College of Technological Studies

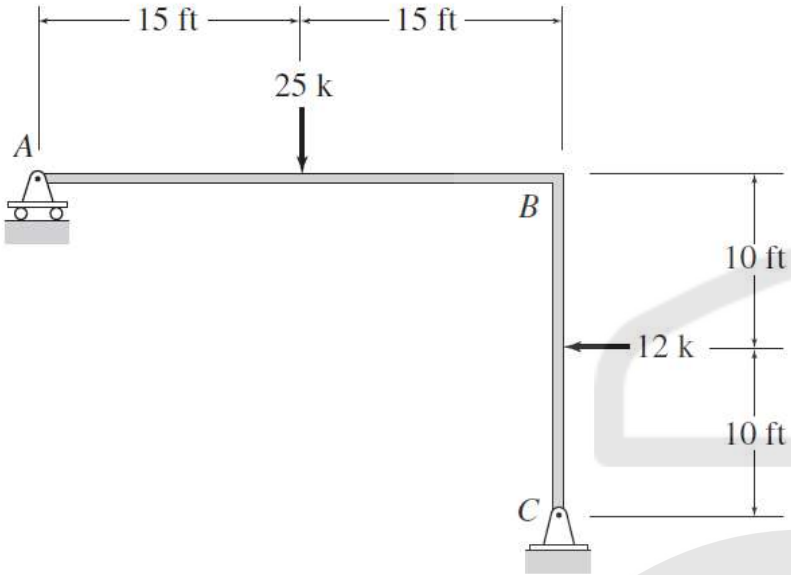
Department of Civil Engineering Technology

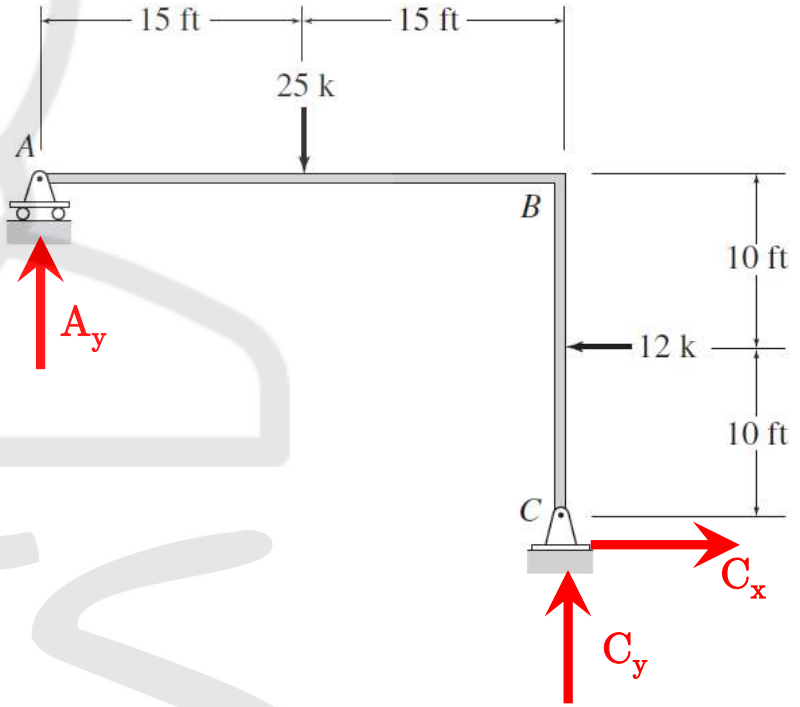
CE 278 Structural Analysis

Tutorial (6)

Internal Forces in Frames

Example (1): Determine the internal forces at point (B) of the frame shown in the figure.

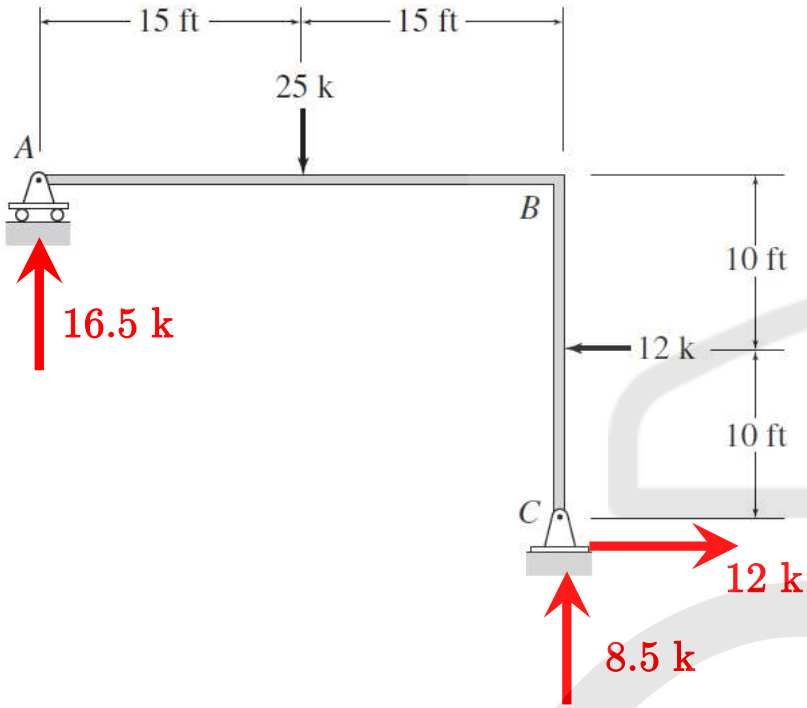


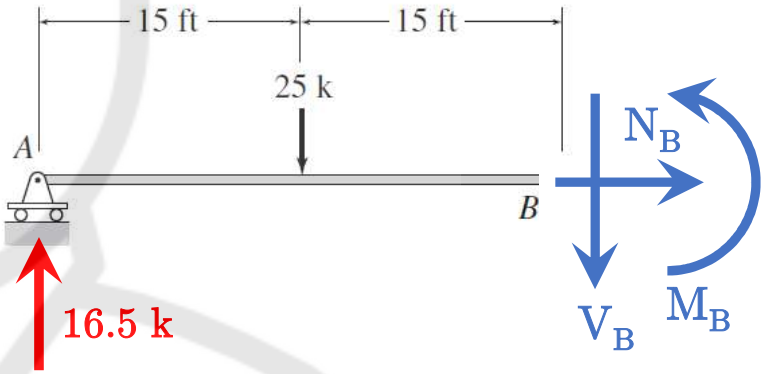


$\sum F_x = 0$ $\quad \boxed{C_x = 12\,\text{k}}$

$\sum M_C = 0$ $\quad (-A_y \times 30) + (25 \times 15) + (12 \times 10) = 0$ $\quad \boxed{A_y = 16.5\,\text{k}}$

$\sum F_y = 0$ $\quad A_y - 25 + C_y = 0$ $\quad 16.5 - 25 + C_y = 0$ $\quad \boxed{C_y = 8.5\,\text{k}}$



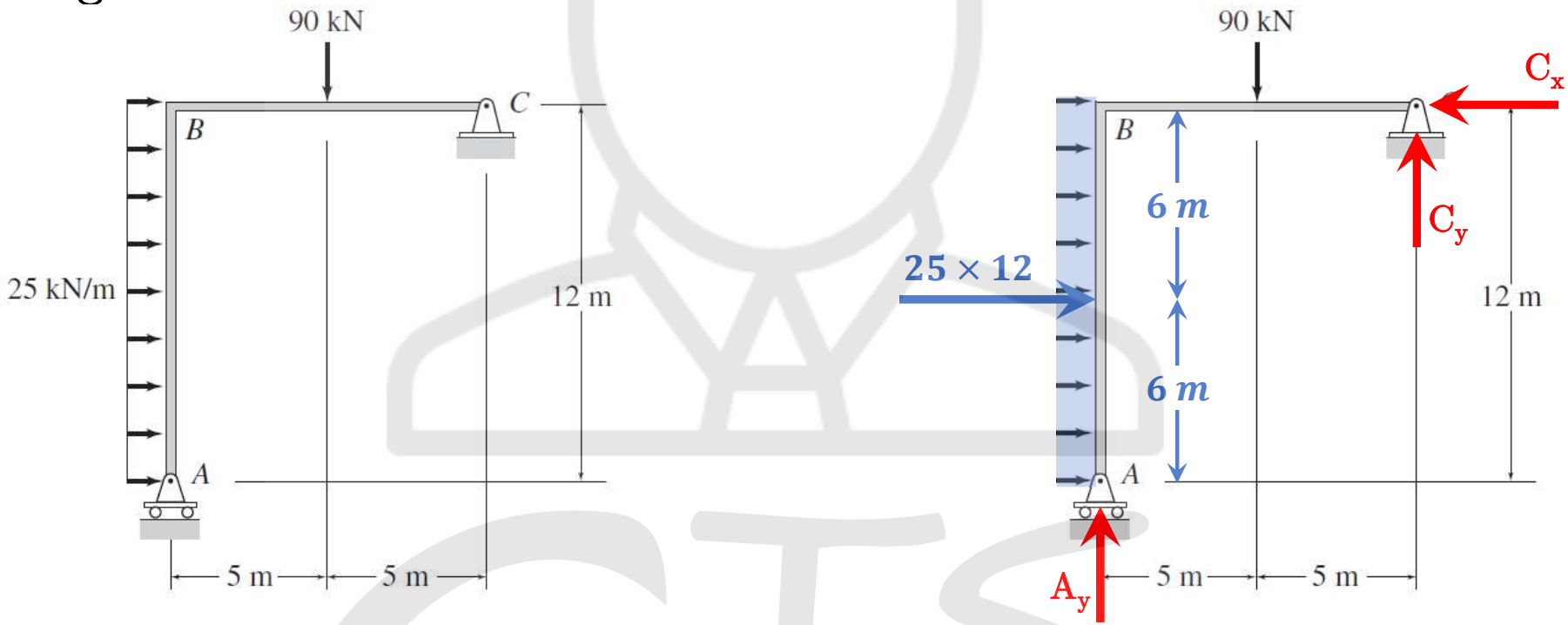


$$\sum F_x = 0 \qquad \boxed{N_B = 0}$$

$$\sum M_B = 0 \qquad M_B + (-16.5 \times 30) + (25 \times 15) = 0 \qquad \boxed{M_B = 120\,\text{k-ft}}$$

$$\sum F_y = 0 \qquad 16.5 - 25 + V_B = 0 \qquad \boxed{V_B = 8.5\,\text{k}}$$

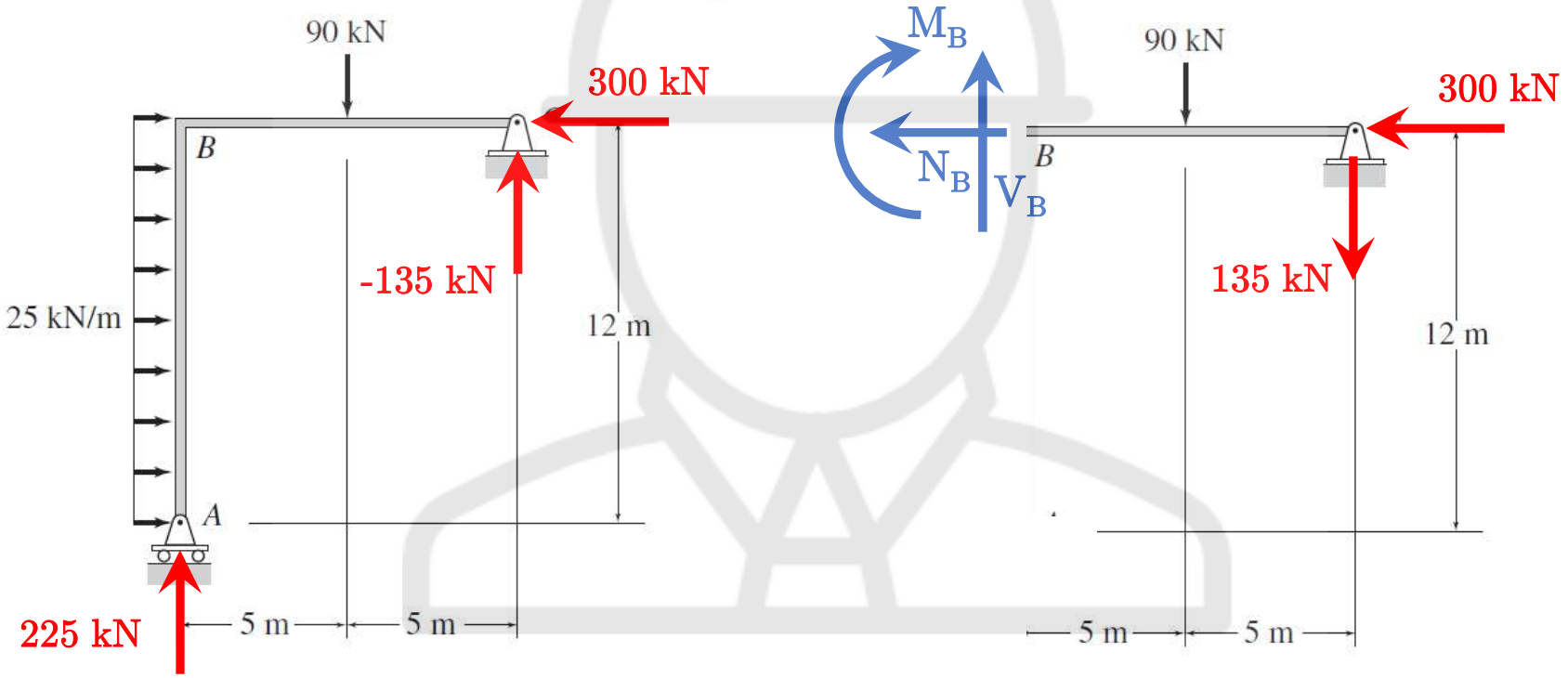
Example (2): Determine the internal forces at point (B) of the frame shown in the figure.



$\sum F_x = 0 \qquad (25\times12) - C_x = 0 \qquad \boxed{C_x = 300\,\text{kN}}$

$\sum M_C = 0 \qquad (-A_y \times 10) + (25\times12\times6) + (90\times5) = 0 \qquad \boxed{A_y = 225\,\text{kN}}$

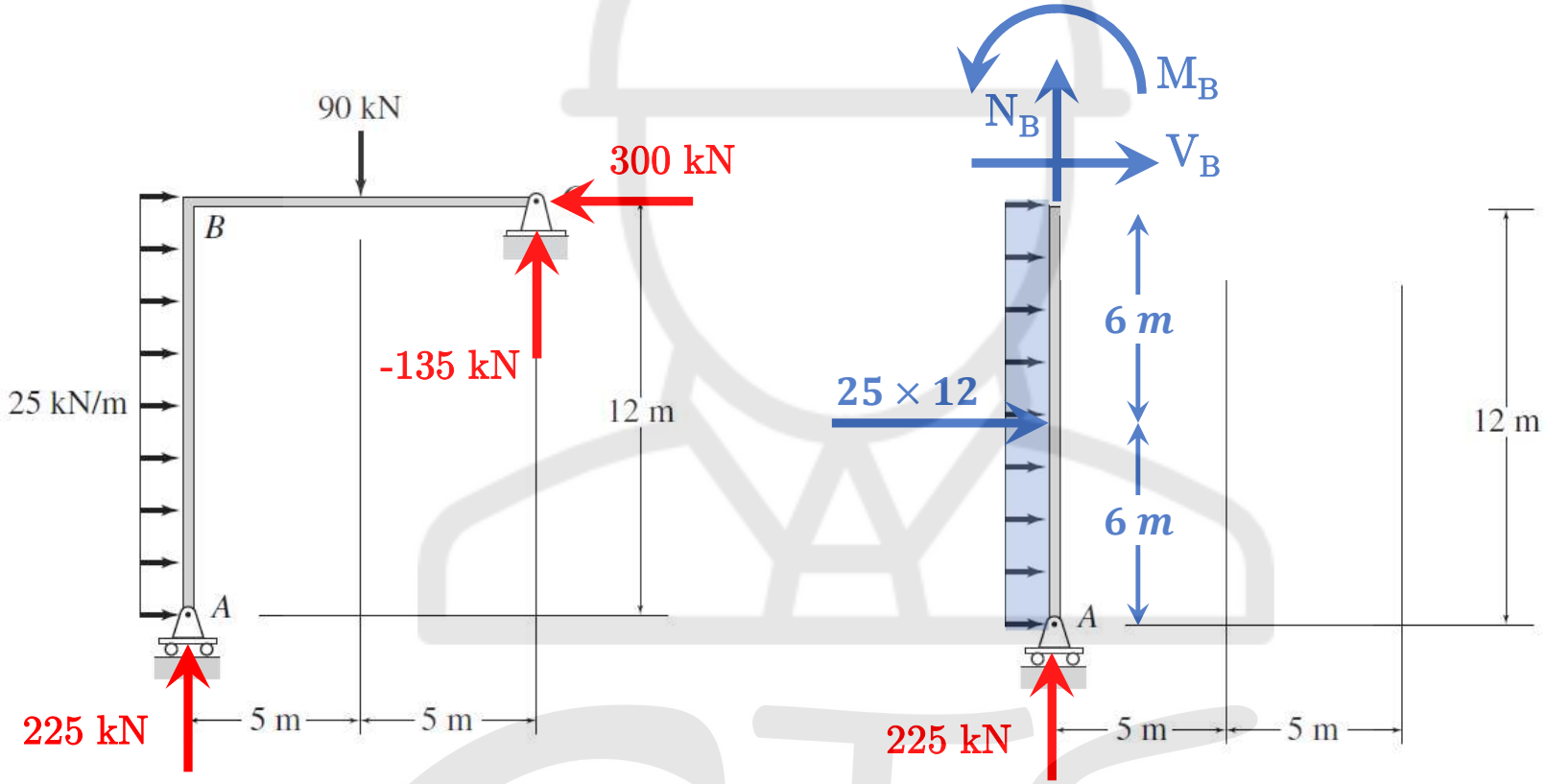
$\sum F_y = 0 \qquad A_y - 90 + C_y = 0 \qquad 225 - 90 + C_y = 0 \qquad C_y = -135\,\text{kN} \qquad \boxed{C_y = 135\,\text{kN}\downarrow}$



$$\sum F_x = 0 \qquad -N_B - 300 = 0 \qquad \boxed{N_B = -300\,\text{kN}}$$

$$\sum F_y = 0 \qquad V_B - 90 - 135 = 0 \qquad \boxed{V_B = 225\,\text{kN}}$$

$$\sum M_B = 0 \qquad -M_B + (-135 \times 10) + (-90 \times 5) = 0 \qquad \boxed{M_B = -1800\,\text{kN-m}}$$

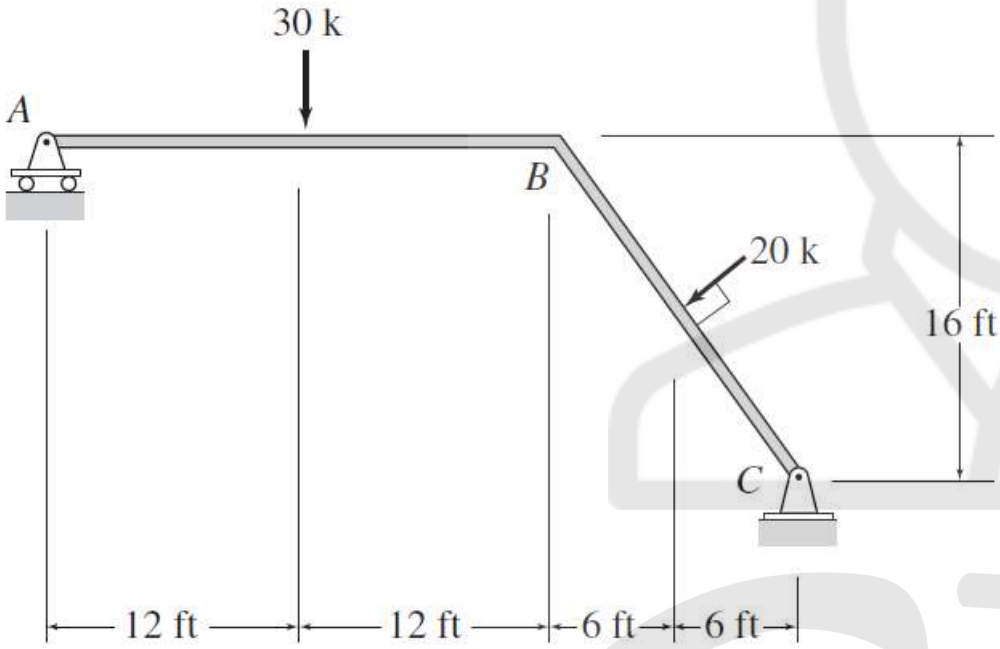


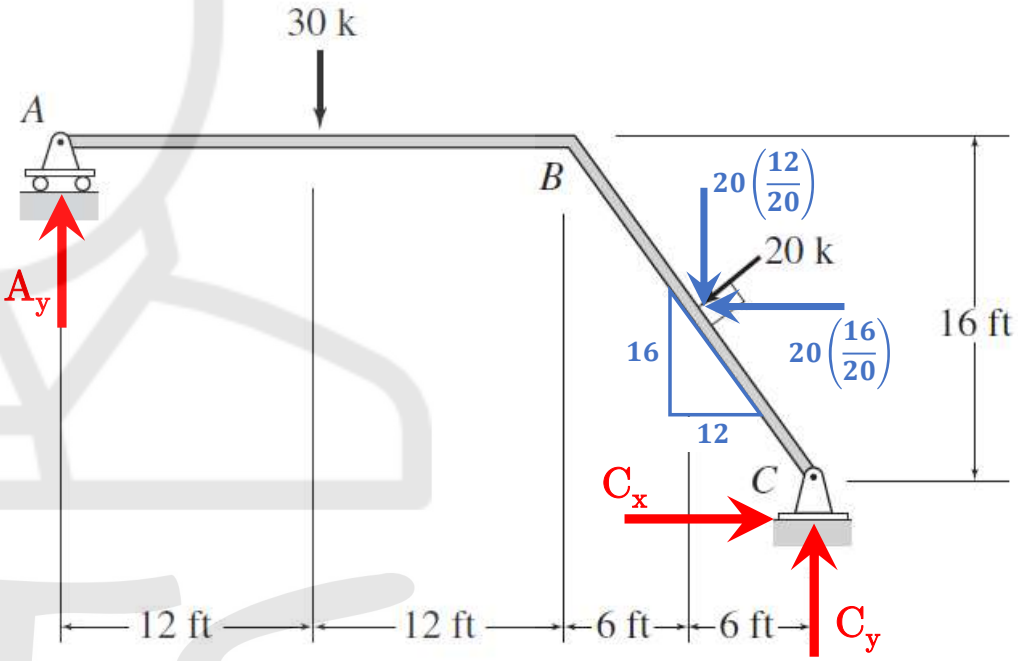
$$\sum F_x = 0 \qquad V_B + (25 \times 12) = 0 \qquad \boxed{V_B = -300\,\text{kN}}$$

$$\sum F_y = 0 \qquad 225 + N_B = 0 \qquad \boxed{N_B = -225\,\text{kN}}$$

$$\sum M_B = 0 \qquad M_B + (25 \times 12 \times 6) = 0 \qquad \boxed{M_B = -1800\,\text{kN-m}}$$

Example (3): Determine the internal forces at point (B) of the frame shown in the figure.

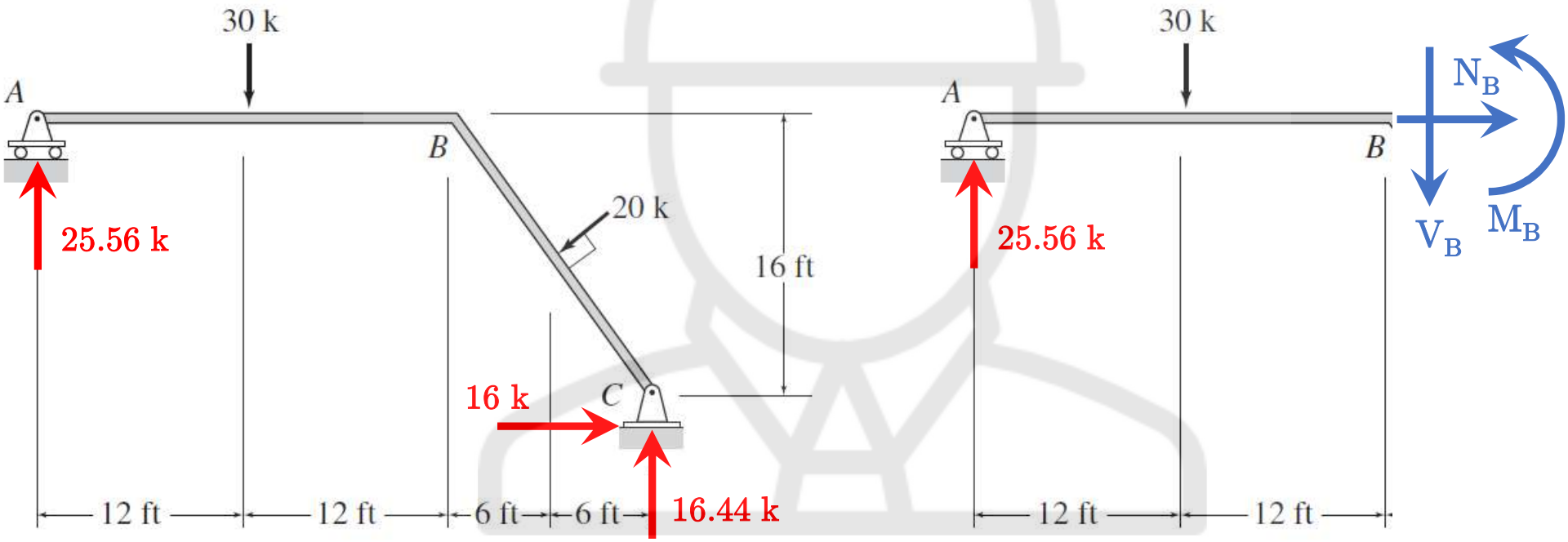




$$\sum F_x = 0 \qquad C_x - 20\left(\frac{16}{20}\right) = 0 \qquad \boxed{C_x = 16\,\text{k}}$$

$$\sum M_C = 0 \qquad \left(-A_y \times 36\right) + \left(30 \times 24\right) + \left(20\left(\frac{12}{20}\right) \times 6\right) + \left(20\left(\frac{16}{20}\right) \times 8\right) = 0 \qquad \boxed{A_y = 25.56\,\text{k}}$$

$$\sum F_y = 0 \qquad A_y - 30 - 20\left(\frac{12}{20}\right) + C_y = 0 \qquad 25.56 - 30 - 20\left(\frac{12}{20}\right) + C_y = 0 \qquad \boxed{C_y = 16.44\,\text{k}}$$

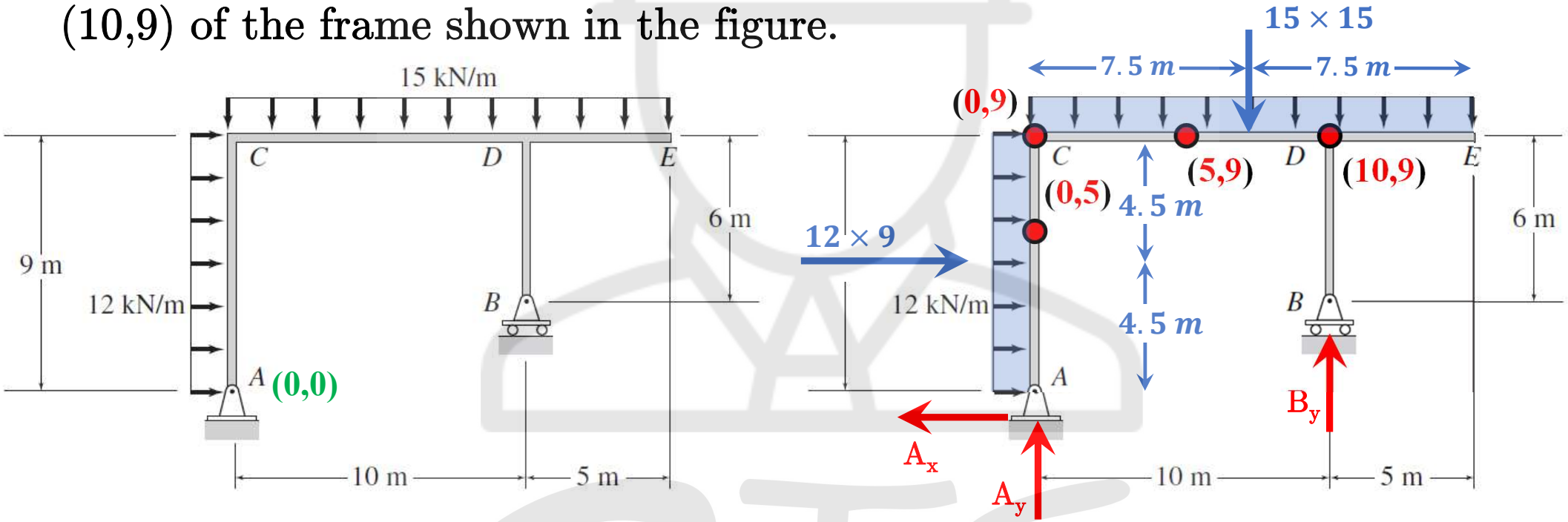


$$\sum F_x = 0 \qquad \boxed{N_B = 0\,\text{k}}$$

$$\sum F_y = 0 \qquad 25.56 - 30 - V_B = 0 \qquad \boxed{V_B = -4.44\,\text{k}}$$

$$\sum M_B = 0 \qquad M_B + (-25.56 \times 24) + (30 \times 12) = 0 \qquad \boxed{M_B = -253.44\,\text{k-ft}}$$

Example (4): Determine the internal forces at points: (0,5), (0,9), (5,9) and (10,9) of the frame shown in the figure.

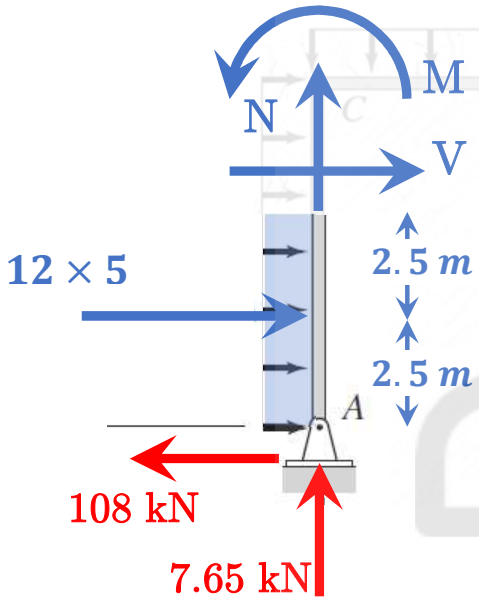


$$\sum F_x = 0 \qquad -A_x + (12 \times 9) = 0 \qquad \boxed{A_x = 108\,\text{kN}}$$

$$\sum M_A = 0 \qquad (B_y \times 10) + (-15 \times 15 \times 7.5) + (-12 \times 9 \times 4.5) = 0 \qquad \boxed{B_y = 217.35\,\text{kN}}$$

$$\sum F_y = 0 \qquad A_y - (15 \times 15) + B_y = 0 \qquad A_y - (15 \times 15) + 217.35 = 0 \qquad \boxed{A_y = 7.65\,\text{kN}}$$

Point (0,5)



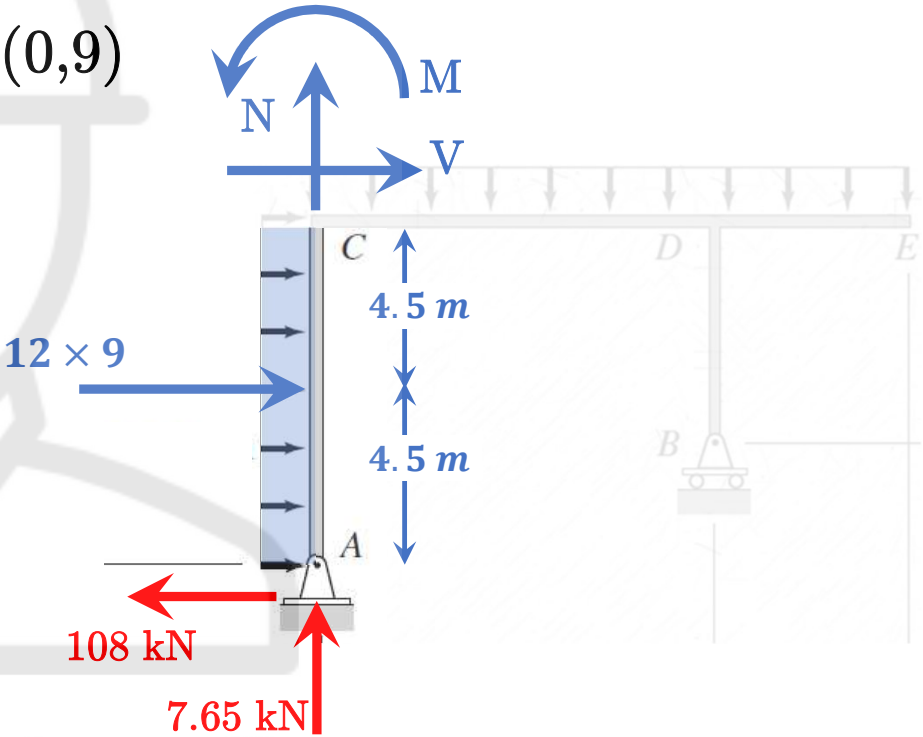
$$\sum F_x = 0 \qquad -108 + (12 \times 5) + V = 0 \qquad \boxed{V = 48\,\text{kN}}$$

$$\sum F_y = 0 \qquad 7.65 + N = 0 \qquad \boxed{N = -7.65\,\text{kN}}$$

$$\sum M = 0 \qquad M + (-108 \times 5) + (12 \times 5 \times 2.5) = 0$$

$$\boxed{M = 390\,\text{kN-m}}$$

Point (0,9)



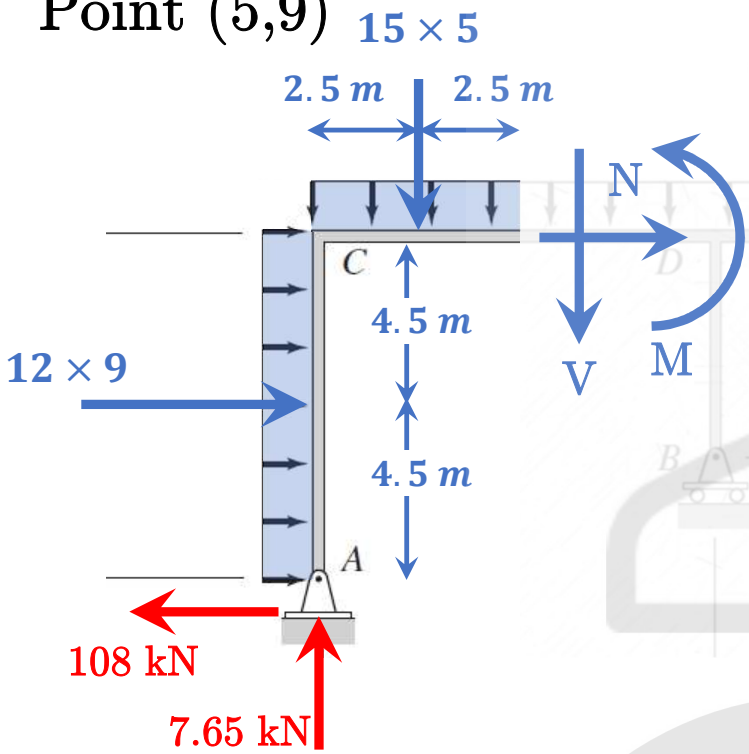
$$\sum F_x = 0 \qquad -108 + (12 \times 9) + V = 0 \qquad \boxed{V = 0\,\text{kN}}$$

$$\sum F_y = 0 \qquad 7.65 + N = 0 \qquad \boxed{N = -7.65\,\text{kN}}$$

$$\sum M = 0 \qquad M + (-108 \times 9) + (12 \times 9 \times 4.5) = 0$$

$$\boxed{M = 486\,\text{kN-m}}$$

Point (5,9)



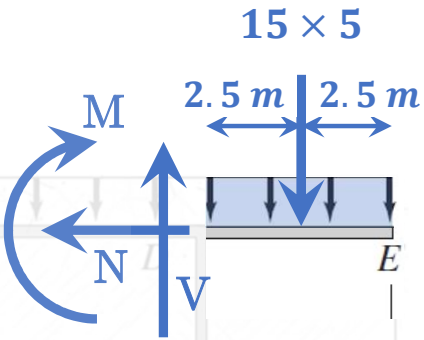
$\sum F_x = 0 \qquad -108 + (12 \times 9) + N = 0 \qquad \boxed{N = 0\,\text{kN}}$

$\sum F_y = 0 \qquad 7.65 - (15 \times 5) - V = 0 \qquad \boxed{V = 67.35\,\text{kN}}$

$\sum M = 0 \qquad M + (-108 \times 9) + (-7.65 \times 5) + (12 \times 9 \times 4.5)$

$+ (15 \times 5 \times 2.5) = 0 \qquad \boxed{M = 336.75\,\text{kN-m}}$

Point (10,9)



$\sum F_x = 0 \qquad \boxed{N = 0\,\text{kN}}$

$\sum F_y = 0 \qquad V - (15 \times 5) = 0 \qquad \boxed{V = 75\,\text{kN}}$

$\sum M = 0 \qquad -M + (-15 \times 5 \times 2.5) = 0$

$\boxed{M = 187.5\,\text{kN-m}}$

Questions?