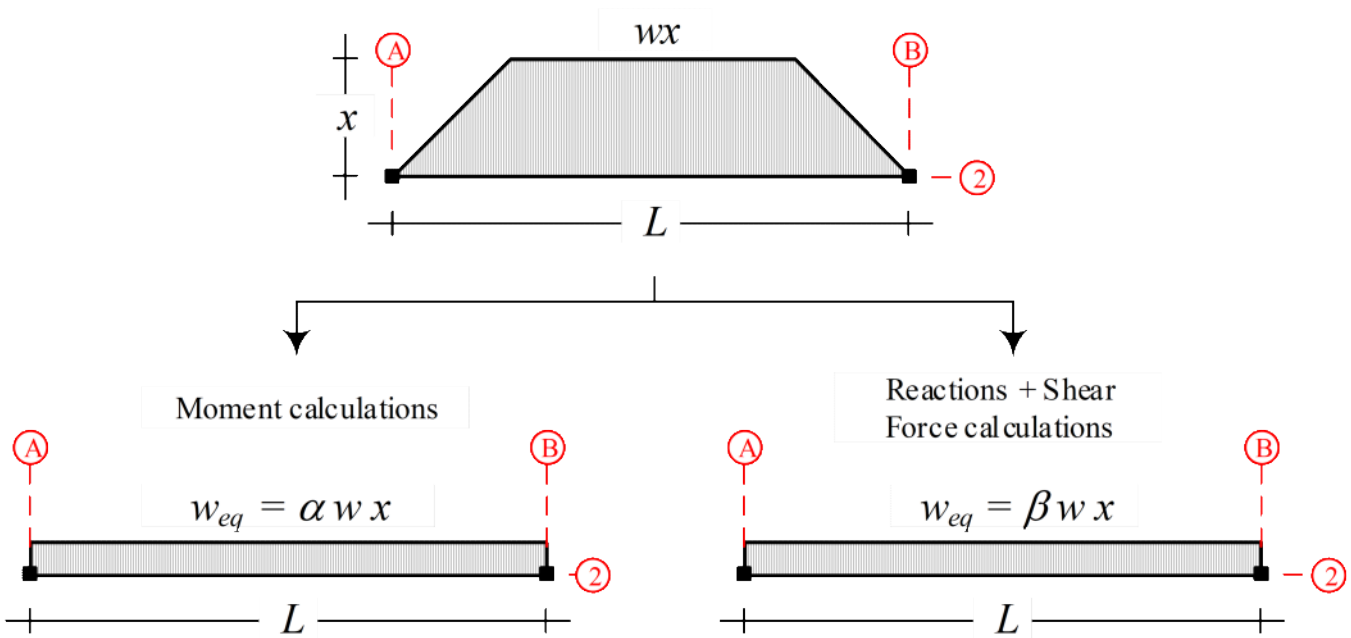
x: Maximum distance of loading to the desired beam

α *w:* Equivalent load for **bending moment** calculations under the condition that the load is distributed over the total span of the beam with the maximum intensity at mid span.

β *w:* Equivalent load for **reaction and shear force** calculations for conditions not satisfied above.



$$\alpha = 1 - \frac{1}{3}\left(\frac{2x}{L}\right)^2 \qquad \beta = 1 - \frac{x}{L}$$



$$\alpha = 1 - \frac{1}{3}\left(\frac{2x}{L}\right)^2$$

$$\beta = 1 - \frac{x}{L}$$

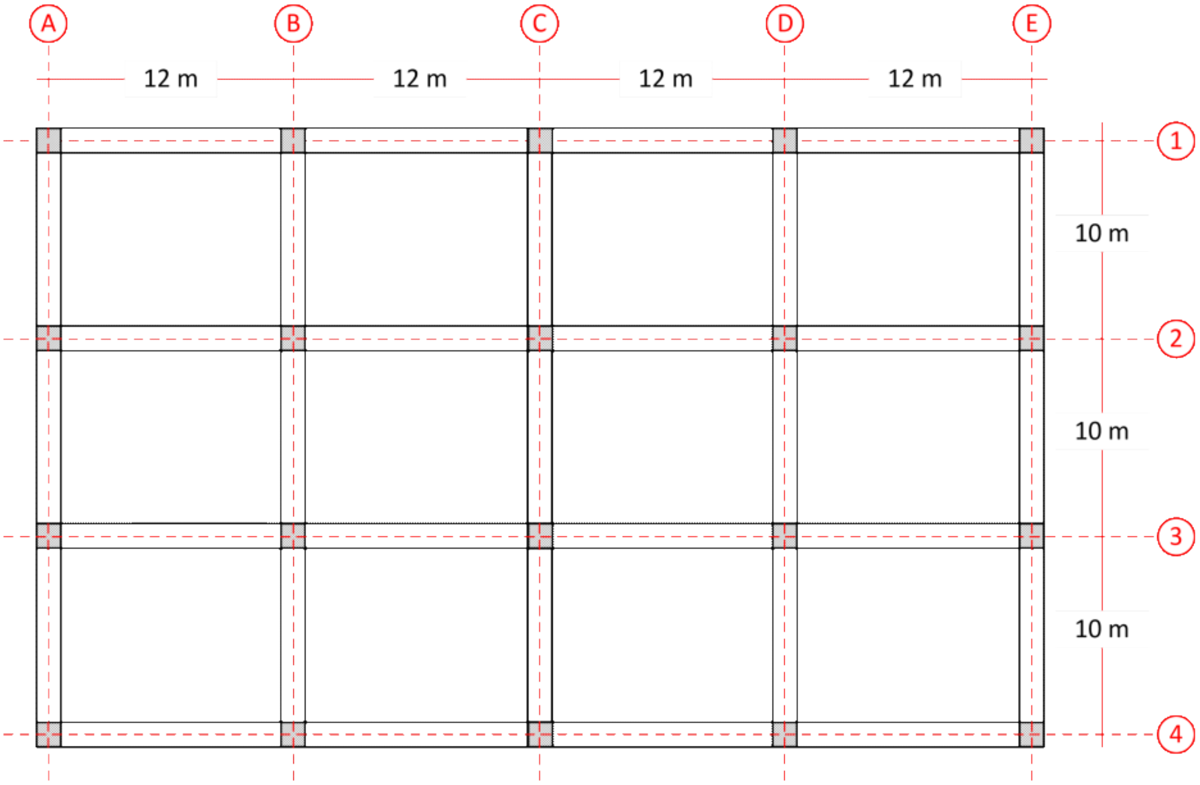
Some tabulated values for (α & β)

L/2x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.667	0.725	0.769	0.803	0.830	0.853	0.870	0.885	0.897	0.908	0.917
β	0.5	0.544	0.583	0.615	0.642	0.667	0.688	0.706	0.722	0.737	0.75

Example (1) - D is known

For the floor plan shown, if D = 3.4 kN/m^2 and L = 2.4 kN/m^2, find the ultimate loads on

- Columns A4, B3, and C4
- Beams A1-B1, D1-D2, and A3-E3



Example (1) - Cont.

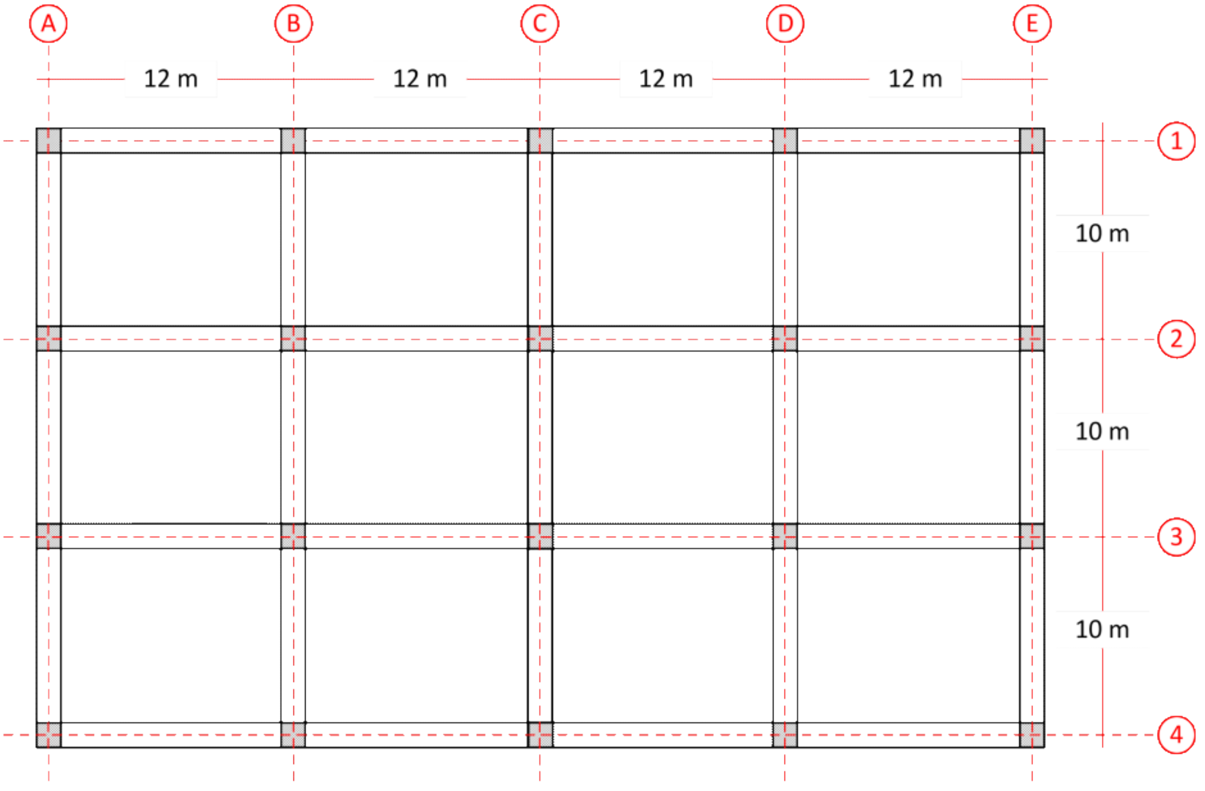
Example (2) - D is unknown

For the Floor plan shown, assuming L = 2.4 kN/m^2, all slabs are 12 cm thick and:

- Concrete density (ρ_c) = 24 kN/m^3
- Mechanical, Electrical, and Piping = 0.6 kN/m^2
- Ceiling system = 0.35 kN/m^2
- Roofing = 0.30 kN/m^2
- Flooring = 0.50 kN/m^2

Find loads on:

- Columns A4, B3, and C4
- Beams A1-B1, D1-D2, and A3-E3

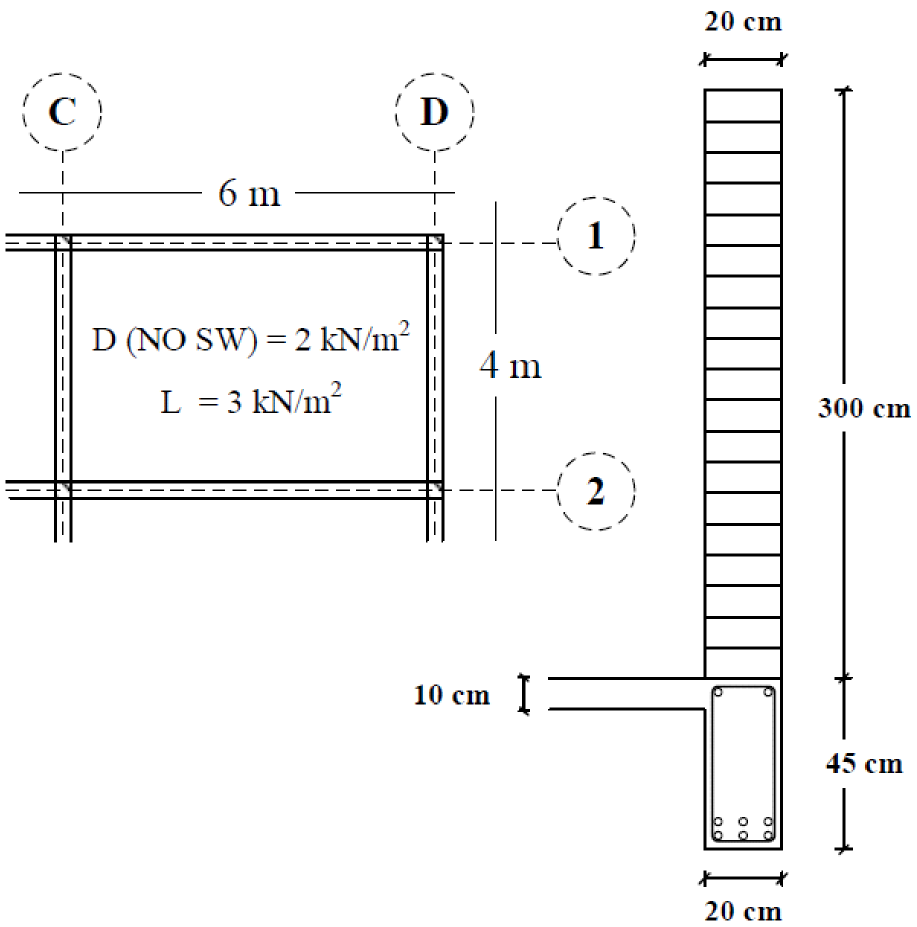


Example (2) - Cont.

Example (3) - D is unknown + Wall

Calculate the ultimate load on the beam (C1-D1) shown in the figure assuming:

- Reinforced concrete (ρ_c) = 25 kN/m^3
- Exterior wall (ρ_{ew})= 16.50 kN/m^3



Example (3) - Cont.