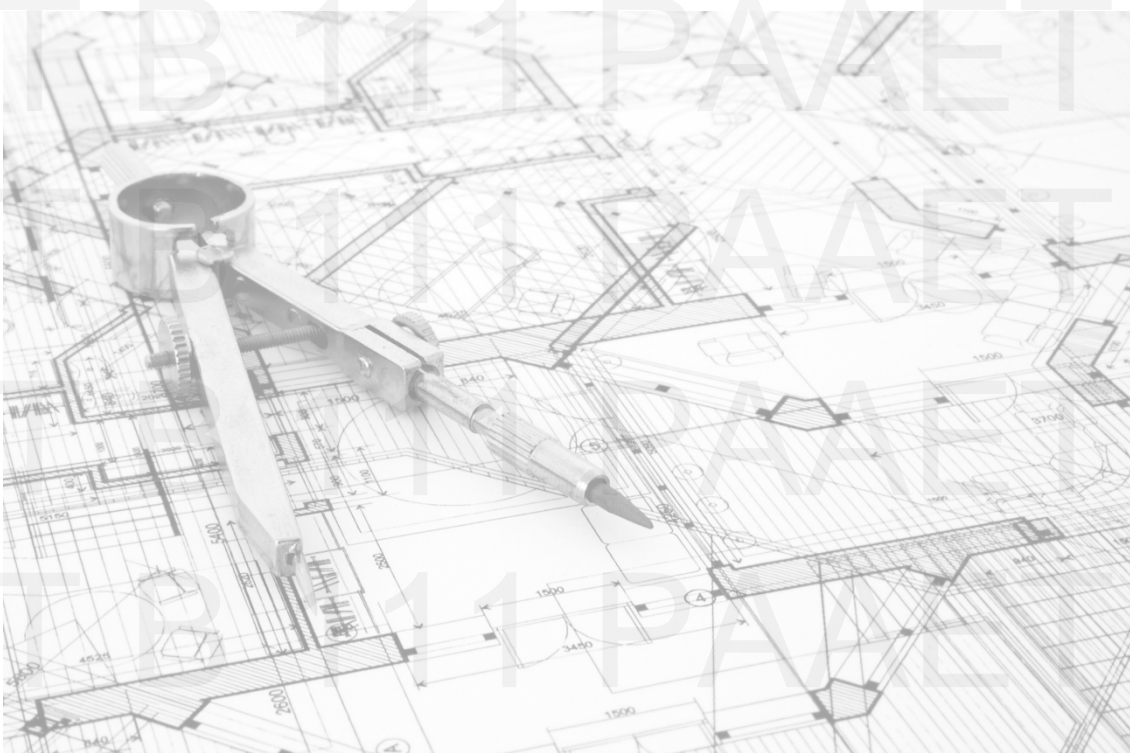




**The Public Authority for Applied Education and Training**

**College of Technological Studies**

**Department of Civil Engineering Technology**



**(CE 161 / B 111) Engineering Statics**

**(Class Notes)**

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**Disclaimer:**

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## Chapter (1): Units and Units System

### 1.1 Units of Measurements:

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	<b>newton*</b> N $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot ft	second s	<b>slug*</b> $\left(\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}\right)$	pound lb

\*Derived unit.

Table 1–1: Systems of Units

- The four basic quantities length, time, mass, and force are not all independent from one another.
- They are related by Newton's second law of motion,  $\mathbf{F} = \mathbf{ma}$ .
- The equality  $\mathbf{F} = \mathbf{ma}$  is maintained only if three of the four units, called base units, are defined and the fourth unit is then derived from the equation.

#### 1.1.1 SI System of Units:

- As shown in Table 1–1, the SI system defines length in meters (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a newton (N), and it is derived from  $\mathbf{F} = \mathbf{ma}$ .
- If we define the weight as the “Force” of gravity, then the weight will have units of (N).
- If the mass of the object ( $m$ ) is given along with the acceleration due to gravity ( $g$ ), the weight of an object can be calculated by equation (1–1) as:

$$W = mg \quad (1-1)$$

Where,  $g = 9.80665 \text{ m/s}^2$ . From now on, we will set  $g = 9.81 \text{ m/s}^2$ .

#### 1.1.2 US Customary Units:

- In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in pounds (lb) as in Table 1–1.
- The unit of mass, called a slug, is derived from  $\mathbf{F} = \mathbf{ma}$ .
- Hence, 1 slug is equal to the amount of matter accelerated at  $1 \text{ ft/s}^2$  when acted upon by a force of 1 lb (slug =  $\text{lb-s}^2/\text{ft}$ ).
- $g = 32.2 \text{ ft/s}^2$ .

### 1.1.3 SI Units Prefixes:

- When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix.
- Some of the prefixes used in the SI system are shown in Table 1–2.
- Each represents a multiple or submultiple of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place.
- For example,  $4000000 \text{ N} = 4000 \text{ kN}$  (kilo–newton)  $= 4 \text{ MN}$  (mega–newton), or  $0.005 \text{ m} = 5 \text{ mm}$  (milli–meter).

The Prefixes Used with SI Units

Prefix	Symbol	Value	Scientific Notation
exa–	E	1,000,000,000,000,000,000	$10^{18}$
peta–	P	1,000,000,000,000,000	$10^{15}$
tera–	T	1,000,000,000,000	$10^{12}$
giga–	G	1,000,000,000	$10^9$
mega–	M	1,000,000	$10^6$
kilo–	k	1,000	$10^3$
hecto–	h	100	$10^2$
deka–	da	10	$10^1$
–	–	1	
deci–	d	0.1	$10^{-1}$
centi–	c	0.01	$10^{-2}$
milli–	m	0.001	$10^{-3}$
micro–	$\mu$	0.000 001	$10^{-6}$
nano–	n	0.000 000 001	$10^{-9}$
pico–	p	0.000 000 000 001	$10^{-12}$
femto–	f	0.000 000 000 000 001	$10^{-15}$
atto–	a	0.000 000 000 000 000 001	$10^{-18}$

Table 1–2: SI Units Prefixes

### 1.2 Unit Conversion:

Quantity	Units of Measurements (FPS)	Equals	Units of Measurements (FPS)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

Table 1–3: Conversion Factors

Table 1–3 provides a set of direct conversion factors between FPS and SI units for the basic quantities. Also, in the FPS system, recall that  $1 \text{ ft} = 12 \text{ in.}$  (inches),  $5280 \text{ ft} = 1 \text{ mi}$  (mile),  $1000 \text{ lb} = 1 \text{ kip}$  (kilo–pound), and  $2000 \text{ lb} = 1 \text{ ton}$ .





**Example (3):**

Convert 2 km/h to m/s How many ft/s is this?

**Solution:**

**Example (4):**

If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

**Solution:**

**Example (5):**

The specific weight (wt./vol.) of brass is  $520 \text{ lb/ft}^3$ . Determine its density (mass/vol.) in SI units. Use an appropriate prefix.

**Solution:**

**Example (6):**

Convert each of the following and express the answer using an appropriate prefix: (a)  $175 \text{ lb/ft}^3$  to  $\text{kN/m}^3$ , (b)  $6 \text{ ft/h}$  to  $\text{mm/s}$ , and (c)  $835 \text{ lb} \cdot \text{ft}$  to  $\text{kN} \cdot \text{m}$ .

**Solution:**

## Chapter (2): Force Vectors

### 2.1 Scalars and Vectors:

#### Scalar:

- A scalar is any positive or negative physical quantity that can be completely specified by its magnitude.
- Examples of scalar quantities include **length**, **mass**, and **time**.

#### Vector:

- A vector is any physical quantity that requires both a magnitude and a direction for its complete description.
- Examples of vectors encountered in statics are **force**, **position**, and **moment**.
- A vector is shown graphically by an arrow.
  - The length of the arrow represents the magnitude of the vector
  - The angle  $\theta$  between the vector and a fixed axis defines the direction of its line of action.
  - The head or tip of the arrow indicates the sense of direction of the vector.

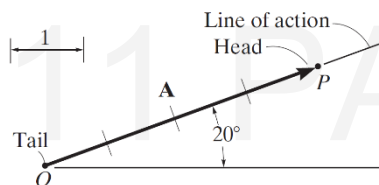


Figure 2-1: Elements of a vector

### 2.2 Vector Operations:

#### 2.2.1 Multiplication and Division of a Vector by a Scalar:

- If a vector is multiplied by a positive scalar, its magnitude is increased by that amount.
- Multiplying by a negative scalar will also change the directional sense of the vector.

Graphic examples of these operations are shown in Figure 2-2.

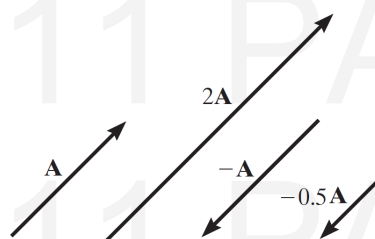


Figure 2-2: Multiplication and Division of a Vector by a Scalar

### 2.2.2 Vector Addition:

#### Parallelogram Method: (Tail-to-Tail)

All vector quantities obey the parallelogram law of addition. To illustrate, the two “component” vectors **A** and **B** in (Figure 2–3a) are added to form a “resultant” vector **R**.

Where **R** can be expressed as:

$$\vec{R} = \vec{A} + \vec{B} \quad (2-1)$$

Using the following procedure:

- First join the tails of the components at a point to make them concurrent, Figure 2–3b.
- From the head of **B**, draw a line parallel to **A**. Draw another line from the head of **A** that is parallel to **B**. These two lines intersect at point **P** to form the adjacent sides of a **parallelogram**.
- The diagonal of this parallelogram that extends to **P** forms **R**, which then represents the resultant vector  $\mathbf{R} = \mathbf{A} + \mathbf{B}$ , Figure 2–3c.

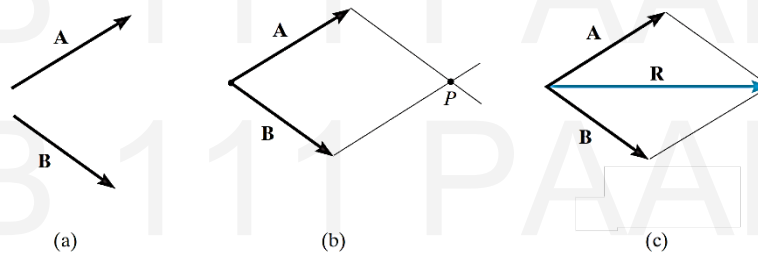


Figure 2–3: Vector Addition

#### Triangular Method: (Head-to-Tail)

- We can also add **B** to **A**, Figure 2–4a, using the triangle rule, which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a “head-to-tail” fashion Figure 2–4b.
- The resultant **R** extends from the tail of **A** to the head of **B**.
- In a similar manner, **R** can also be obtained by adding **A** to **B**, Figure 2–4c.
- By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e.,  $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ .

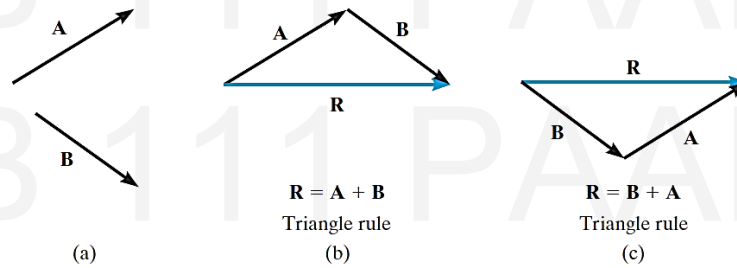


Figure 2-4: Triangular Method

### 2.2.3 Vector Subtraction:

The resultant of the difference between two vectors A and B of the same type may be expressed as:

$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \quad (2-2)$$

This vector sum is shown graphically in Figure 2-5. **Subtraction is therefore defined as a special case of addition**, so the rules of vector addition also apply to vector subtraction.

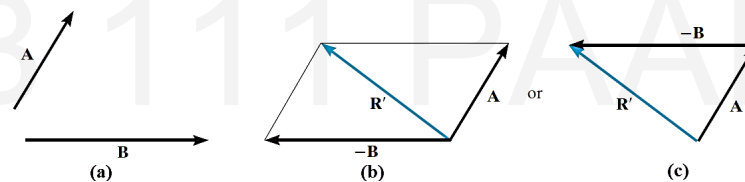


Figure 2-5: Vector Subtraction

### 2.3 Forces as Vectors:

- Forces can be fully expressed with *magnitude* and *direction*. Hence, **they are vectors**.
- As a result, the rules discussed in section (2.2) can be applied.
- Not only resultants can be found by adding forces, **a resultant force can be resolved into components as well**.
- Basically, we “work backwards” from the *Parallelogram law of addition* or the *triangle rule of addition* to resolve the resultant force vector into components along a two axes to components as in Figure 2-6.

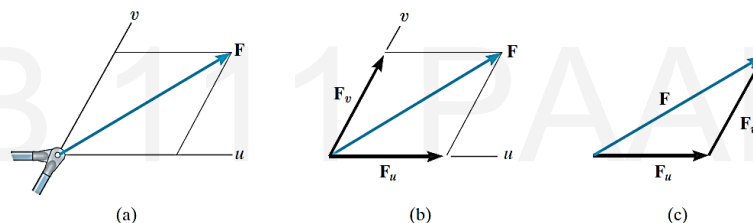


Figure 2-6: Resolving Vectors Into Components

Note:

Solving for resultant vector or resolving one will transfer the problem from vector addition (or subtraction) into **“Trigonometry”**. Therefore, a knowledge of such subject is a necessity in solving problems.

### 2.3.1 Lami’s Theorem:

Lami’s theorem states that “if a body is in equilibrium under the action forces, then each force is proportional to the sin of the angle between the other two forces”. Based on Figure 2-7,

$$\frac{\bar{F}_1}{\sin(\alpha)} = \frac{\bar{F}_2}{\sin(\beta)} = \frac{\bar{F}_3}{\sin(\gamma)} \quad (2-3)$$

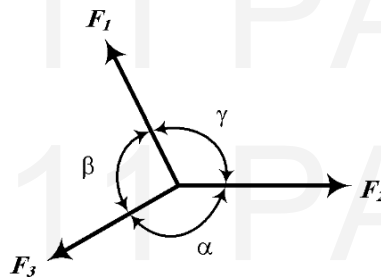


Figure 2-7: Lami’s Theorem

### 2.3.2 Law of Cosines and Law of Sines:

Two important trigonometric laws should be presented here; the Law of Cosines and the Law of Sines. Referring to Figure 2-8:

**Law of Sines:**

$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)} \quad (2-4)$$

**Law of Cosines:**

$$C = \sqrt{A^2 + B^2 - 2AB \cos(c)} \quad (2-5)$$

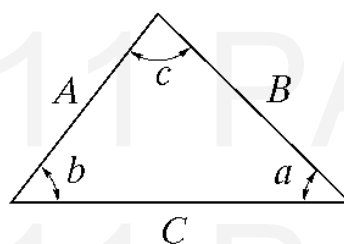


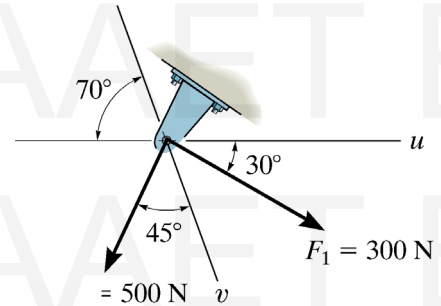
Figure 2-8: Law of Sines and Law of Cosines Lengths and Angles

### 2.3.3 Examples:

#### Example (1):

Resolve the force  $\mathbf{F}_1$  into components acting along the  $u$  and  $v$  axes and determine the magnitudes of the components.

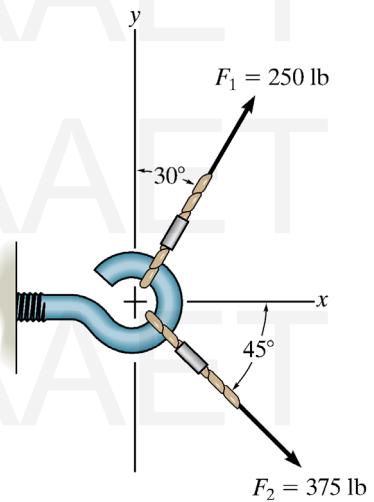
**Solution:**



#### Example (2):

Determine the magnitude of the resultant force  $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$  and its direction, measured counterclockwise from the positive  $x$  axis.

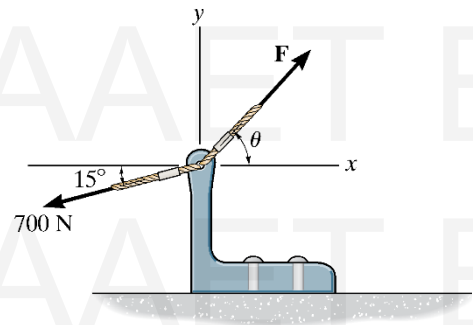
**Solution:**



**Example (3):**

If the magnitude of the resultant force is to be 500 N, directed along the positive  $y$  axis, determine the magnitude of force  $F$  and its direction  $\theta$ .

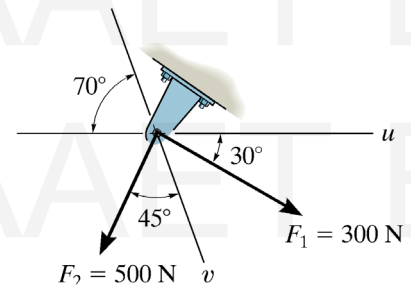
**Solution:**



**Example (4):**

Determine the magnitude of the resultant force  $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$  and its direction, measured clockwise from the positive  $u$  axis.

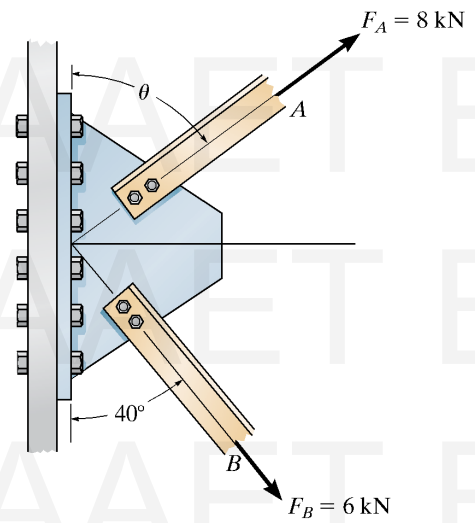
**Solution:**





**Example (5):**

The plate is subjected to the two forces at  $A$  and  $B$  as shown. If  $\theta = 60^\circ$ , determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

**Solution:**

## 2.4 Addition of a System of Coplanar Forces Using Scalar Notation:

### 2.4.1 Using “Angles”:

Referring to Figure (1.9), a vector  $\mathbf{F}$  can be resolved into two rectangular components  $\mathbf{F}_x$  and  $\mathbf{F}_y$  using the parallelogram law so that  $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ .

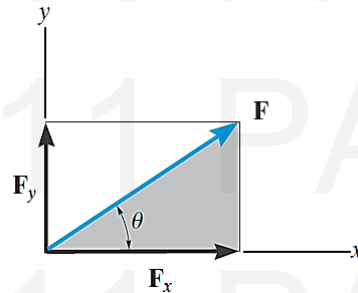


Figure 2-9: Resolving a Vector Into Components Using Angles

With the aid of the angle  $\theta$ , the components of the vector  $\mathbf{F}$  can be presented as:

$$\begin{aligned} F_x &= F \cos(\theta) \\ F_y &= F \sin(\theta) \end{aligned} \quad (2-6)$$

### 2.4.2 Using “Slope”:

The direction of  $\mathbf{F}$  can also be defined using a “slope” triangle as shown in Figure 2-10.

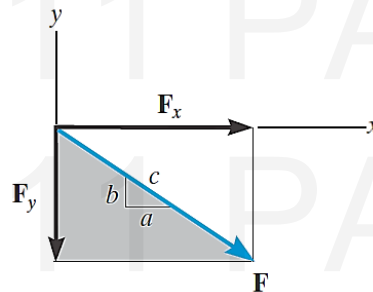


Figure 2-10: Resolving a Vector Into Components Using Slope

Using the slope, the components of the vector  $\mathbf{F}$  can be presented as:

$$\begin{aligned} F_x &= F \left( \frac{a}{c} \right) \\ F_y &= -F \left( \frac{b}{c} \right) \end{aligned} \quad (2-7)$$

## 2.5 Resultant of Coplanar Forces:

We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the  $x$  and  $y$  components of all the forces

$$\begin{aligned} (F_R)_x &= \sum F_x \\ (F_R)_y &= \sum F_y \end{aligned} \quad (2-8)$$

To illustrate, let us try to obtain the resultant force of multiple concurrent forces as shown in Figure 2–11a using the Cartesian vector notation. Each force is first represented as a Cartesian vector as presented in Figure 2–1b:

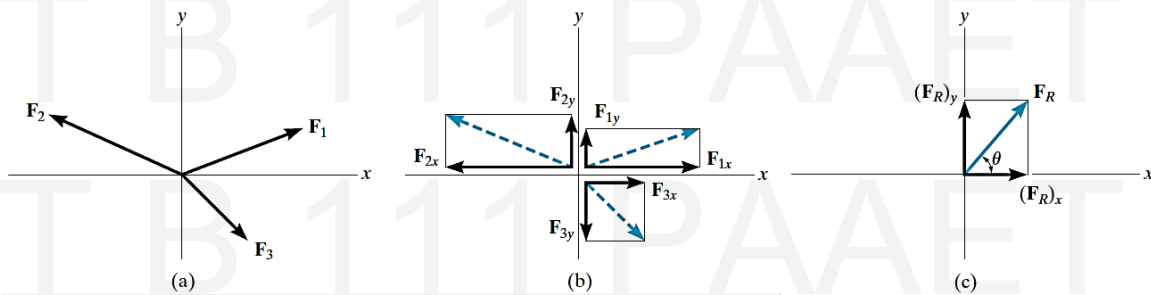


Figure 2–11: (a) Three concurrent forces, (b)  $x$  and  $y$  components of concurrent forces, (c) Finding  $F_R$  from  $(F_R)_x$  and  $(F_R)_y$

When  $(F_R)_x$  and  $(F_R)_y$  are determined, we use Pythagorean theorem to determine the magnitude of  $F_R$ , as shown in Figure 2–11c.

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} \quad (2-9)$$

The angle that specifies the direction of  $F_R$  can be calculated from trigonometry as:

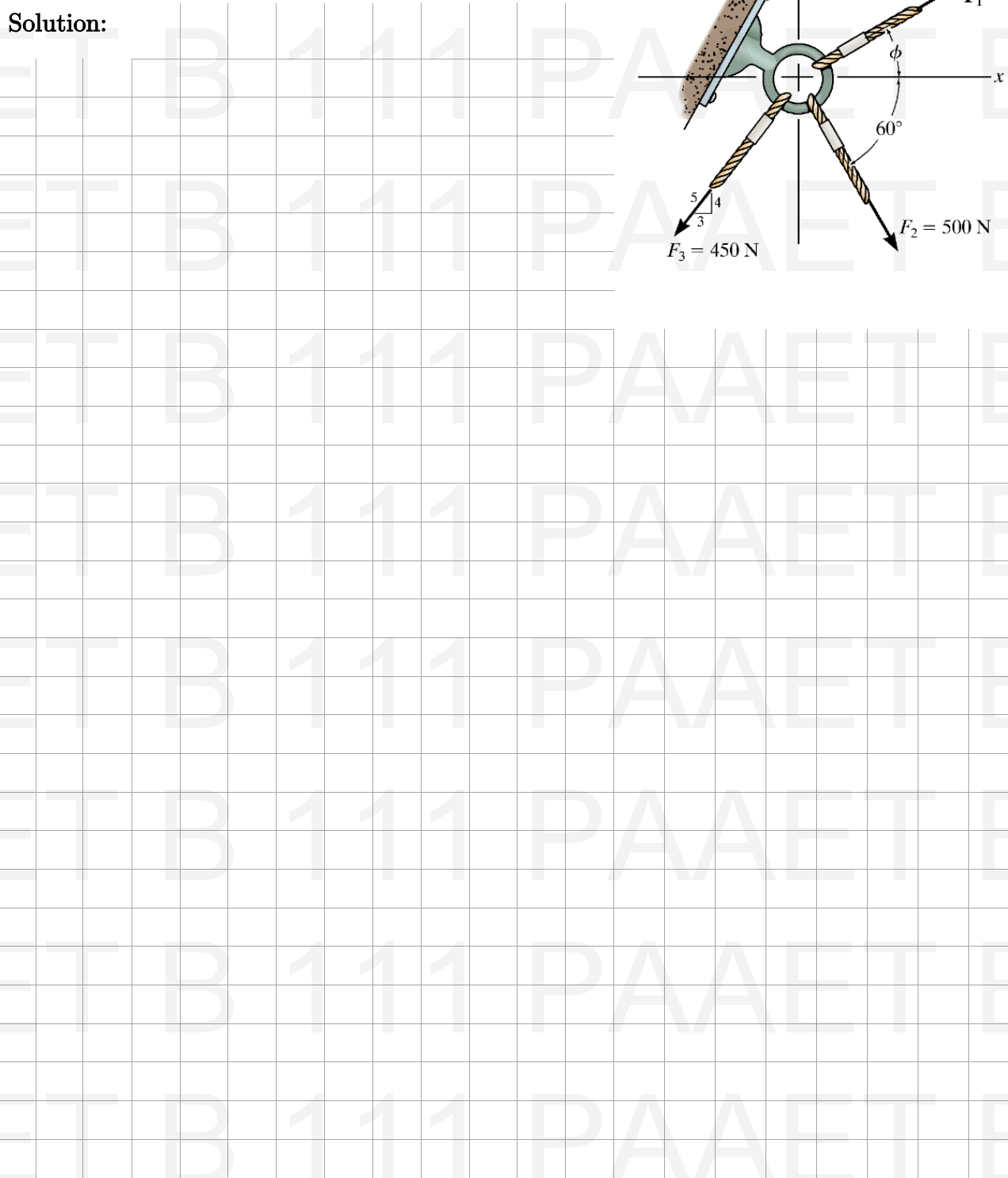
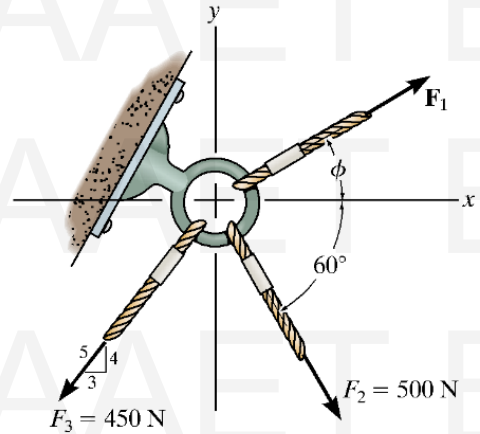
$$\theta = \tan^{-1} \left| \frac{(F_R)_x}{(F_R)_y} \right| \quad (2-10)$$

2.5.1 Examples:

Example (1):

If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive  $x$  axis is  $\theta = 30^\circ$ , determine the magnitude of  $F_1$  and the angle  $\phi$ .

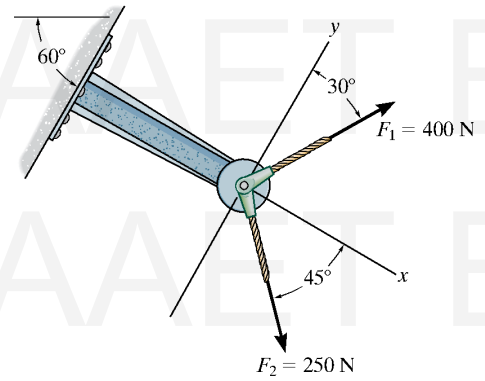
Solution:



**Example (2):**

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive  $x$  axis.

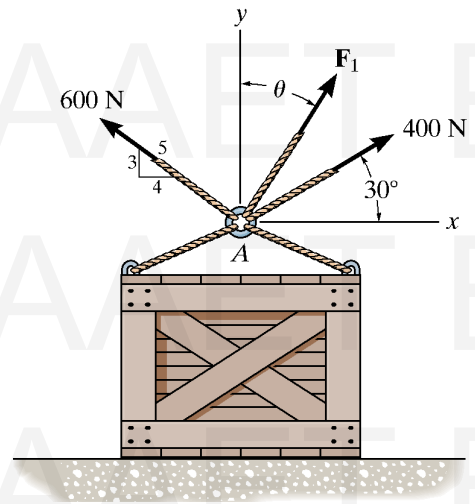
**Solution:**



**Example (3):**

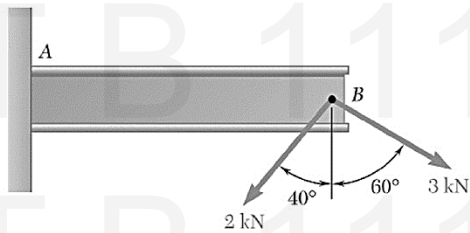
Determine the magnitude and direction measured counterclockwise from the positive  $x$  axis of the resultant force of the three forces acting on the ring  $A$ . Take  $F_1 = 500$  N and  $\theta = 20^\circ$ .

**Solution:**



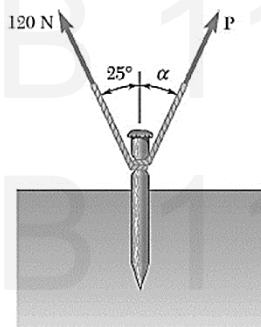
2.6 Problems:

Question (2.1)



Two forces are applied at point  $B$  of beam  $AB$ . Determine graphically the magnitude and direction of their resultant.

Question (2.2)

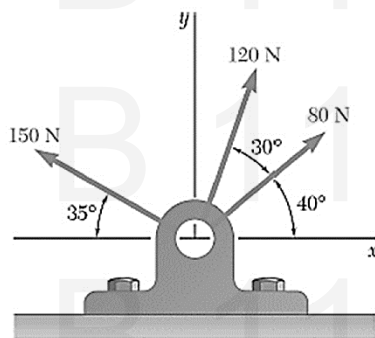


A stake is being pulled out of the ground by means of two ropes as shown.

Knowing that  $\alpha = 30^\circ$ , determine by trigonometry

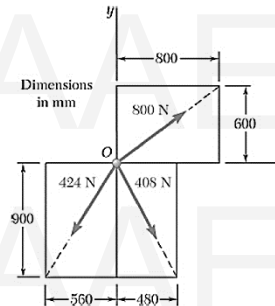
- (a) The magnitude of the force  $P$  so that the resultant force exerted on the stake is vertical
- (b) The corresponding magnitude of the resultant.

Question (2.3)



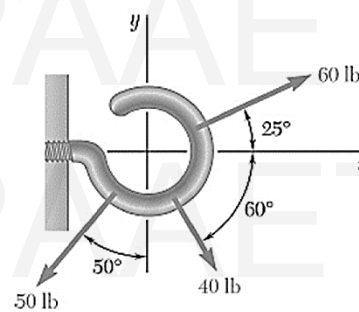
Determine the  $x$  and  $y$  components of each of the forces shown.

Question (2.4)



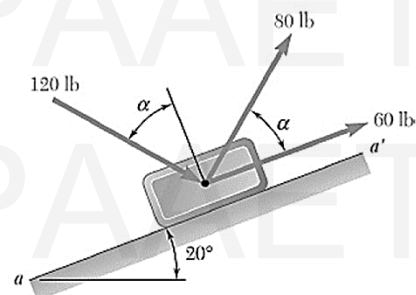
Determine the  $x$  and  $y$  components of each of the forces shown.

Question (2.5)



Determine the resultant of the three forces of

Question (2.6)



Knowing that  $\alpha = 75^\circ$ , determine the resultant of the three forces shown.

## Chapter (3): Equilibrium of a Particle

### 3.1 Condition for the Equilibrium of a Particle

- A particle is said to be in equilibrium if it remains at rest if originally at rest, or has a constant velocity if originally in motion.
- Static equilibrium is used to describe an object at rest.
- To maintain equilibrium, it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as:

$$\sum F = 0 \quad (3-1)$$

Where  $\sum F$  is the vector sum of all the forces acting on the particle.

### 3.2 The Free-Body Diagram:

- To apply the equation of equilibrium, we must account for all the known and unknown forces which act on the particle.
- The best way to do this is to think of the particle as isolated and "free" from its surroundings.
- A drawing that shows the particle with all the forces that act on it is called a free-body diagram (FBD).

#### 3.2.1 Cables and Pulleys:

- Unless otherwise stated, all cables (or cords) will be assumed to have negligible weight and they cannot stretch.
- Also, a cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable.
- **For any angle  $\theta$** , shown in Figure 3-1, the cable is subjected to a **constant tension  $T$**  throughout its length.

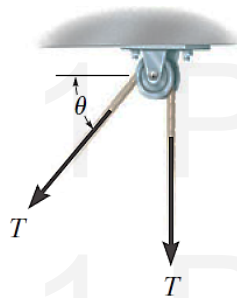


Figure 3-1: Cable is in Tension



### 3.2.2 Procedure for Drawing a Free-Body Diagram:

- Draw Outlined Shape.
  - Imagine the particle to be isolated or cut “free” from its surroundings by drawing its outlined shape.
- Show All Forces.
  - Indicate on this sketch all the forces that act on the particle.
  - These forces can be active forces, which tend to set the particle in motion, or they can be reactive forces which are the result of the constraints or supports that tend to prevent motion.
- Identify Each Force.
  - The forces that are known should be labeled with their proper magnitudes and directions.
  - Letters are used to represent the magnitudes and directions of forces that are unknown.

### 3.3 Coplanar Force Systems:

- If a particle is subjected to a system of coplanar forces that lie in the  $x$ - $y$  plane, as in Figure 3-2, then each force can be resolved into its perpendicular components.
- For equilibrium, these forces must sum to produce a zero force resultant, i.e.,

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0\end{aligned}\quad (3-2)$$

These two equations can be solved for at most two unknowns, generally represented as angles (or slopes) and magnitudes of forces shown on the particle's free-body diagram.

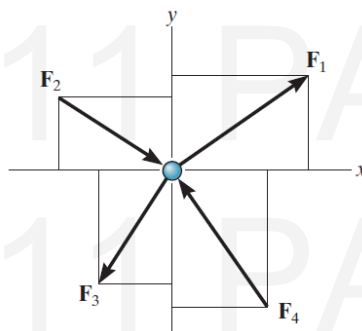


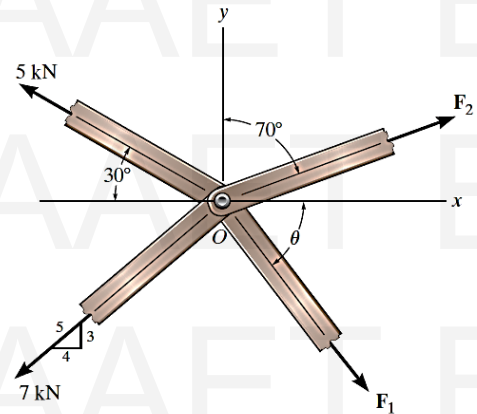
Figure 3-2: System of Coplanar Vectors Acting on a Particle

### 3.3.1 Examples:

#### Example (1):

The members of a truss are pin connected at joint  $O$ . Determine the magnitudes of  $F_1$  and  $F_2$  for equilibrium. Set  $\theta = 60^\circ$ .

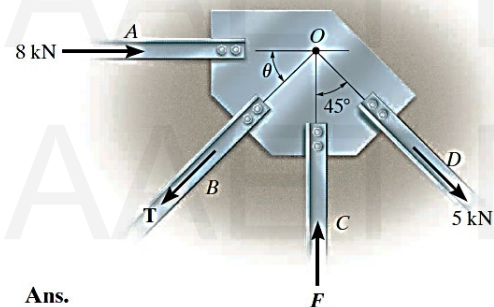
#### Solution:



#### Example (2):

The members of a truss are connected to the gusset plate. If the forces are concurrent at point  $O$ , determine the magnitudes of  $F$  and  $T$  for equilibrium. Take  $\theta = 30^\circ$ .

#### Solution:

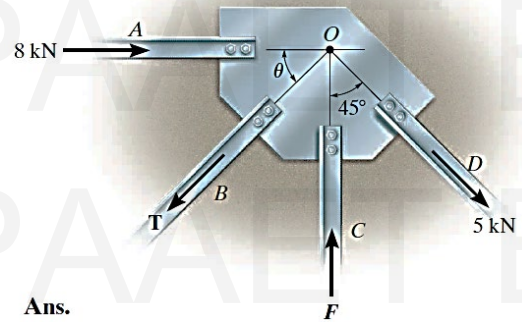


Ans.

**Example (3):**

The gusset plate is subjected to the forces of four members. Determine the force in member *B* and its proper orientation  $\theta$  for equilibrium. The forces are concurrent at point *O*. Take  $F = 12$  kN.

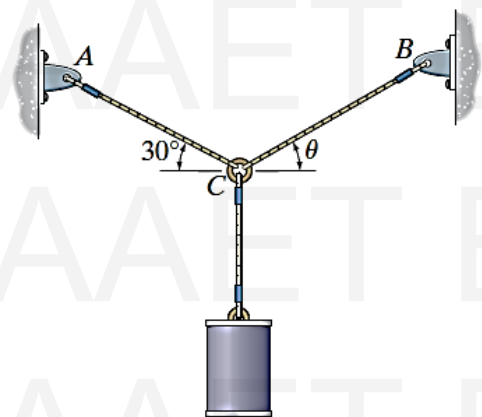
**Solution:**



**Example (4):**

Determine the tension developed in wires *CA* and *CB* required for equilibrium of the 10-kg cylinder. Take  $\theta = 40^\circ$ .

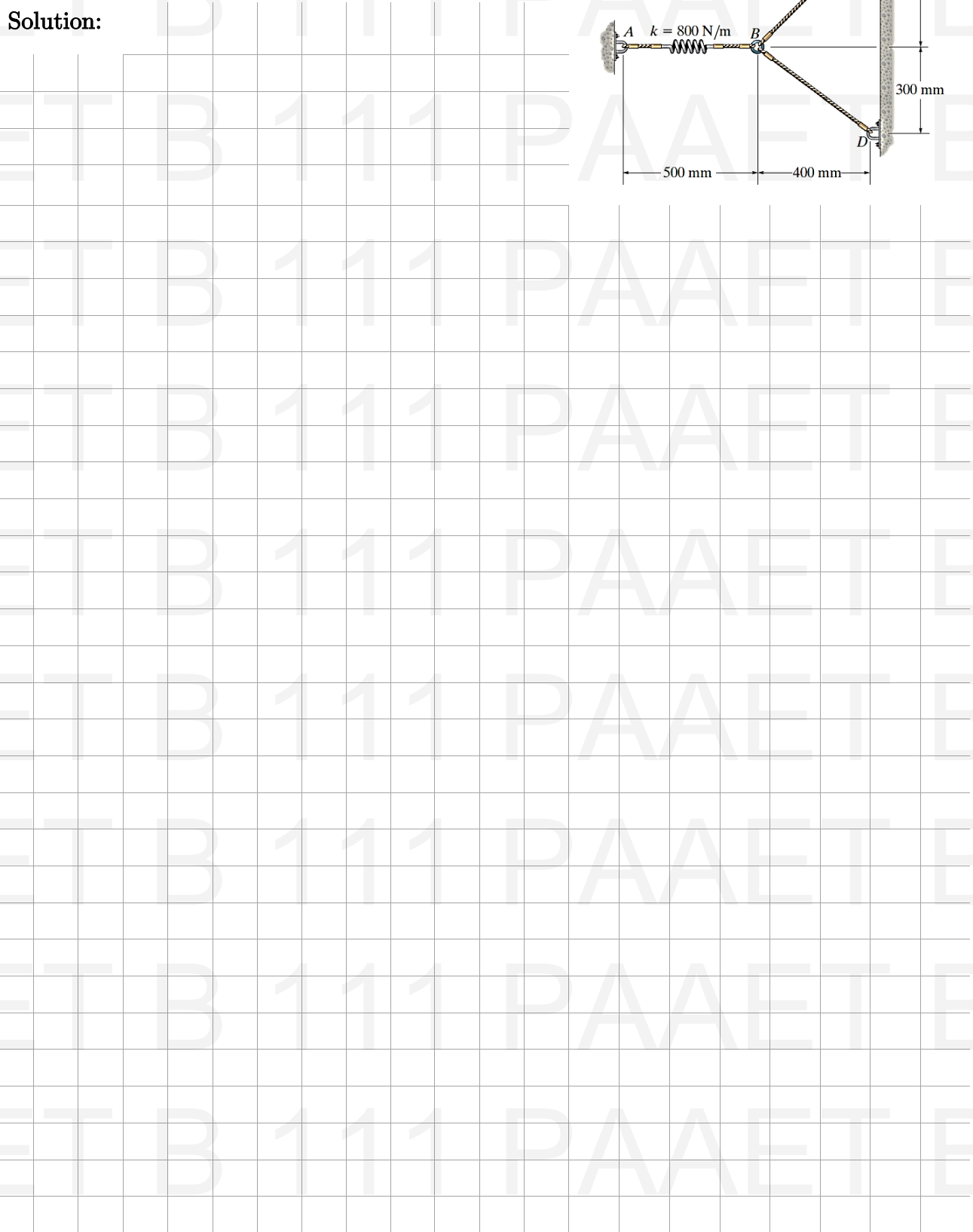
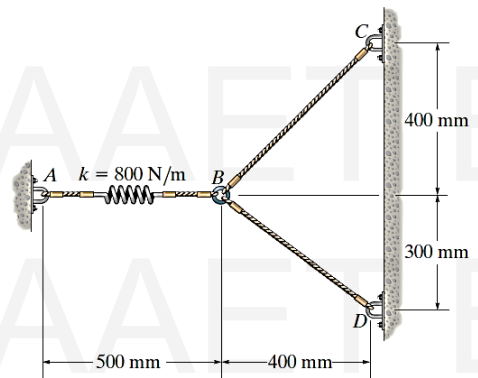
**Solution:**



**Example (5):**

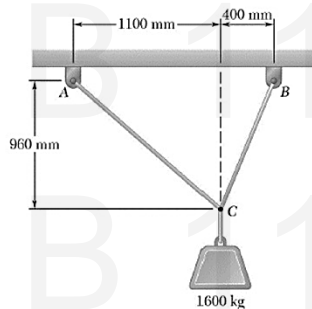
The spring has a stiffness of  $k = 800 \text{ N/m}$  and an unstretched length of 200 mm. Determine the force in cables  $BC$  and  $BD$  when the spring is held in the position shown.

**Solution:**



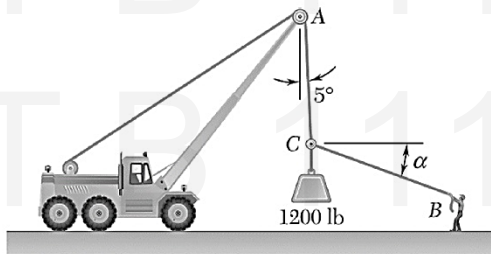
3.4 Problems:

Question (3.1)



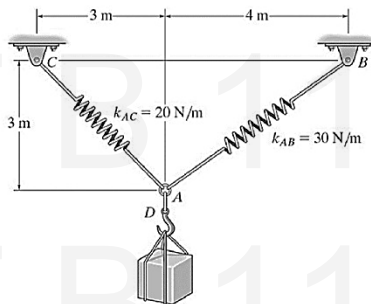
Two cables are tied together at  $C$  and are loaded as shown. Determine the tension in cables  $AC$  and  $BC$ .

Question (3.2)



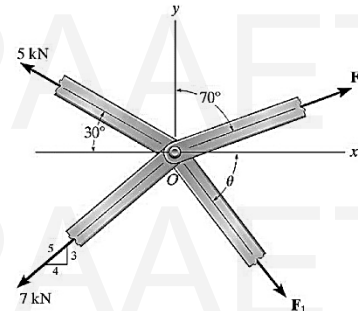
Knowing that  $\alpha = 20^\circ$ , determine the tension in cables  $AC$  and  $BC$ .

Question (3.3)



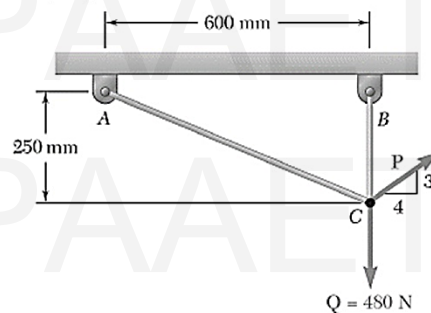
- Determine the stretched length in springs  $AC$  and  $AB$  for equilibrium of the 2-kg block.
- The un-stretched length of spring  $AB$  is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at  $D$ .

Question (3.4)



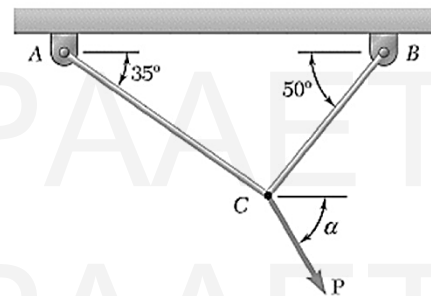
Determine the magnitude  $R$  of  $F_1$  and  $F_2$  for equilibrium. Set  $\theta = 60^\circ$ .

Question (3.5)



Two cables are tied together at  $C$  and loaded as shown. Knowing that  $P = 360$  N, determine the tension in cables  $AC$  and  $BC$ .

Question (3.6)



- Two cables tied together at  $C$  are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine:
- the magnitude of the largest force  $P$  that can be applied at  $C$ .
  - the corresponding value of  $\alpha$ .

## Chapter (4): Force System Resultants

### 4.1 Moment of a Force – Scalar Formulation:

- When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force.
- This tendency to rotate is sometimes called a torque, but most often it is called the **moment of a force** or simply the **moment**.

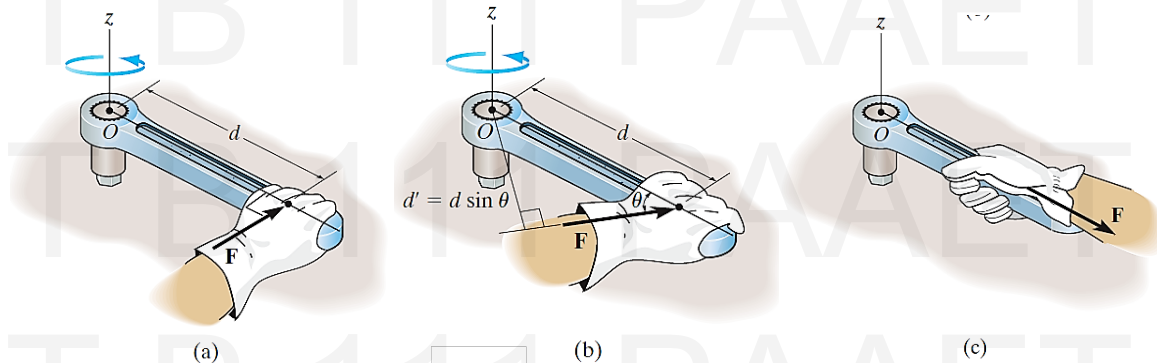


Figure 4-1: Force and Moment Arms

For example, consider a wrench used to unscrew the bolt in Figure 4-1.

- If a force is applied to the handle of the wrench it will tend to turn the bolt about point  $O$  (or the  $z$  axis).
- The magnitude of the moment is directly proportional to the magnitude of  $\mathbf{F}$  and the perpendicular distance or moment arm  $d$ .
- **The larger the force or the longer the moment arm, the greater the moment or turning effect.**
- Note that if the force  $\mathbf{F}$  is applied at an angle  $\theta$  that is not 90 degrees, Figure 4-1b , then it will be more difficult to turn the bolt since the moment arm  $d' = d \sin (\theta)$  will be smaller than  $d$ .
- If  $\mathbf{F}$  is applied along the wrench, Figure 4-1c, its **moment arm will be zero** since the line of action of  $\mathbf{F}$  will **intersect point  $O$**  (the  $z$  axis). As a result, the moment of  $\mathbf{F}$  about  $O$  is also zero and no turning can occur.

#### 4.1.1 Moment Magnitude, Direction, Sense of Rotation, & Resultant Moment:

##### Magnitude:

The magnitude of  $M_O$  is expressed as:

$$M_O = Fd \quad (4-1)$$

where  $d$  is the moment arm or **perpendicular distance** from the axis at point  $O$  to the line of action of the force. Units of moment magnitude consist of **force times distance** (N–m or lb–ft).

#### Direction & Sense of Rotation:

- The direction of  $M_o$  is defined by its moment axis, which is perpendicular to the plane that contains the force  $\mathbf{F}$  and its moment arm  $d$ .
- The right–hand rule is used to establish the sense of direction of  $M_o$ .
- According to this rule, the natural curl of the fingers of the right hand, as they are drawn towards the palm, represent the rotation, or if no movement is possible, there is a tendency for rotation caused by the moment.
- As this action is performed, the thumb of the right hand will give the directional sense of  $M_o$ , Figure 4–2a.
- Notice that the moment vector is represented three–dimensionally by a curl around an arrow.
- In two dimensions this vector is represented only by the curl as in Figure 4–2b.
- Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

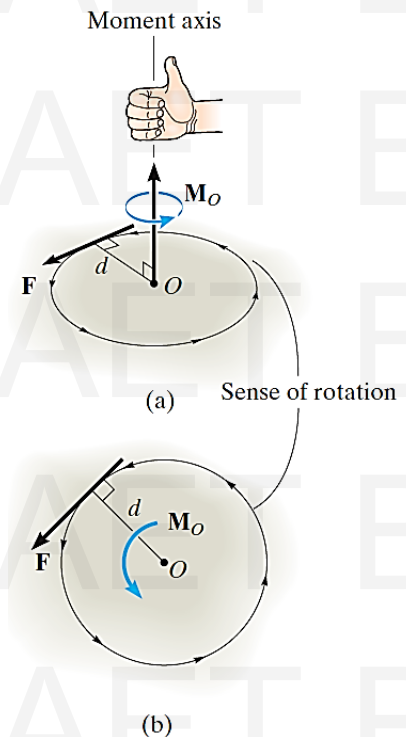


Figure 4–2: Moment Direction & Sense of Rotation

**Resultant Moment:**

- For two-dimensional problems, where all the forces lie within the  $x$ - $y$  plane, Figure 4-3, the resultant moment  $(M_R)_O$  about point  $O$  can be determined by finding the algebraic sum of the moments caused by all the forces in the system.
- As a convention, we will generally consider **positive moments as counterclockwise** since they are directed along the positive  $z$  axis (out of the page) and **Clockwise moments will be negative**.
- Using this sign convention, the resultant moment in Figure 4-3 is therefore:

$$(M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$$

If the numerical result of this sum is a **positive** scalar,  $(M_R)_O$  will be a **counterclockwise** moment (out of the page); and if the result is **negative**,  $(M_R)_O$  will be a **clockwise** moment (into the page).

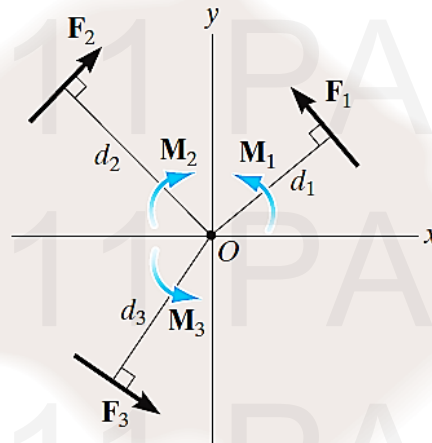


Figure 4-3: Resultant Moment

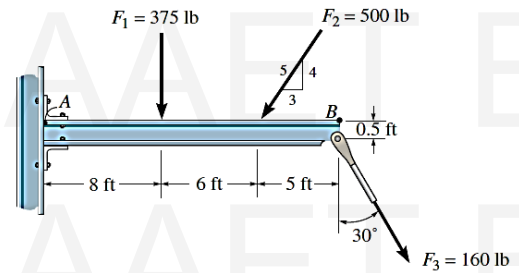




**Example (2):**

Determine the moment about point  $A$  of each of the three forces acting on the beam.

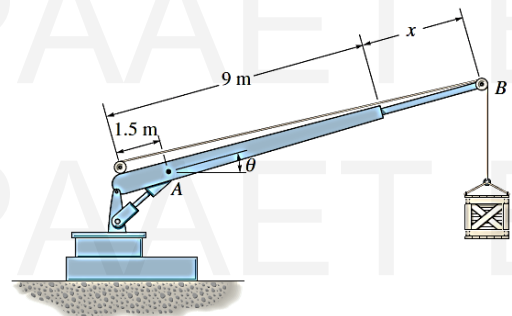
**Solution:**



**Example (3):**

The crane can be adjusted for any angle  $0^\circ \leq \theta \leq 90^\circ$  and any extension  $0 \leq x \leq 5$  m. For a suspended mass of 120 kg, determine the moment developed at  $A$  as a function of  $x$  and  $\theta$ . What values of both  $x$  and  $\theta$  develop the maximum possible moment at  $A$ ? Compute this moment. Neglect the size of the pulley at  $B$ .

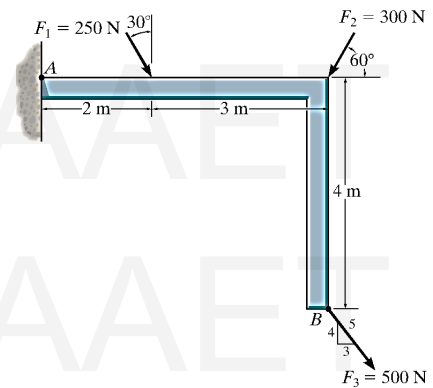
**Solution:**



**Example (4):**

Determine the moment of each of the three forces about point  $A$ .

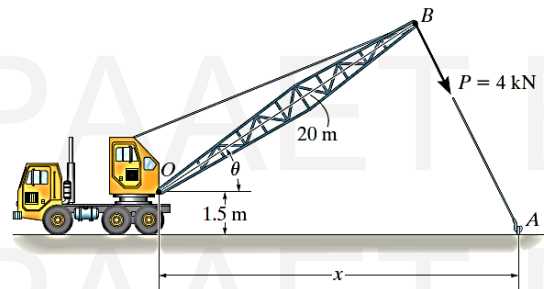
**Solution:**



**Example (5):**

The towline exerts a force of  $P = 4\text{ kN}$  at the end of the  $20\text{-m}$ -long crane boom. If  $\theta = 30^\circ$ , determine the placement  $x$  of the hook at  $A$  so that this force creates a maximum moment about point  $O$ . What is this moment?

**Solution:**



#### 4.2 Moment of a Couple:

- A couple is defined as two **parallel** forces that have the **same magnitude**, but **opposite directions**, and are separated by a **perpendicular distance  $d$** , Figure 4–4.
- Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible.

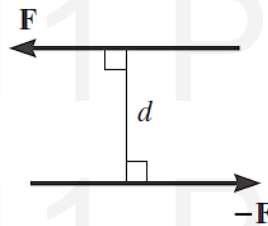


Figure 4–4: Moment of a Couple

##### 4.2.1 Scalar Formulation:

- The moment of a couple,  $M$ , Figure 4–4, is defined as having a magnitude of

$$M = Fd \quad (4-2)$$

- $F$  is the magnitude of one of the forces and  $d$  is the **perpendicular distance** or **moment arm** between the forces.
- The direction and sense of the couple moment are determined by the right–hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces.
- In all cases,  $M$  will act perpendicular to the plane containing these forces.

##### 4.2.2 Examples:

###### Example (1):

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

**Solution:**

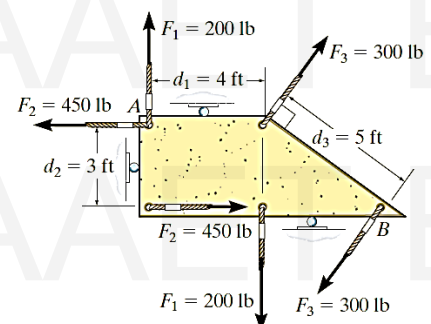
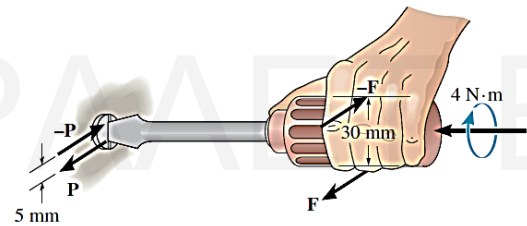


Fig. 4–30

**Example (2):**

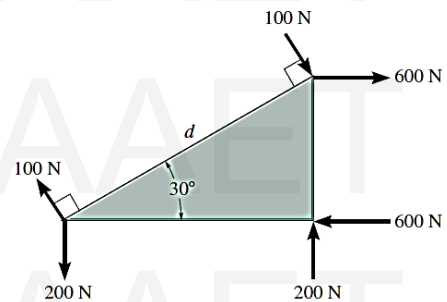
A twist of  $4 \text{ N} \cdot \text{m}$  is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces  $F$  exerted on the handle and  $P$  exerted on the blade.



**Solution:**

**Example (3):**

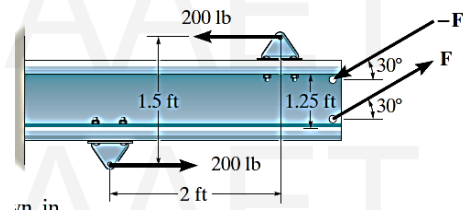
The ends of the triangular plate are subjected to three couples. Determine the plate dimension  $d$  so that the resultant couple is  $350 \text{ N} \cdot \text{m}$  clockwise.



**Solution:**

**Example (4):**

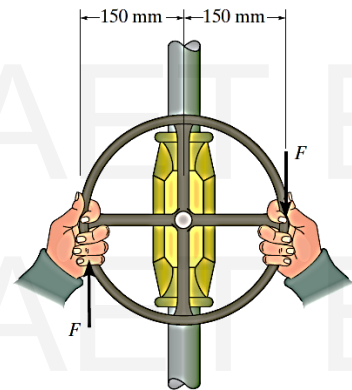
Two couples act on the beam. If  $F = 125 \text{ lb}$ , determine the resultant couple moment.



**Solution:**

**Example (5):**

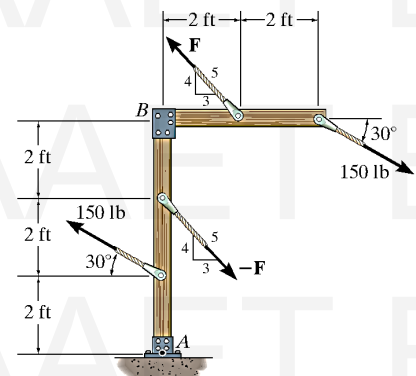
If the valve can be opened with a couple moment of  $25 \text{ N} \cdot \text{m}$ , determine the required magnitude of each couple force which must be applied to the wheel.



**Solution:**

**Example (6):**

Determine the required magnitude of force  $F$  if the resultant couple moment on the frame is  $200 \text{ lb} \cdot \text{ft}$ , clockwise.



**Solution:**

### 4.3 Simplification of a Force and Couple System:

- Sometimes it is **convenient to reduce a system** of forces and couple moments acting on a body to a simpler form by replacing it with an **equivalent system, consisting of a single resultant force acting at a specific point and a resultant couple moment.**
- A system is **equivalent** if the **external effects** it **produces** on a body are **the same as those caused by the original force and couple moment system.**
- In this context, the external effects of a system refer to the translating and rotating motion of the body if the body is free to move, or it refers to the reactive forces at the supports if the body is held fixed.
- This demonstrates the principle of transmissibility, which states that a force acting on a body (stick) is a sliding vector since it can be applied at any point along its line of action.
- If the force is to be moved NOT along the line of action, it can be moved provided a couple moment  $M$  is added to maintain equivalent system.

### 4.4 System of Forces and Couple Moments:

A system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point  $O$  and a resultant couple moment.

$$\begin{aligned}(F_R)_x &= \sum F_x \\ (F_R)_y &= \sum F_y \\ (M_R)_O &= \sum M_O + \sum M\end{aligned}\quad (4-3)$$

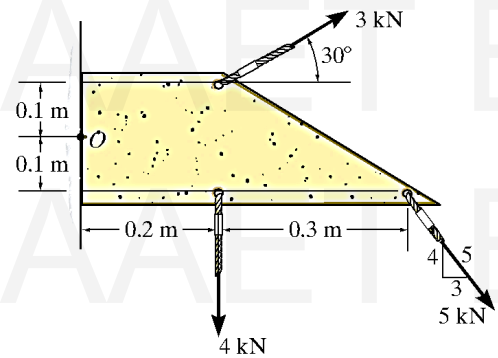
- The first and second equations states that the resultant force of the system is equivalent to the sum of all the forces
- The third equation states that the resultant couple moment of the system is equivalent to the sum of all the couple moments  $\sum M$  plus the moments of all the forces  $\sum M_O$  about point  $O$ .

4.4.1 Examples:

Example (1):

Replace the force and couple system shown in Fig. 4-37a by an equivalent resultant force and couple moment acting at point  $O$ .

Solution:

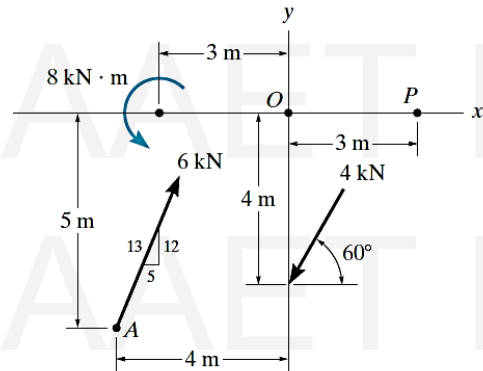




**Example (2):**

Replace the force and couple system by an equivalent force and couple moment at point  $O$ .

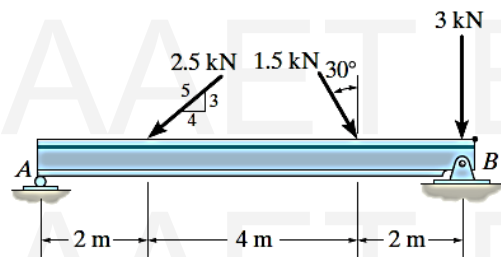
**Solution:**



**Example (3):**

Replace the force system acting on the beam by an equivalent force and couple moment at point  $A$ .

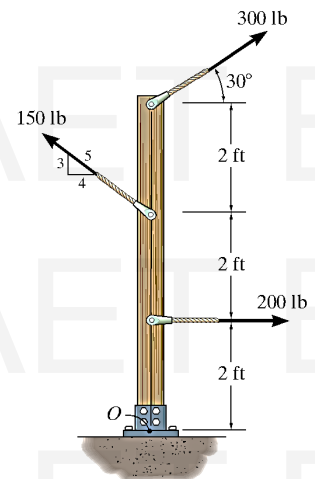
**Solution:**



**Example (4):**

Replace the force system acting on the post by a resultant force and couple moment at point  $O$ .

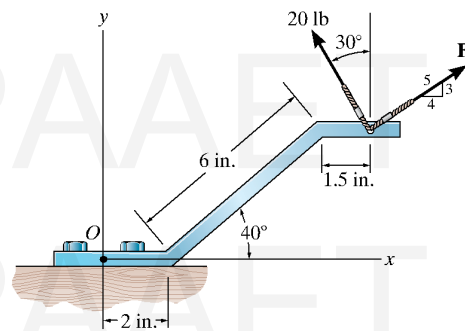
**Solution:**



**Example (5):**

Replace the two forces by an equivalent resultant force and couple moment at point  $O$ . Set  $F = 20$  lb.

**Solution:**



#### 4.5 Reduction of Distributed Loads:

#### 4.6 Loading Types:

The loading on beams and frames can be categorized to (Figure 4–5):

- Concentrated Load
  - Concentrated Force
  - Concentrated Moment
- Distributed Load
  - Uniformly Distributed Load (UDL)
  - Linearly Varying Distributed Load (LVDU)

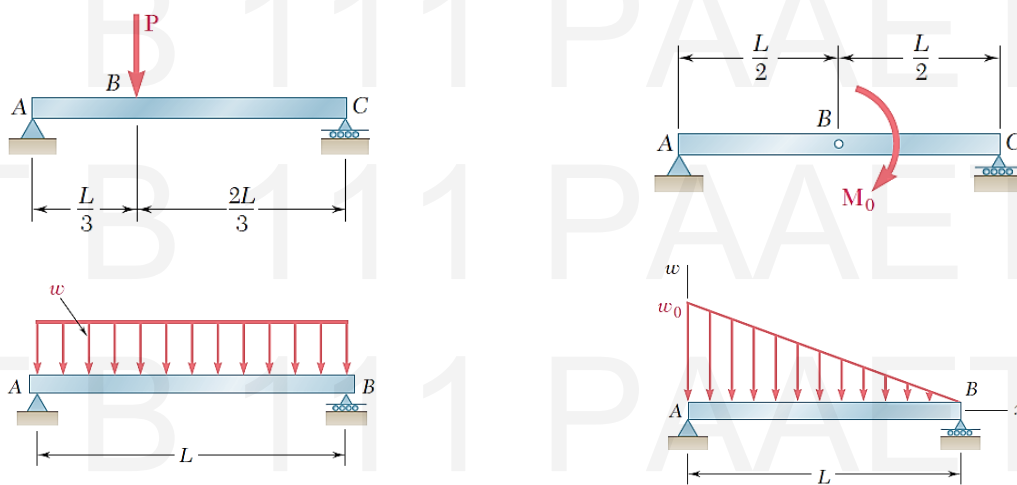


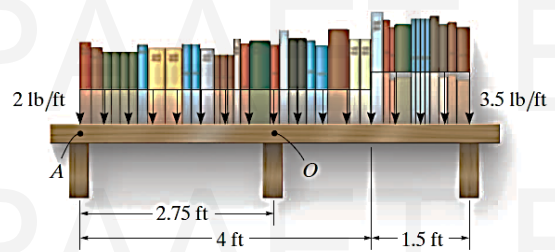
Figure 4–5: Loading On Beams

- When the above loading types are combined on a single structure, **they can be reduced single concentrated force (equivalent)** that will produce the same internal reactions as if the original loading was applied on the structure.
- The **distributed loads** shown in Figure 4–5, can be **reduced to concentrated forces** with a **magnitude equal to area under the loading diagram**.
- The **line of action** of this concentrated force **passes through the geometric center of the area under the loading diagram**. ( $L/2$  for rectangular load,  $L/3$  for triangular load from the higher value of the triangle)

4.6.1 Examples:

Example (1):

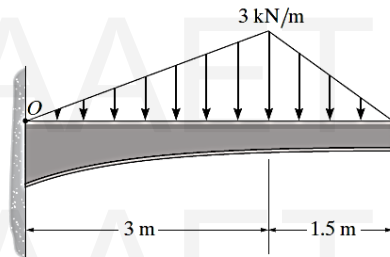
The loading on the bookshelf is distributed as shown. Determine the magnitude of the equivalent resultant location, measured from point  $O$ .



Solution:

Example (2):

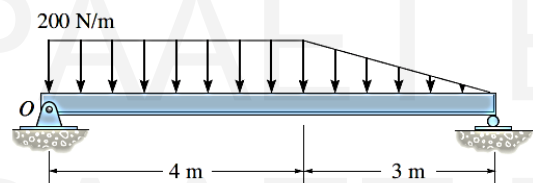
Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point  $O$ .



Solution:

Example (3):

Replace the loading by an equivalent force and couple moment acting at point  $O$ .

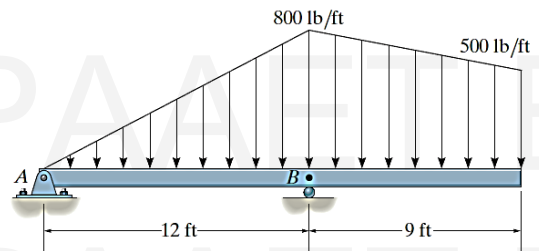


Solution:

**Example (4):**

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point *B*.

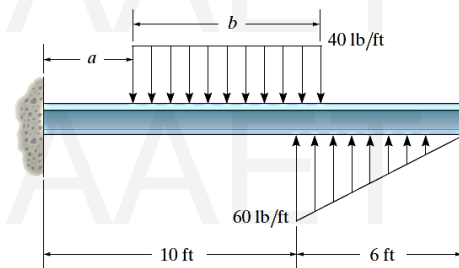
**Solution:**



**Example (5):**

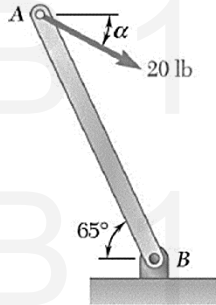
The beam is subjected to the distributed loading. Determine the length *b* of the uniform load and its position *a* on the beam such that the resultant force and couple moment acting on the beam are zero.

**Solution:**



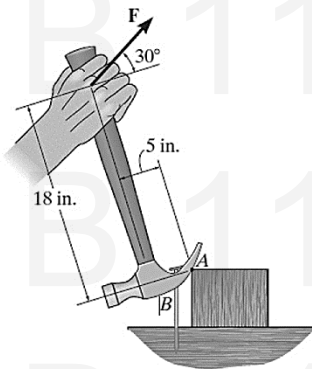
4.7 Problems:

Question (4.1)



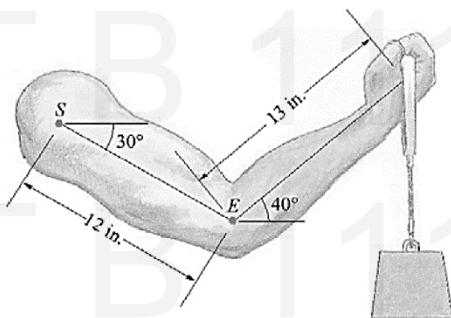
- (a) A 20-lb force is applied to the control rod  $AB$  as shown. Knowing that the length of the rod is 9 in. and that  $\alpha = 25^\circ$ , determine the moment of the force about Point  $B$ .
- (b) If the moment of the 20-lb force about  $B$  is  $120 \text{ lb} \cdot \text{in.}$  clockwise, determine the value of  $\alpha$ .

Question (4.2)



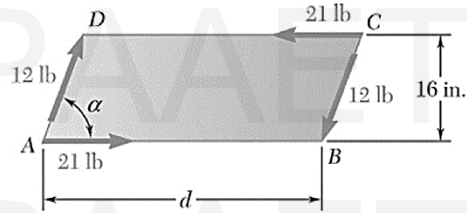
The handle of the hammer is subjected to the force of  $F = 20 \text{ lb}$ . Determine the moment of this force about the point  $A$ .

Question (4.3)



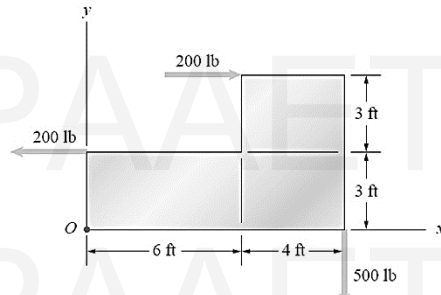
The moment exerted about point  $E$  by the weight is  $299 \text{ in-lb}$ . What moment does the weight exert about point  $S$ ?

Question (4.4)



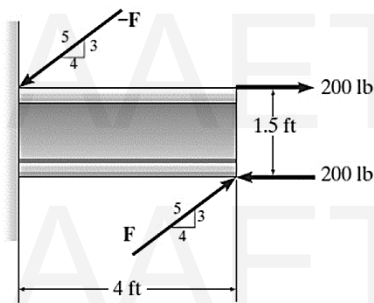
- A plate in the shape of a parallelogram is acted upon by two couples. Determine:
- (a) the moment of the couple formed by the two 21-lb forces
  - (b) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero
  - (c) the value of  $\alpha$  if the resultant couple is  $72 \text{ lb} \cdot \text{in.}$  clockwise and  $d$  is 42 in.

Question (4.5)



Determine the sum of the moments about point  $O$  by the couple and the 500 lb force.

Question (4.6)



Two couples act on the beam as shown. Determine the magnitude of  $F$  so that the resultant couple moment is  $300 \text{ lb-ft}$  counterclockwise. Where on the beam does the resultant couple act?

## Chapter (5): Elementary Structural Analysis

### 5.1 Introduction:

Structural analysis can be defined as the process of finding internal forces developed in structural members (such as beams, frames, trusses, columns, cables, etc.) due to external applied loads. The determination of such loads will aid the design process of the structure.

In this chapter, we will start with loads on structural beams.

### 5.2 Loading Types:

The loading on beam and frames can be categorized to (Figure 5-1):

- Concentrated Load
  - Concentrated Force
  - Concentrated Moment
- Distributed Load
  - Uniformly Distributed Load (UDL)
  - Linearly Varying Distributed Load (LVDU)

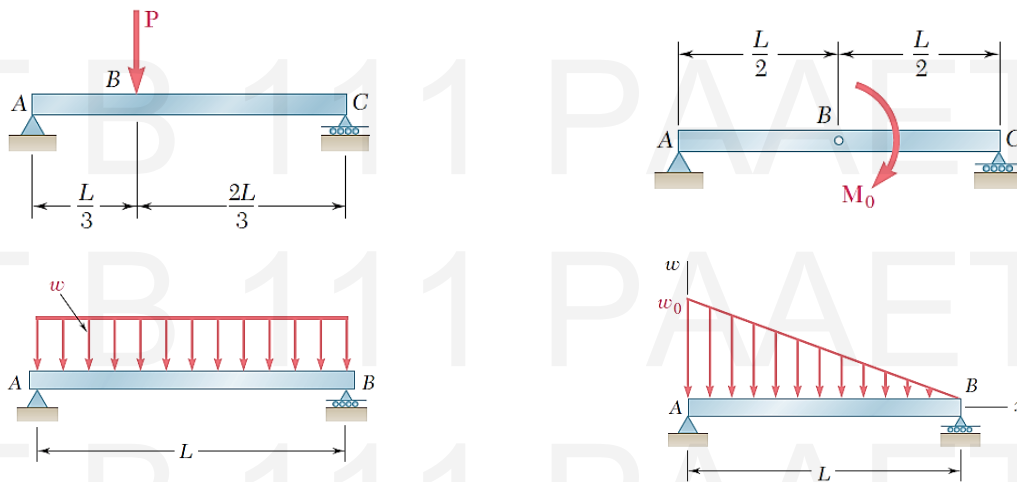


Figure 5-1: Loading types on beams

### 5.3 Support Types:

Supports on beams transfer the loads to the following structural member (usually a column)

Three major types (Figure 5–2):

- Roller → Vertical reaction only
- Hinge → Vertical and horizontal reaction
- Fixed → Vertical and horizontal reaction + a bending moment


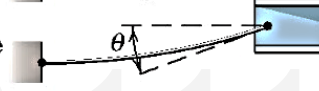
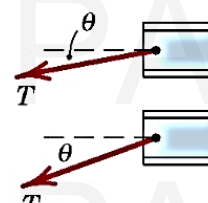

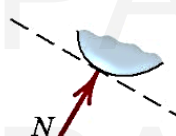

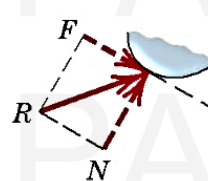
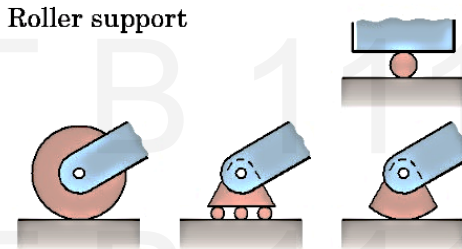
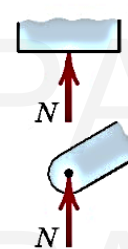
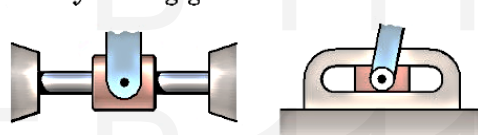
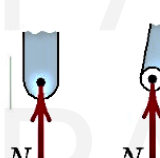
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Flexible cable, belt, chain, or rope</p> <p>Weight of cable negligible</p>  <p>Weight of cable not negligible</p> 	 <p>Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.</p>
<p>2. Smooth surfaces</p> 	 <p>Contact force is compressive and is normal to the surface.</p>
<p>3. Rough surfaces</p> 	 <p>Rough surfaces are capable of supporting a tangential component <math>F</math> (frictional force) as well as a normal component <math>N</math> of the resultant contact force <math>R</math>.</p>
<p>4. Roller support</p> 	 <p>Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.</p>
<p>5. Freely sliding guide</p> 	 <p>Collar or slider free to move along smooth guides; can support force normal to guide only.</p>

Figure 5–2: Beam Reaction Types




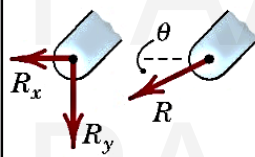
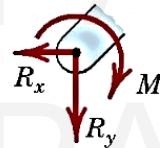
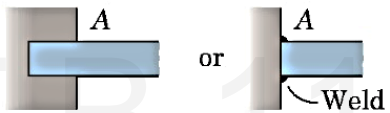
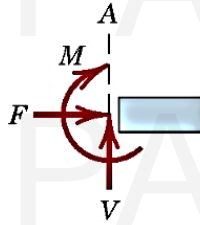
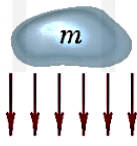
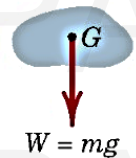
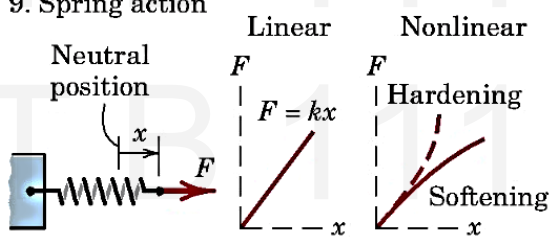

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>6. Pin connection</p> 	<p>Pin free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components <math>R_x</math> and <math>R_y</math> or a magnitude <math>R</math> and direction <math>\theta</math>. A pin not free to turn also supports a couple <math>M</math>.</p> <p>Pin not free to turn </p>
<p>7. Built-in or fixed support</p> 	 <p>A built-in or fixed support is capable of supporting an axial force <math>F</math>, a transverse force <math>V</math> (shear force), and a couple <math>M</math> (bending moment) to prevent rotation.</p>
<p>8. Gravitational attraction</p> 	 <p>The resultant of gravitational attraction on all elements of a body of mass <math>m</math> is the weight <math>W = mg</math> and acts toward the center of the earth through the center mass <math>G</math>.</p>
<p>9. Spring action</p> 	 <p>Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness <math>k</math> is the force required to deform the spring a unit distance.</p>

Figure 5-3: Beam Reaction Types (Continued)

### 5.4 Beam Types:

Beams can be divided into (Figure 5-4):

- **Statically determinate beams:**
  - Simply supported beams
  - One-sided over-hanging beam
  - Two-sided over-hanging beam
  - Cantilever beam
- **Statically indeterminate beams:**
  - Continuous beam
  - End-supported cantilever
  - Fixed at both ends

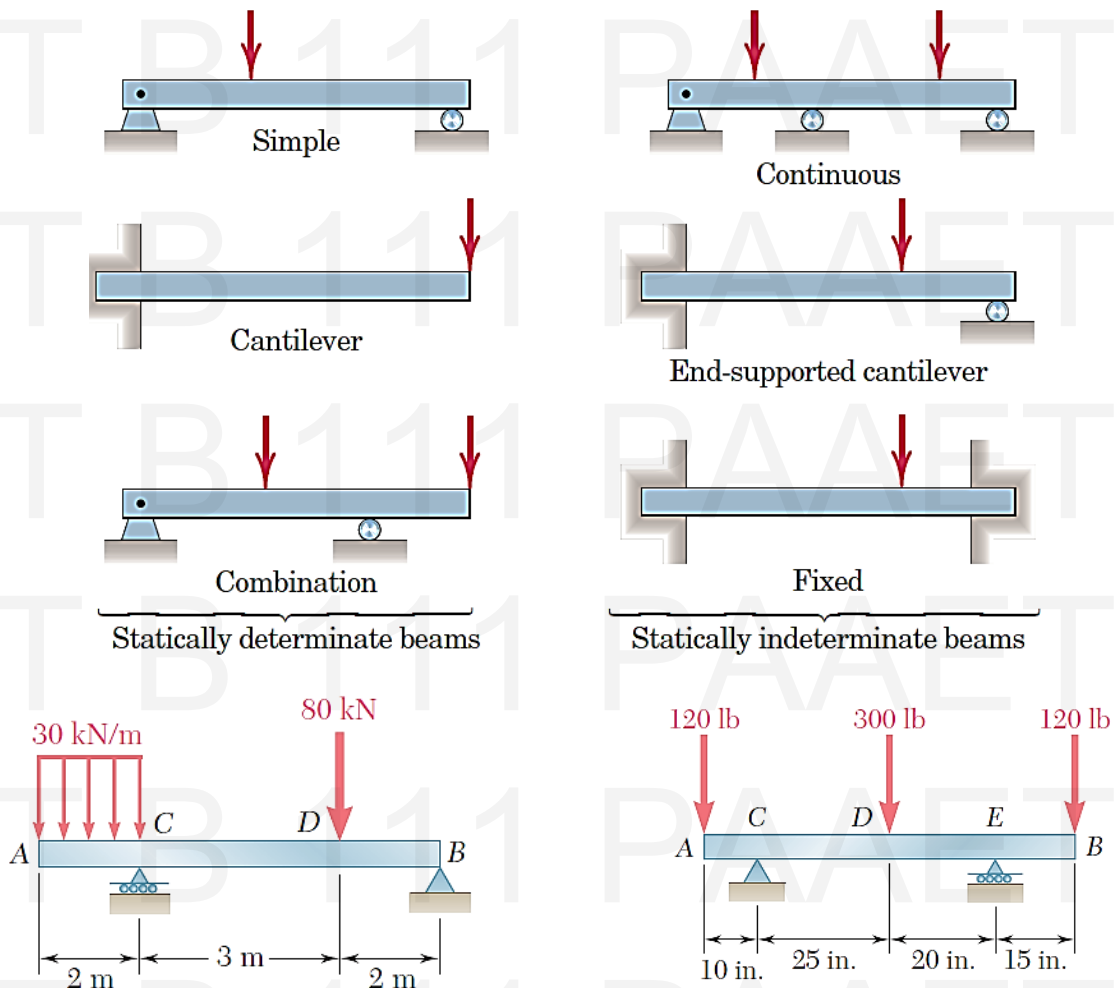


Figure 5-4: Beam types

### 5.5 Beam Reactions:

- Reactions on beams are developed due to the applications of the various loads on the beam.
- The reactions can be calculated (determinate beams only) by applying the three equations of equilibrium after drawing the free body diagram of the beam.
- The three equations of equilibrium are:

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M &= 0\end{aligned}\tag{5-1}$$

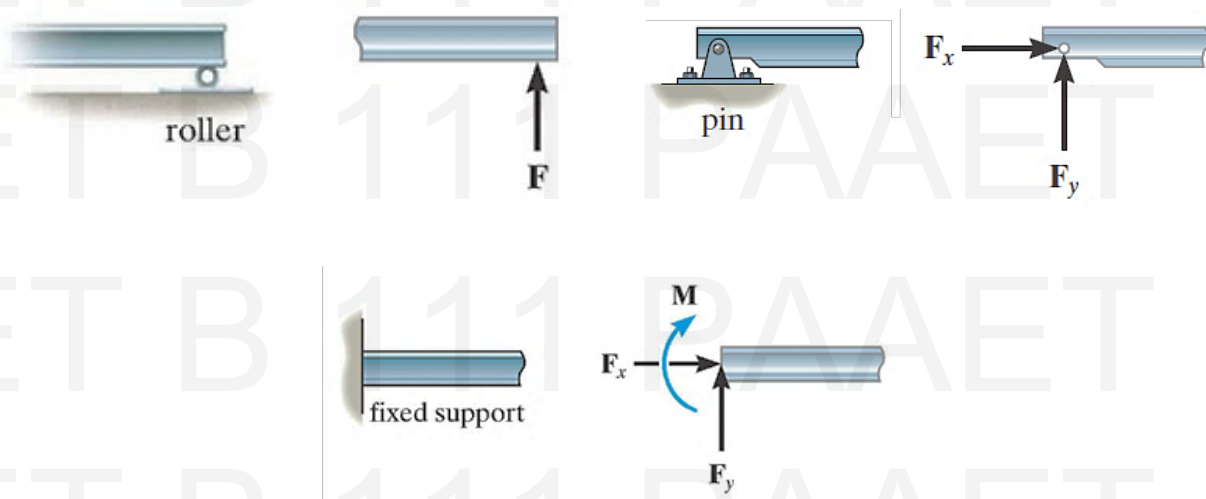


Figure 5-5: Beam reaction types

### 5.6 Sign Convention:

The positive sign convention used throughout the course is summarized in Figure 5-6. The positive  $x$ -direction is taken to the right, the positive  $y$ -direction is taken upward, and the positive moment is taken in the counter-clockwise direction.

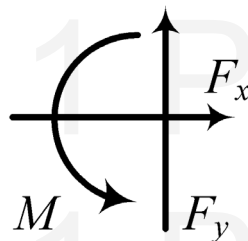
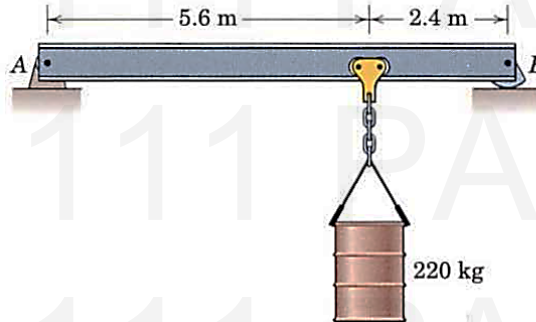


Figure 5-6: The positive sign convention for forces and moment

5.7 Examples:

Example (1):

The 450-kg uniform I-beam supports the load shown.  
Determine the reactions at the supports.

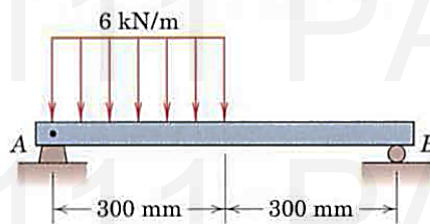


Solution:

Example (2):

Determine the reactions at A and B for the beam  
subjected to the uniform load distribution.

Ans.  $R_A = 1.35 \text{ kN}$ ,  $R_B = 0.45 \text{ kN}$

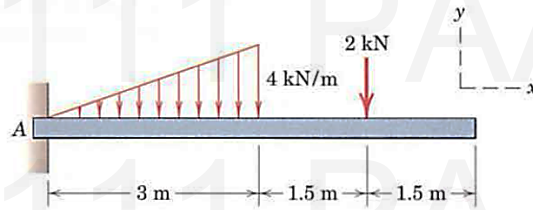


Solution:

**Example (3):**

**5/97** Determine the reactions at A for the cantilever beam subjected to the distributed and concentrated loads.

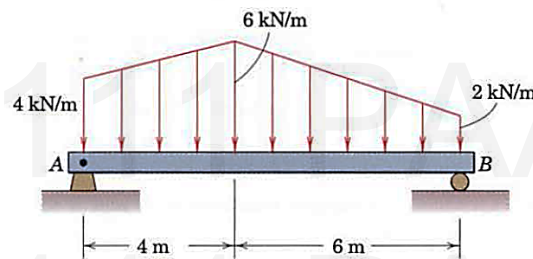
*Ans.*  $A_x = 0$ ,  $A_y = 8 \text{ kN}$ ,  $M_A = 21 \text{ kN} \cdot \text{m}$



**Solution:**

**Example (4):**

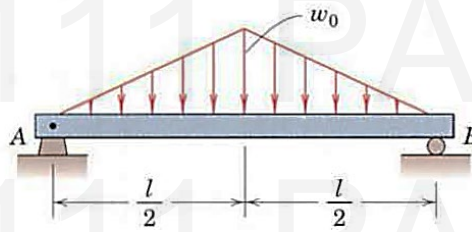
**5/100** Calculate the support reactions at A and B for the beam subjected to the two linearly varying load distributions.



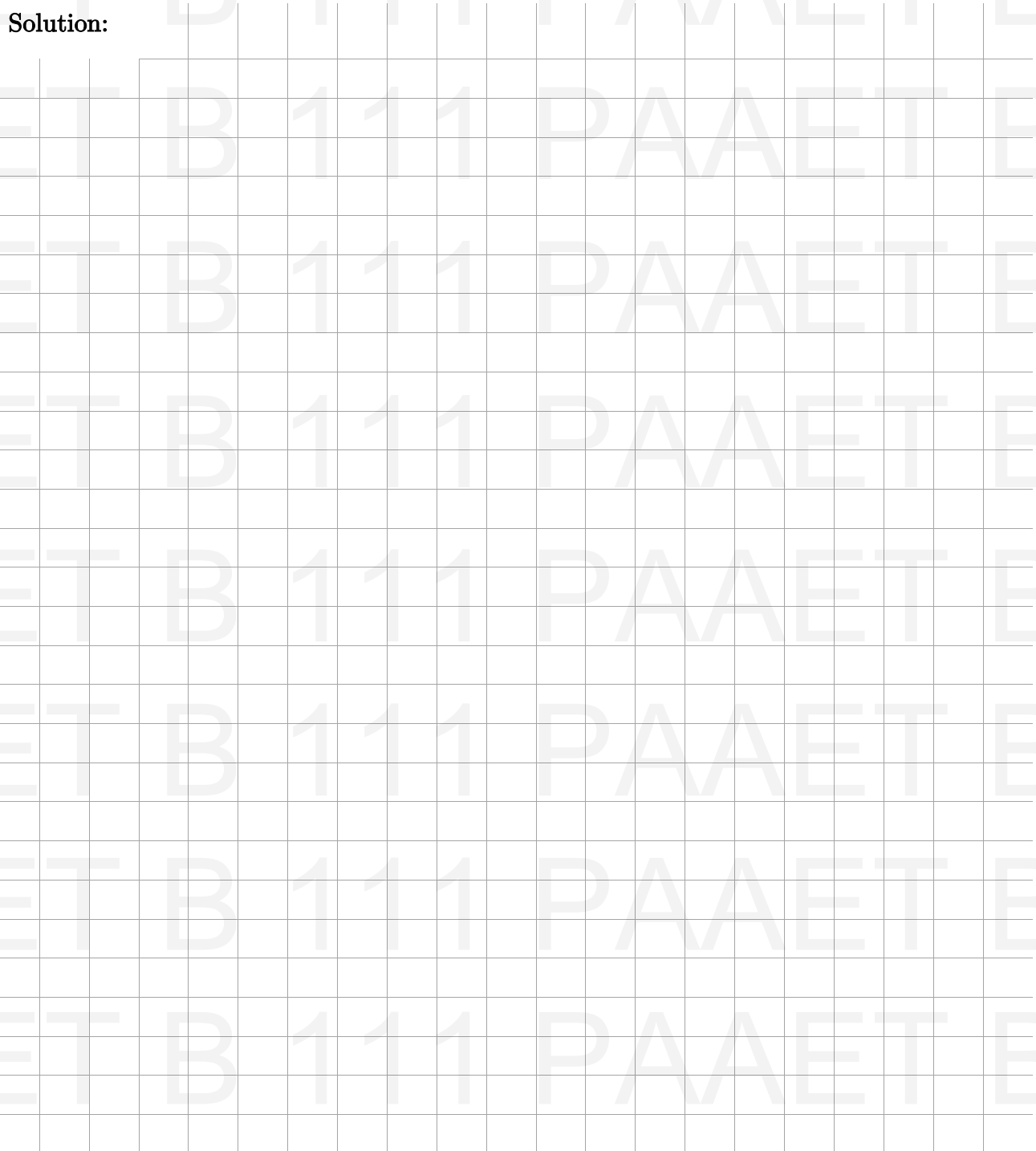
**Solution:**

Example (5):

5/94 Determine the reactions at the supports  $A$  and  $B$  for the beam loaded as shown.

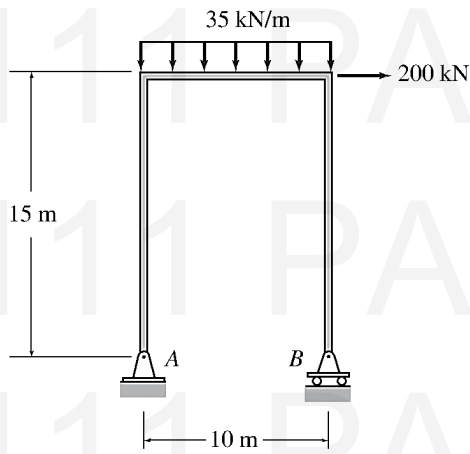


Solution:

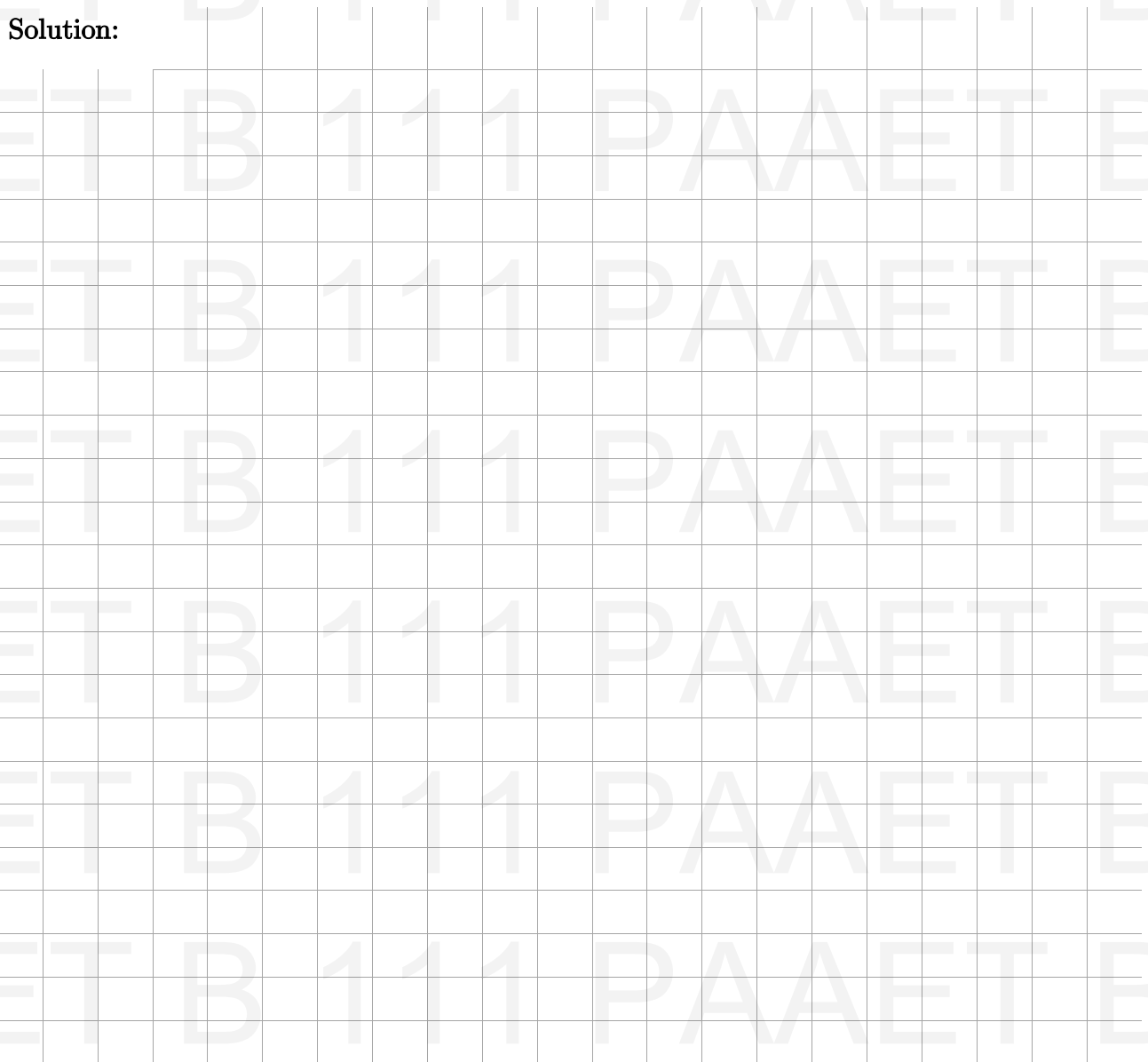


For the following examples, calculate the reactions at the frame supports.

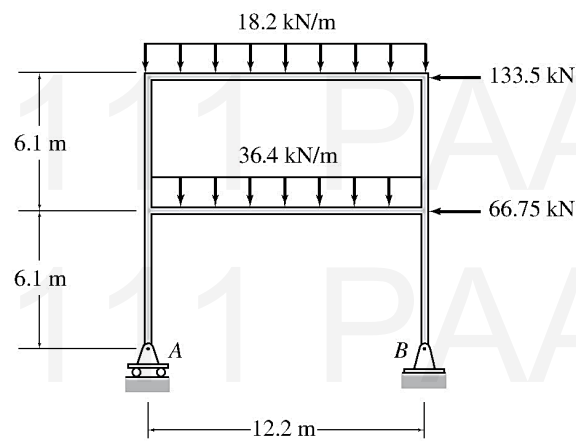
**Example (6):**



**Solution:**



Example (7):

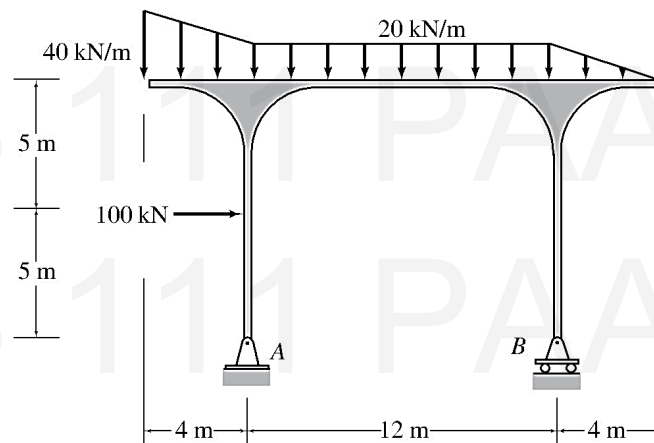


Solution:





Example (8):



Solution:



### 5.8 Internal Forces in Structural Members:

Internal forces were defined as the forces and couples exerted on a portion of the structure by the rest of the structure.

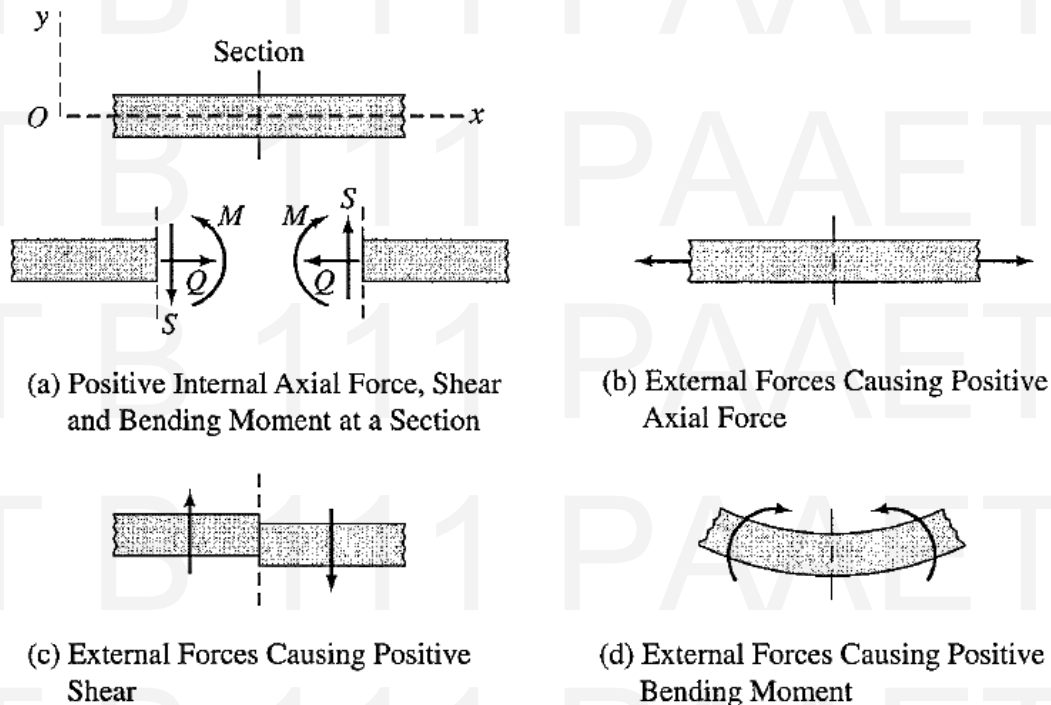


Figure 5-7: Sign convention for axial force, shear force, and bending moment

#### 5.8.1 Procedure for Analysis

The procedure for determining internal forces at a specified location on a beam can be summarized as follows:

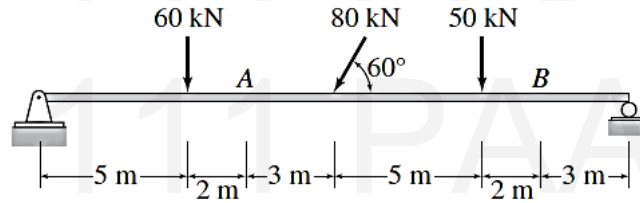
- 1- Compute the support reactions by applying the equations of equilibrium and condition (if any) to the free body of the entire beam. In cantilever beams, this step can be avoided by selecting the free, or externally unsupported, portion of the beam for analysis.
- 2- Pass a section perpendicular to the centroidal axis of the beam at the point where the internal forces are desired, thereby cutting the beam into two portions.
- 3- Although either of the two portions of the beam can be used for computing internal forces, we should select the portion that will require the least amount of computational effort, such as the portion that does not have any reactions acting on it or that has the least number of external loads and reactions applied to it.
- 4- Determine the axial force at the section by algebraically summing the components in the direction parallel to the axis of the beam of all the external loads and support reactions acting on the selected portion.

- 5- Determine the shear at the section by algebraically summing the components in the direction perpendicular to the axis of the beam of all the external loads and reactions acting on the selected portion.
- 6- Determine the bending moment at the section by algebraically summing the moments about the section of all the external forces plus the moments of any external couples acting on the selected portion.
- 7- To check the calculations, values of some or all of the internal forces may be computed by using the portion of the beam not utilized in steps 4 through 6. If the analysis has been performed correctly, then the results based on both left and right portions must be identical.

For the following examples, determine the axial forces, shears, and bending moments at points *A* and *B* of the structure shown.

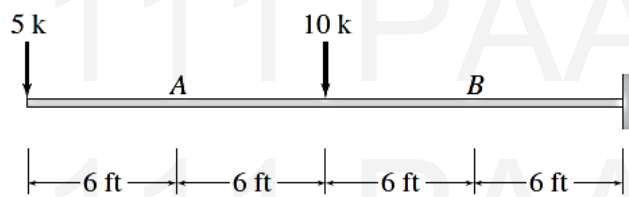
**5.9 Examples:**

**Example (1):**



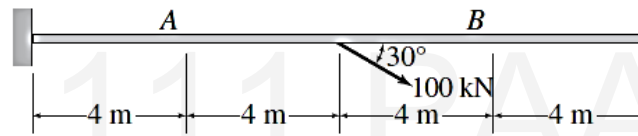
**Solution:**

**Example (2):**



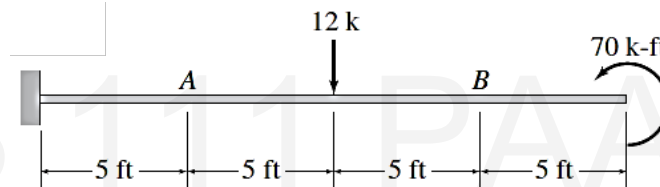
**Solution:**

**Example (3):**



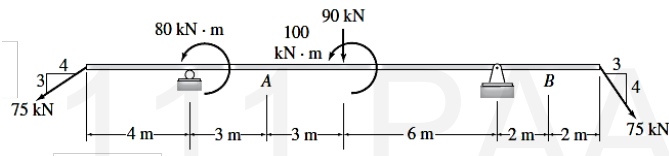
**Solution:**

**Example (4):**

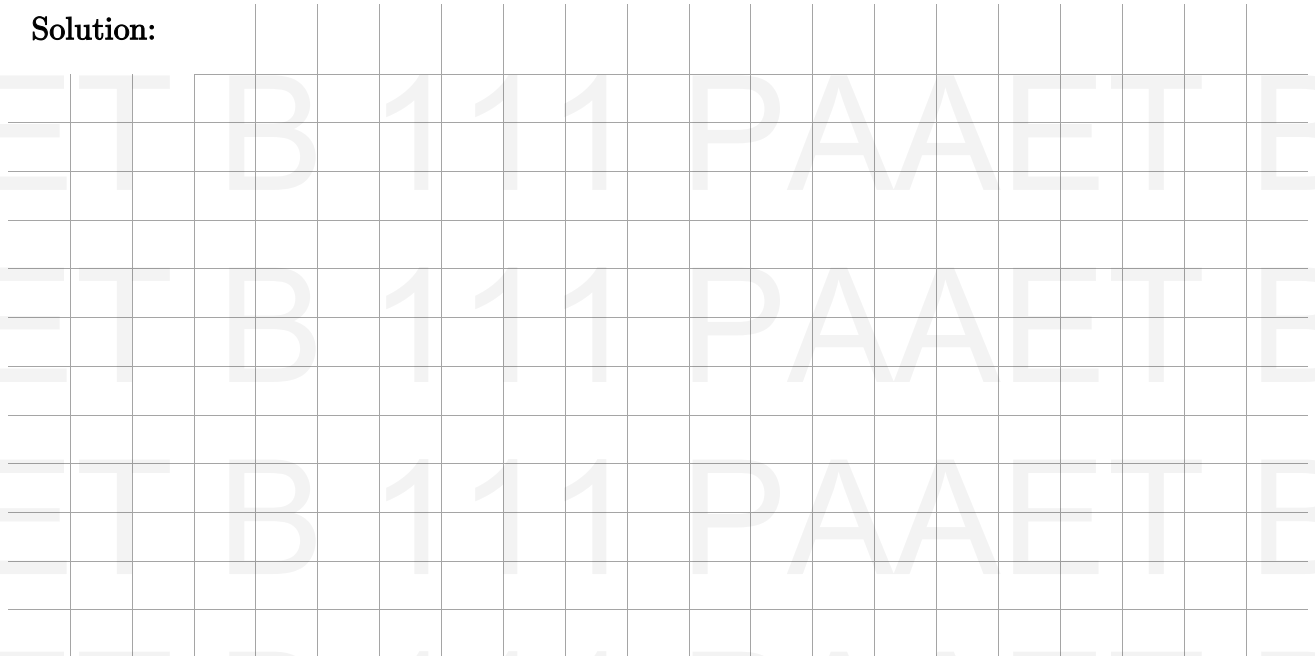


**Solution:**

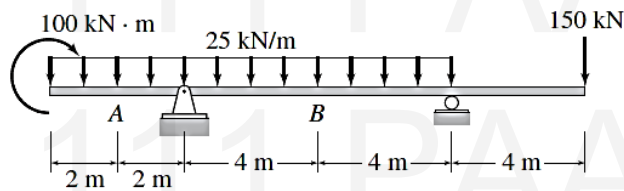
**Example (5):**



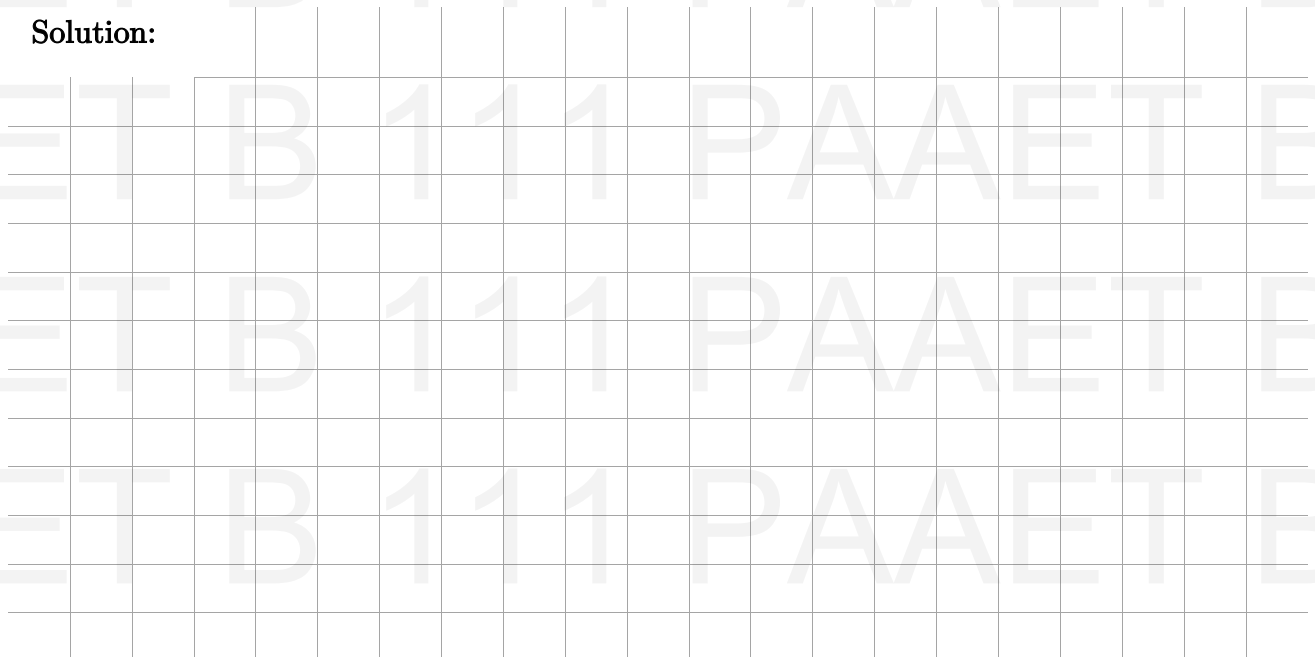
**Solution:**



**Example (6):**

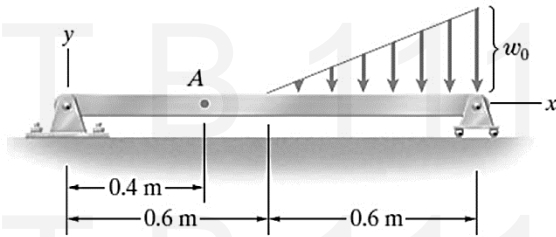


**Solution:**



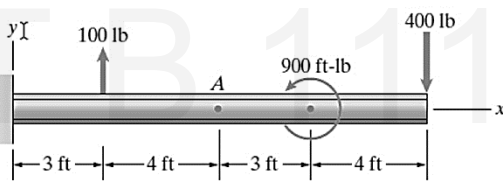
5.10 Problems:

Question (5.1)



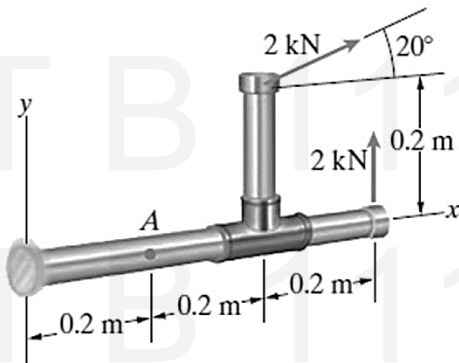
The magnitude of the triangular distributed load is  $w_0 = 2 \text{ kN/m}$ . Determine the internal forces and moment at  $A$ .

Question (5.2)



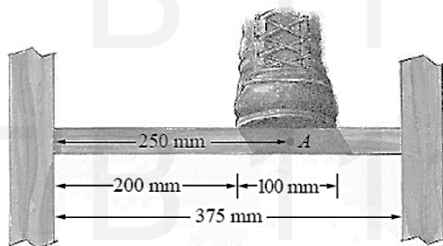
Determine the internal forces and moment at  $A$ .

Question (5.3)



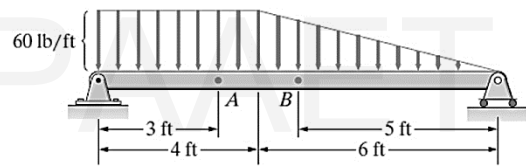
The pipe has a fixed support at the left end. Determine the internal forces and moment at  $A$ .

Question (5.4)



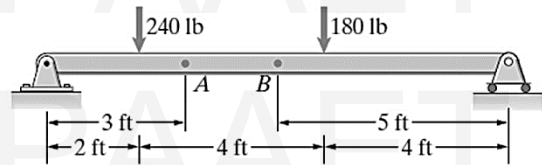
Model the ladder rung as a simply supported (pin supported) beam and assume that the 750-N load exerted by the person's shoe is uniformly distributed. Determine the internal forces and moment at  $A$ .

Question (5.5)



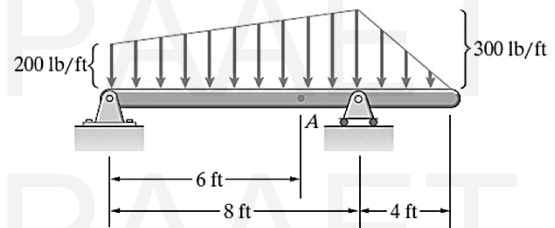
Determine the internal forces and moment at  $A$  and  $B$ .

Question (5.6)



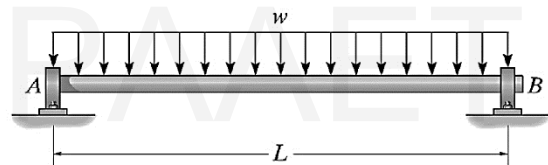
Determine the internal forces and moment at  $A$  and  $B$ .

Question (5.7)



Determine the internal forces and moment at  $A$ .

Question (5.8)



For the beam shown, What is the shear force and bending moment at mid-span? Assume support  $A$  is a hinge and  $B$  is a roller.

## Chapter (6): Truss Analysis

### 6.1 Introduction:

Truss is an assemblage of straight members connected at their ends by flexible connections to form a rigid configuration. Because of their light weight and high strength, trusses are widely used, and their applications range from supporting bridges and roofs of buildings to being support structures in space stations. Modern trusses are constructed by connecting members, which usually consist of structural steel or aluminum shapes or wood struts, to gusset plates by bolted or welded connections.

If all the members of a truss and the applied loads lie in a single plane, the truss is called a plane truss. Plane trusses are commonly used for supporting decks of bridges and roofs of buildings.

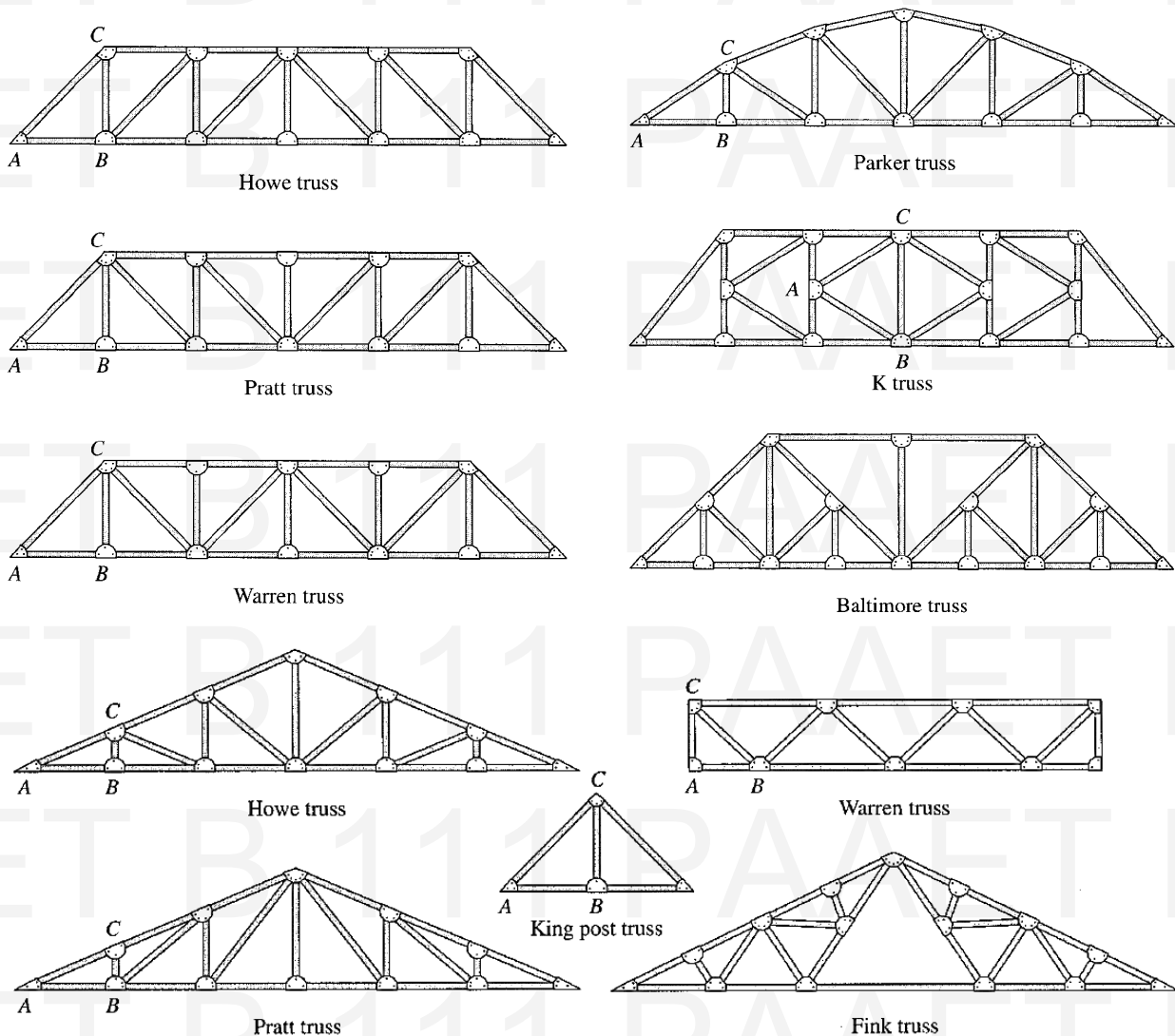


Figure 6-1: Common roof trusses



## 6.2 Assumptions for Analysis of Trusses:

The analysis of trusses is usually based on the following simplifying assumptions:

- 1- All members are connected only at their ends by frictionless hinges in plane trusses and by frictionless ball-and-socket joints in space trusses.
- 2- All loads and support reactions are applied only at the joints.
- 3- The centroidal axis of each member coincides with the line connecting the centers of the adjacent joints.

## 6.3 Method of Joints:

### 6.3.1 Procedure for Analysis

The following step-by-step procedure can be used for the analysis of statically determinate simple plane trusses by the method of joints.

- 1- Check the truss for static determinacy. If the truss is found to be statically determinate and stable, proceed to step 2. Otherwise, end the analysis at this stage.
- 2- Determine the slopes of the inclined members (except the zero-force members) of the truss.
- 3- Draw a free-body diagram of the whole truss, showing all external loads and reactions.
- 4- Examine the free-body diagram of the truss to select a joint that has no more than **two unknown** forces (which must not be collinear) acting on it. If such a joint is found, then go directly to the next step. Otherwise, determine reactions by applying the three equations of equilibrium and the equations of condition (if any) to the free body of the whole truss; then select a joint with two or fewer unknowns, and go to the next step.
- 5- a. Draw a free-body diagram of the selected joint, showing tensile forces by arrows pulling away from the joint and compressive forces by arrows pushing into the joint.

It is usually convenient to assume the unknown member forces to be tensile.

- b. Determine the unknown forces by applying the two equilibrium equations ( $x$  and  $y$  direction). A positive answer for a member force means that the member is in tension, as initially assumed, whereas a negative answer indicates that the member is in compression.

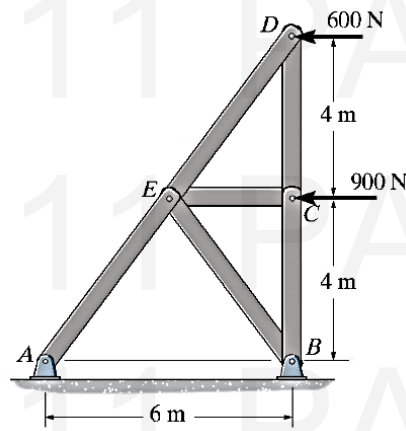
**If at least one of the unknown forces acting at the selected joint is in the horizontal or vertical direction, the unknowns can be conveniently determined by satisfying the two equilibrium equations by inspection of the joint on the free-body diagram of the truss.**

- 6- If all the desired member forces and reactions have been determined, then go to the next step. Otherwise, select another joint with no more than two unknowns, and return to step 5.
- 7- If the reactions were determined in step 4 by using the equations of equilibrium and condition of the whole truss, then apply the remaining joint equilibrium equations that have not been utilized so far to check the calculations. If the reactions were computed by applying the joint equilibrium equations, then use the equilibrium equations of the entire truss to check the calculations. If the analysis has been performed correctly, then these extra equilibrium equations must be satisfied.

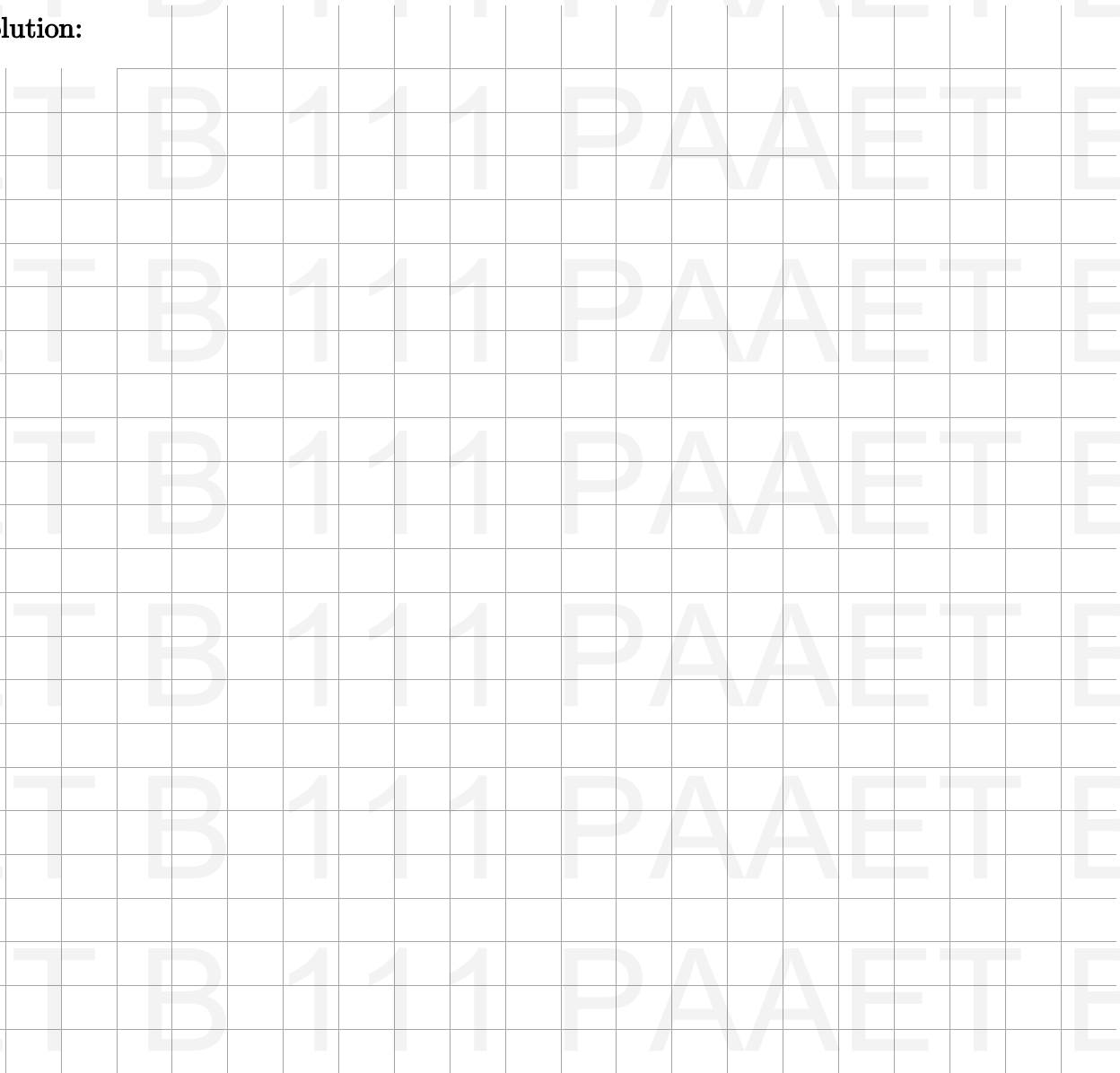
For the following examples, find the forces in the members of the truss and indicate if the member is in tension or compression.

6.3.2 Examples:

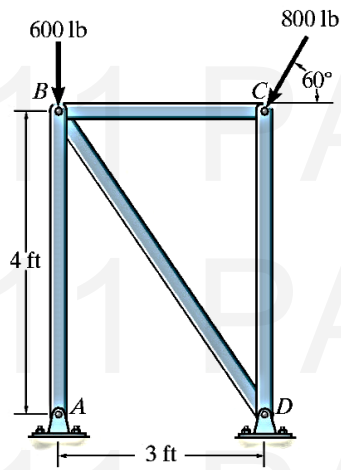
Example (1):



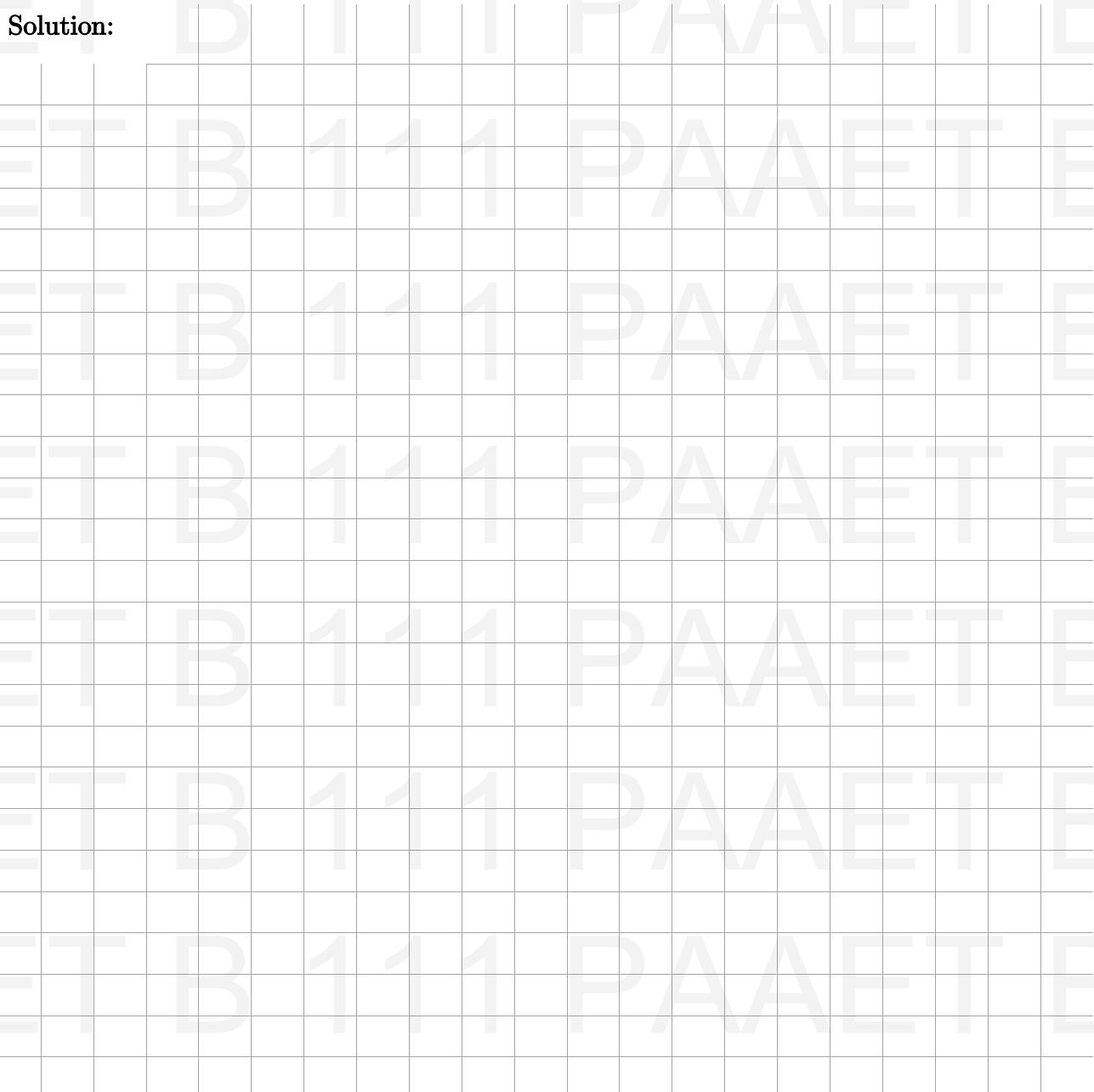
Solution:



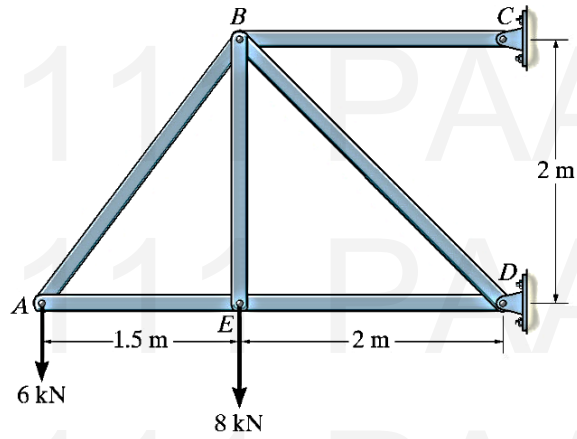
Example (2):



Solution:



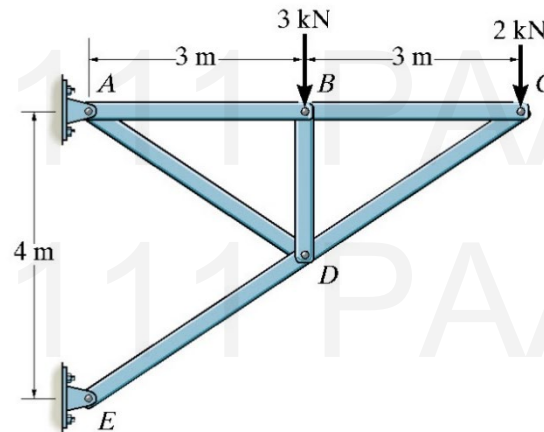
**Example (3):**



**Solution:**



**Example (4):**



**Solution:**



## 6.4 Method of Sections:

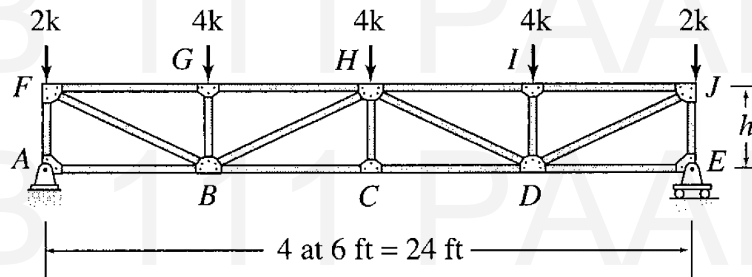
### 6.4.1 Procedure for Analysis:

The following step-by-step procedure can be used for determining the member forces of statically determinate plane trusses by the method of sections.

1. Select a section that passes through as many members as possible whose forces are desired, but not more than three members with unknown forces. The section should cut the truss into two parts.
2. Although either of the two portions of the truss can be used for computing the member forces, we should select the portion that will require the least amount of computational effort in determining the unknown forces. To avoid the necessity for the calculation of reactions, if one of the two portions of the truss does not have any reactions acting on it, then select this portion for the analysis of member forces and go to the next step. If both portions of the truss are attached to external supports, then calculate reactions by applying the equations of equilibrium and condition (if any) to the free body of the entire truss. Next, select the portion of the truss for analysis of member forces that has the least number of external loads and reactions applied to it.
3. Draw the free-body diagram of the portion of the truss selected, showing all external loads and reactions applied to it and the forces in the members that have been cut by the section. The unknown member forces are usually assumed to be tensile and are, therefore, shown on the free-body diagram by arrows pulling away from the joints.
4. Determine the unknown forces by applying the three equations of equilibrium. To avoid solving simultaneous equations, try to apply the equilibrium equations in such a manner that each equation involves only one unknown. This can sometimes be achieved by using the alternative systems of equilibrium equations (Sum of moment equations) instead of the usual two-force summations and a moment summation system of equations.
5. Apply an alternative equilibrium equation, which was not used to compute member forces, to check the calculations. This alternative equation should preferably involve all three member forces determined by the analysis. If the analysis has been performed correctly, then this alternative equilibrium equation must be satisfied.

For the following examples, use the method of sections to solve for the required members (indicated by x) and state whether the members are in tension or compression.

**Example (1):**

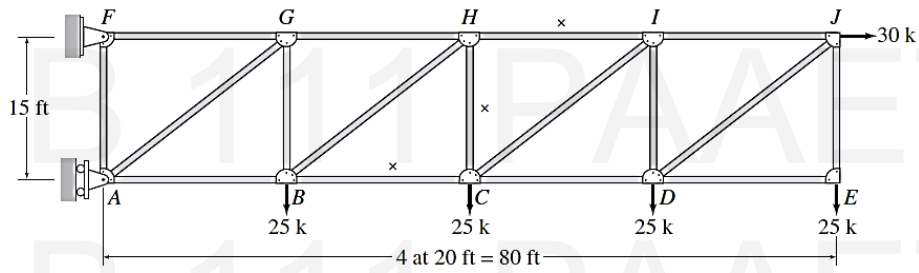


**Solution:**

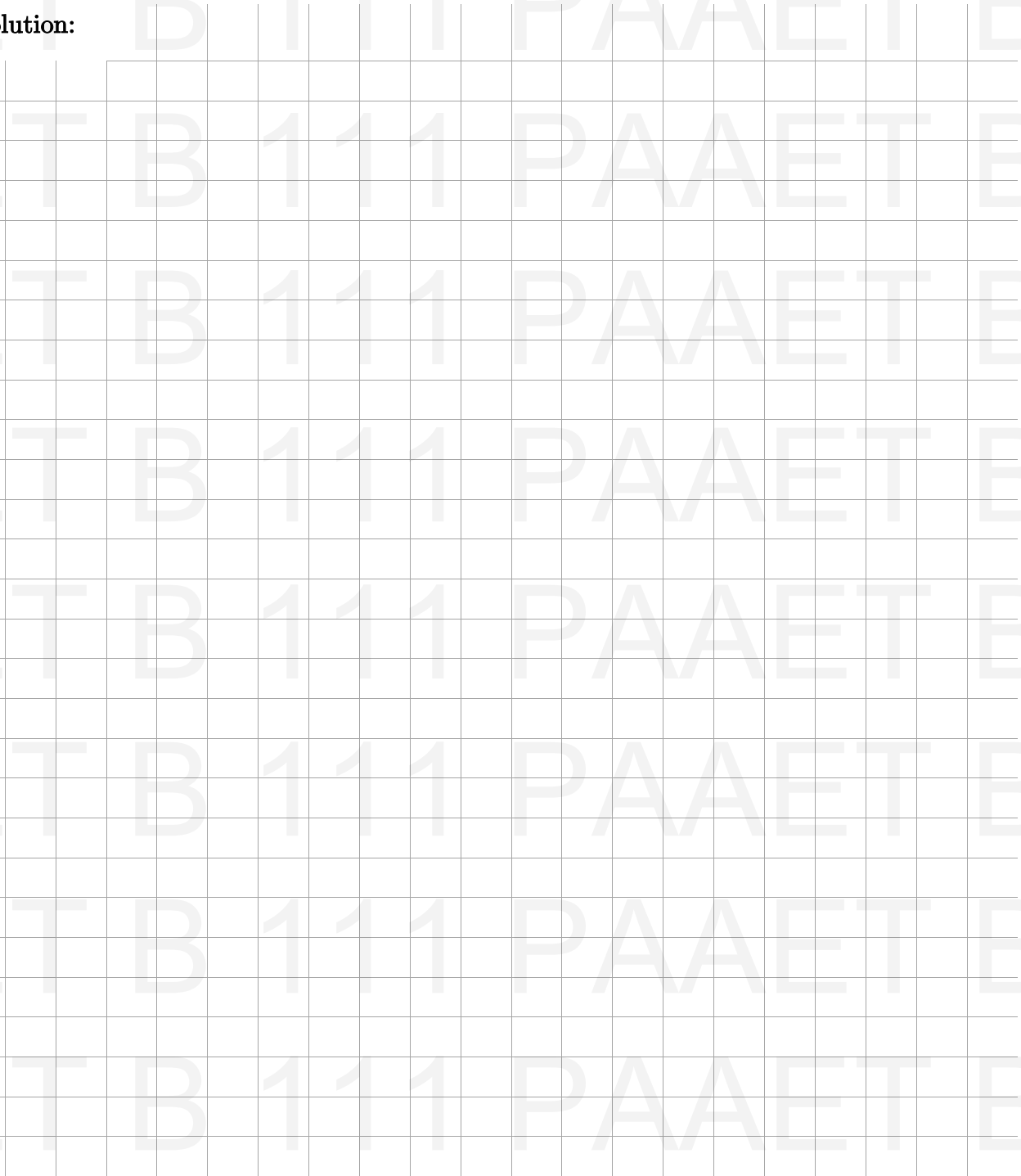
A large grid area provided for the student to show their solution to the truss problem.



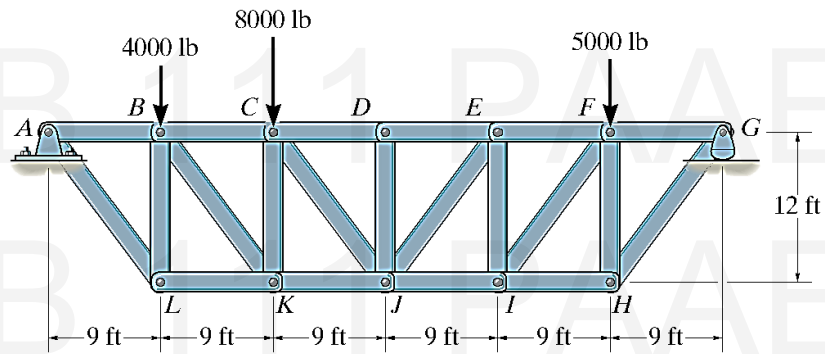
**Example (2):**



**Solution:**

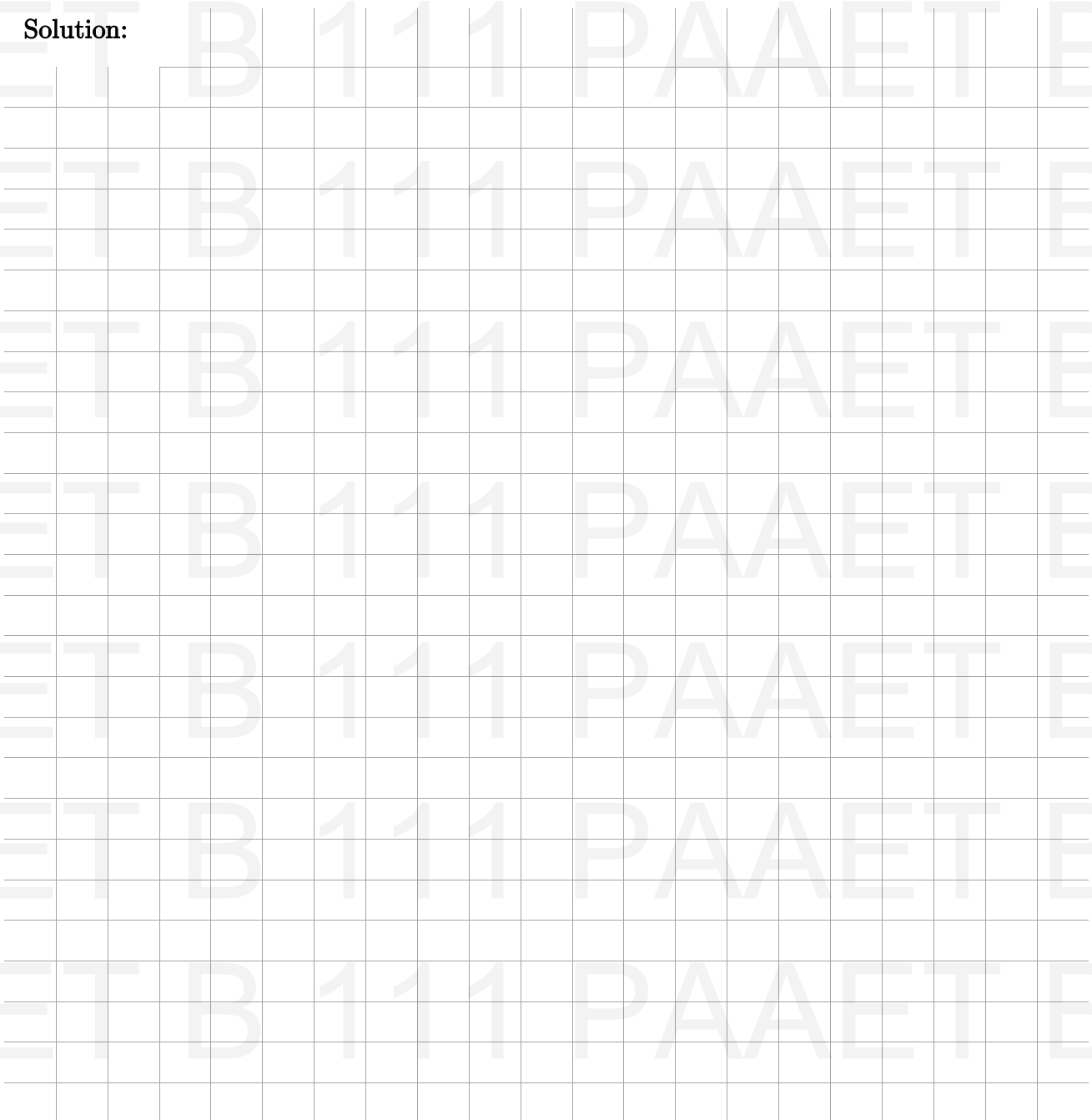


**Example (3):**

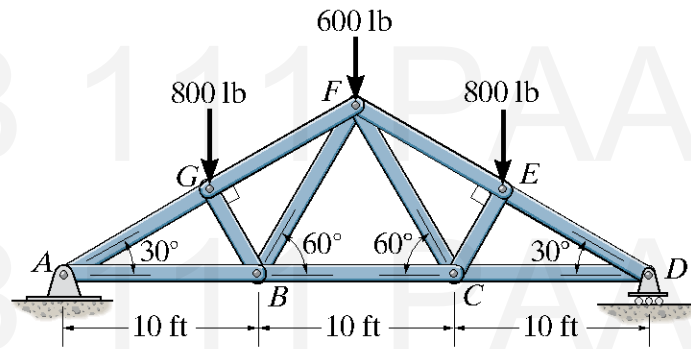


Members:  $EI, JI$

**Solution:**

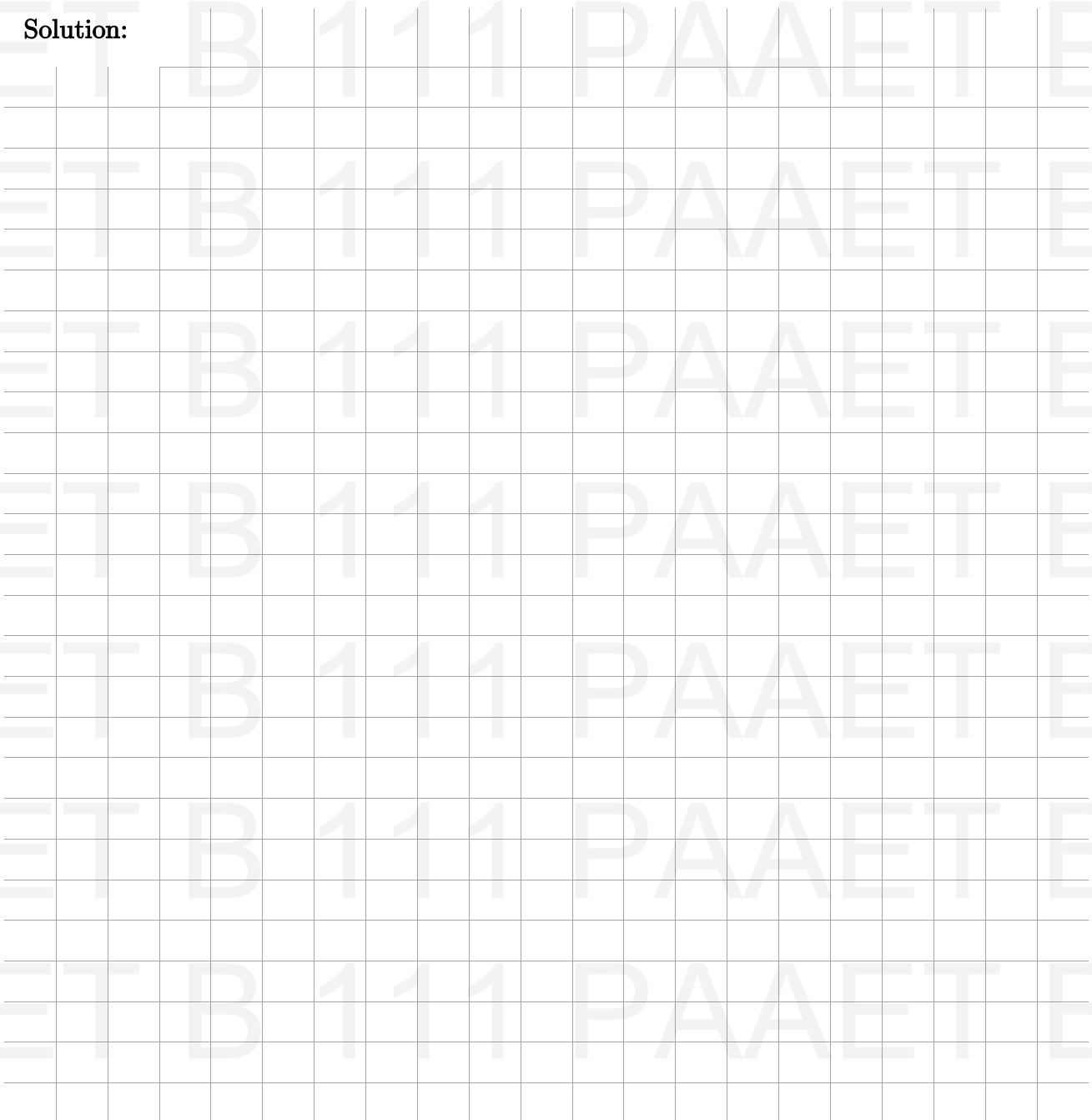


**Example (4):**



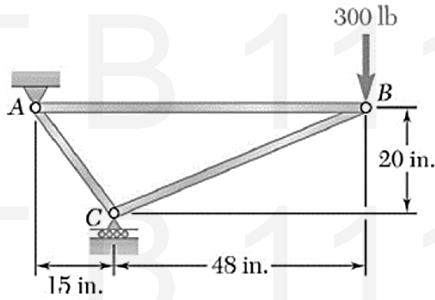
Members:  $FE, EC$

**Solution:**



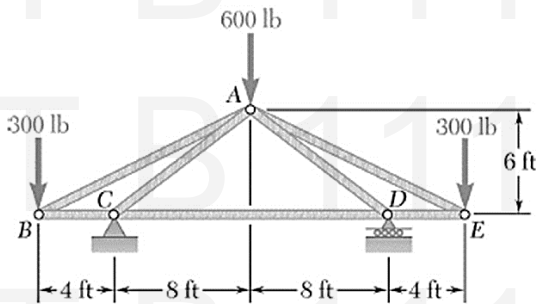
6.5 Problems:

Question (6.1)



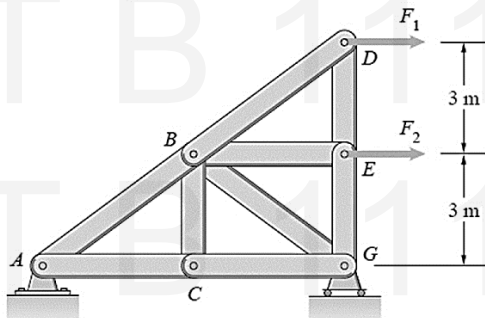
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

Question (6.2)



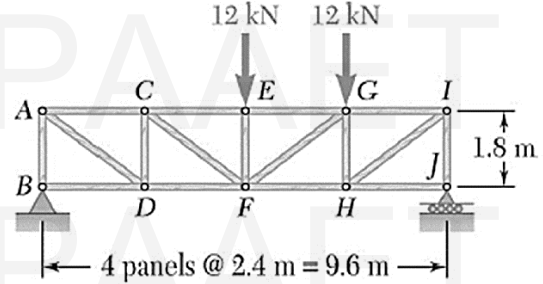
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

Question (6.3)



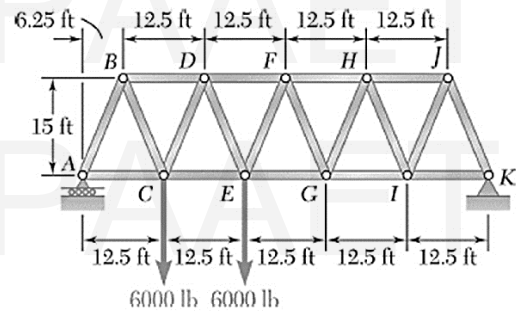
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression. Assume the loads  $F_1 = F_2 = 8 \text{ kN}$ .

Question (6.4)



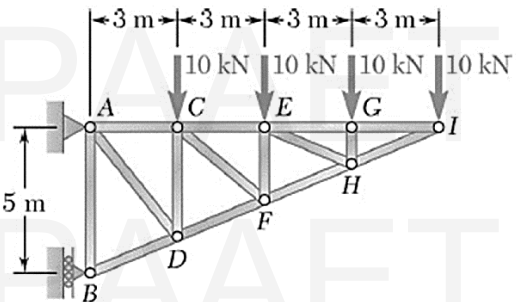
Using the method of sections, determine the force in members  $CD$  and  $DF$ . State whether each member is in tension or compression.

Question (6.5)



Using the method of sections, determine the force in members  $CE$ ,  $DE$  and  $DF$ . State whether each member is in tension or compression.

Question (6.6)



Using the method of sections, determine the force in members  $CD$  and  $DF$ . State whether each member is in tension or compression.

## Chapter (7): Geometric Centroids

### 7.1 Introduction:

- The centroid represents the geometric center of a body.
- This point coincides with the center of mass or the center of gravity only if the material composing the body is uniform or homogeneous.
- Finding the centroid of an area has many usages in engineering.
- The locations of centroids are usually tabulated in engineering references. (Figure 7-1)
- In the following section, we look at “Composite Shapes” and try to find their centroids.

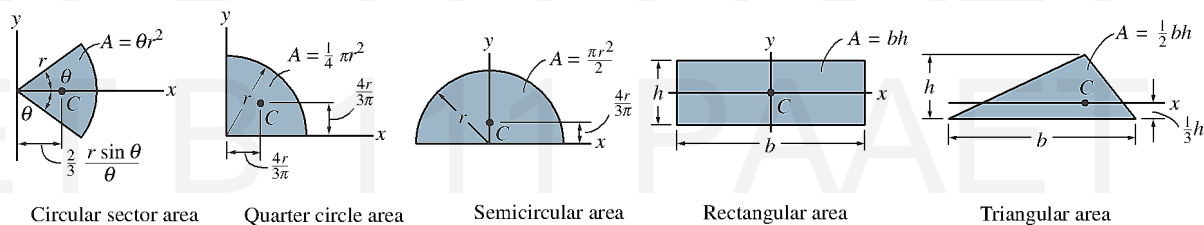


Figure 7-1: Centroidal Locations For Common Geometric Shapes

### 7.2 Composite Shapes:

A composite shape consists of a series of connected “simpler” shapes, which may be rectangular, triangular, semicircular, etc.

Such a shape can often be sectioned or divided into its composite parts and, provided the area and location of the center of gravity of each of these parts are known, the centroid for the entire composite shape can be found.

We apply the following

$$\bar{x} = \frac{\sum Ax}{\sum A}, \quad \bar{y} = \frac{\sum Ay}{\sum A} \quad (7-1)$$

Where:

$x$  : the distance from the local centroid of the “simple” shape to the  $y$ -axis ( $x$  moment arm)

$y$  : the distance from the local centroid of the “simple” shape to the  $x$ -axis ( $y$  moment arm)

$\bar{x}$  : is the  $x$ -coordinate of the centroid location

$\bar{y}$  : is the  $y$ -coordinate of the centroid location

$A$  : is the area of the “simple” shape

### 7.3 Procedure of Calculating Centroid Location:

The location of the center of the centroid of a composite geometrical object represented by an area can be determined using the following procedure.

- **Composite Parts.**
  - Using a sketch, divide the area into a finite number of composite parts that have simpler shapes.
  - If a composite shape has a hole, then consider the composite shape without the hole and consider the hole as an additional composite part having negative area.
- **Moment Arms**
  - Establish the coordinate axes on the sketch and determine the coordinates  $x$ ,  $y$  of the center centroid of each part.
- **Summations**
  - Determine  $\bar{x}$ ,  $\bar{y}$  by applying the centroid equations (7-1).
  - If an area is symmetrical about an axis, the centroid of the area lies on this axis.
  - If desired, the calculations can be arranged in tabular form.

7.4 Examples:

Example (1):

Locate the centroid of the plate area shown in Fig. 9-17a.

Solution:

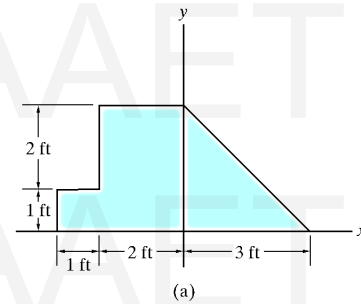


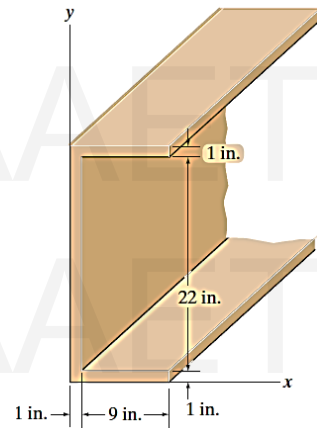
Fig. 9-17

A large grid area for the student to show their solution. The grid is composed of 20 columns and 20 rows of squares. The word 'SOLUTION:' is written at the top left of the grid area.

**Example (2):**

Locate the centroid  $(\bar{x}, \bar{y})$  of the cross-sectional area of the channel.

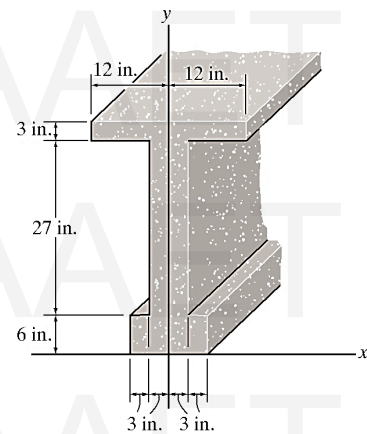
**Solution:**



**Example (3):**

\*9-52. Locate the centroid  $\bar{y}$  of the cross-sectional area of the concrete beam.

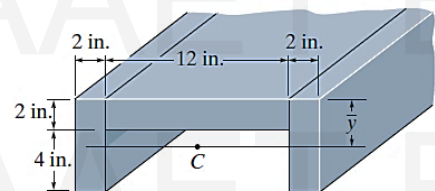
**Solution:**



**Example (4):**

9-54. Locate the centroid  $\bar{y}$  of the channel's cross-sectional area.

**Solution:**

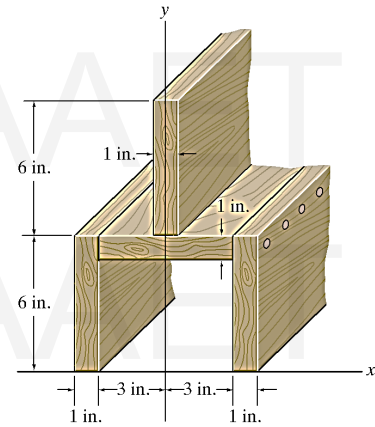




**Example (5):**

Locate the centroid  $\bar{y}$  of the cross-sectional area of the built-up beam.

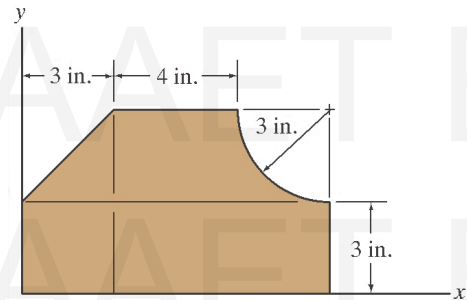
**Solution:**



**Example (6):**

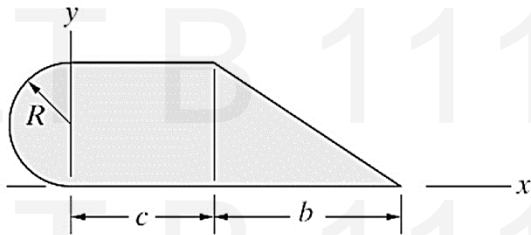
Locate the centroid  $(\bar{x}, \bar{y})$  of the composite area.

**Solution:**



7.5 Problems:

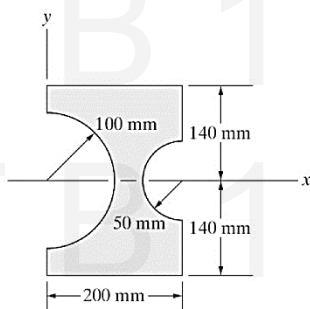
Question (7.1)



Let the dimensions  $R = 6$  in,  $c = 14$  in, and  $b = 18$  in. Determine the  $x$  coordinate of the centroid.

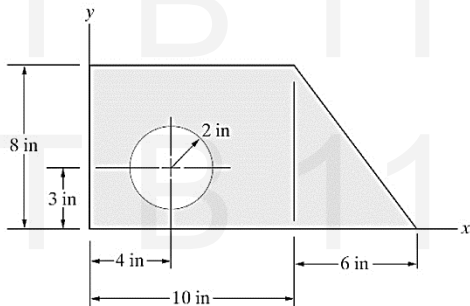
Bonus: Do problem (7.1) symbolically.

Question (7.2)



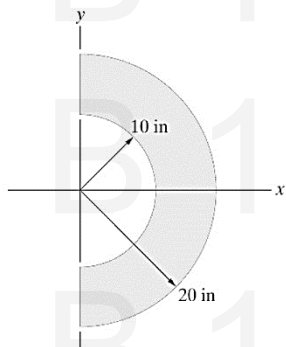
Determine the  $x$  coordinate of the centroid.

Question (7.3)



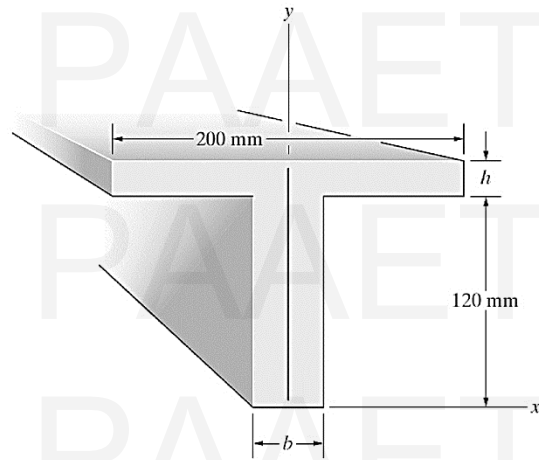
Determine the  $x$  and  $y$  coordinate of the centroid.

Question (7.4)



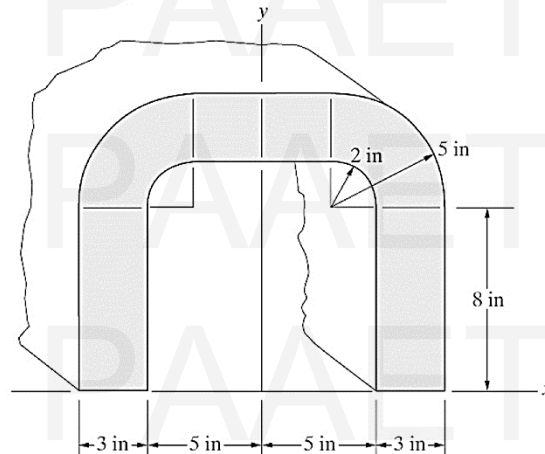
Determine the  $x$  and  $y$  coordinate of the centroid.

Question (7.5)



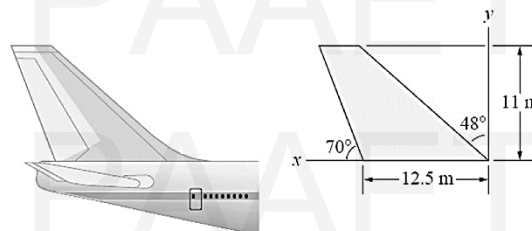
If the cross-sectional area of the beam is  $8400$   $\text{mm}^2$  and the  $y$  coordinate of the centroid of the area is ( $\bar{y} = 90$  mm) what are the dimensions  $b$  and  $h$ ?

Question (7.6)



Determine the  $x$  and  $y$  coordinate of the centroid of the beam's cross-section.

Question (7.7)

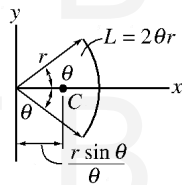


Determine the  $x$  and  $y$  coordinate of the airplane's vertical stabilizer.

## Chapter (8): Useful Formulas

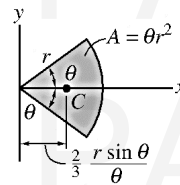
### 8.1 Geometric Properties of Line and Area Elements:

#### Centroid Location



Circular arc segment

#### Centroid Location

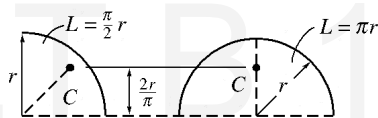


Circular sector area

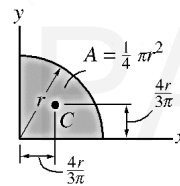
#### Area Moment of Inertia

$$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2} \sin 2\theta)$$

$$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2} \sin 2\theta)$$



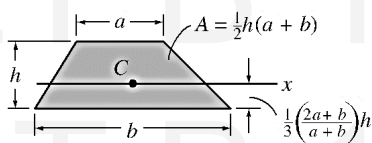
Quarter and semicircle arcs



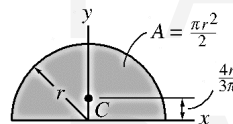
Quarter circle area

$$I_x = \frac{1}{16} \pi r^4$$

$$I_y = \frac{1}{16} \pi r^4$$



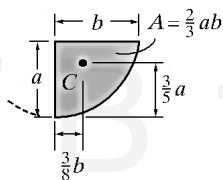
Trapezoidal area



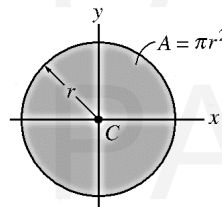
Semicircular area

$$I_x = \frac{1}{8} \pi r^4$$

$$I_y = \frac{1}{8} \pi r^4$$



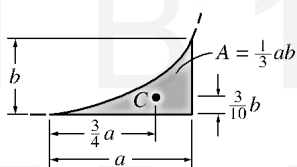
Semiparabolic area



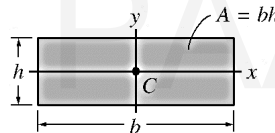
Circular area

$$I_x = \frac{1}{4} \pi r^4$$

$$I_y = \frac{1}{4} \pi r^4$$



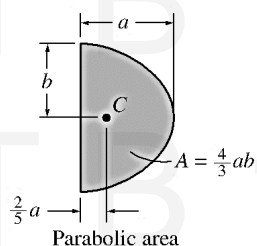
Exparabolic area



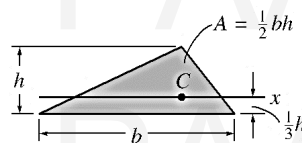
Rectangular area

$$I_x = \frac{1}{12} bh^3$$

$$I_y = \frac{1}{12} hb^3$$



Parabolic area



Triangular area

$$I_x = \frac{1}{36} bh^3$$

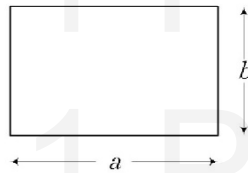
Square



Perimeter:  $P = 4a$

Area:  $A = a^2$

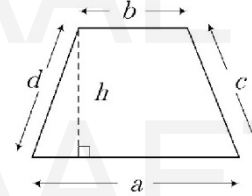
Rectangle



Perimeter:  $P = 2(a+b)$

Area:  $A = ab$

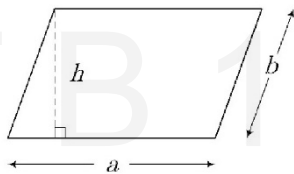
Trapezoid



Perimeter:  $P = a+b+c+d$

Area:  $A = \left(\frac{a+b}{2}\right)h$

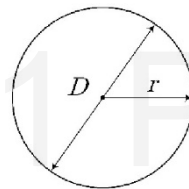
Parallelogram



Perimeter:  $P = 2(a+b)$

Area:  $A = ah$

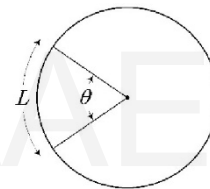
Circle



Perimeter:  $P = 2\pi r = \pi D$

Area:  $A = \pi r^2 = \frac{\pi D^2}{4}$

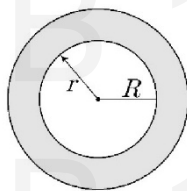
Circular Sector



Arch Length:  $L = \frac{\pi r \theta}{180^\circ}$

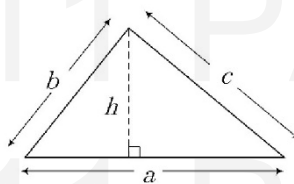
Sector Area:  $A = \frac{\pi r^2 \theta}{360^\circ}$

Circular Ring



Area:  $A = \pi(R^2 - r^2)$

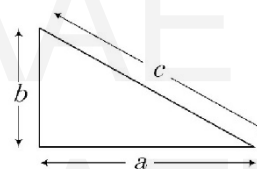
Triangle



Perimeter:  $P = a+b+c$

Area:  $A = \frac{ah}{2}$

Right Triangle

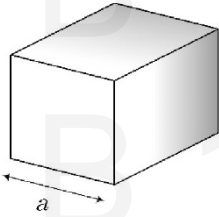


Perimeter:  $P = a+b+c$

Area:  $A = \frac{ah}{2}$

Pythagorean theorem:  
 $c^2 = a^2 + b^2$

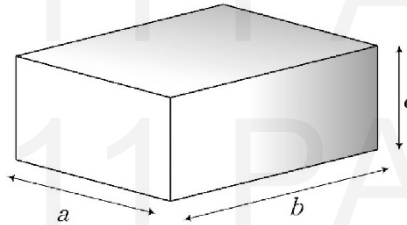
Cube



Surface Area:  $A = 6a^2$

Volume:  $V = a^3$

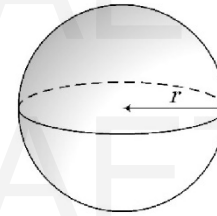
Rectangular Solid



Area:  $A = 2(ab + ac + bc)$

Volume:  $V = abc$

Sphere



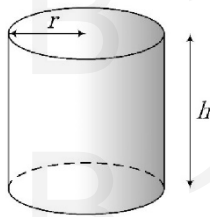
Surface Area:

$$A = 4\pi r^2$$

Volume:

$$V = \frac{4\pi r^3}{3}$$

Cylinder

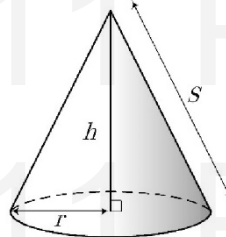


Surface Area:

$$A = 2\pi r(r + h)$$

Volume:  $V = \pi r^2 h$

Right Cone



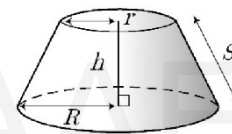
Surface Area:

$$A = \pi r(r + S)$$

$$S = \sqrt{r^2 + h^2}$$

Volume:  $V = \frac{\pi r^2 h}{3}$

Frustum of a Cone



Area:

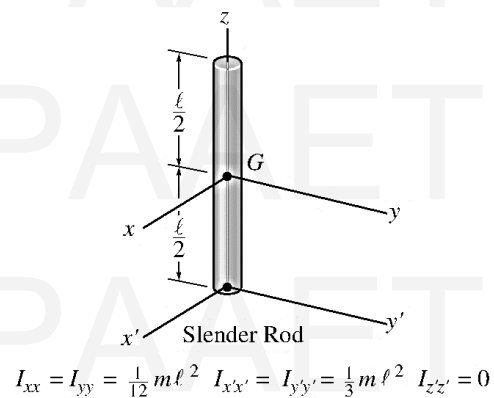
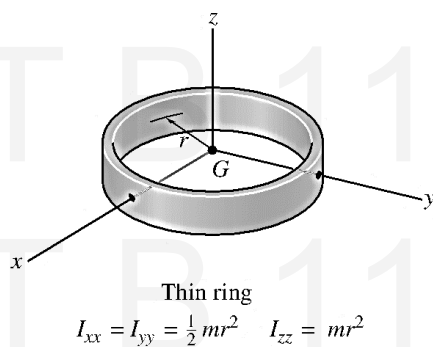
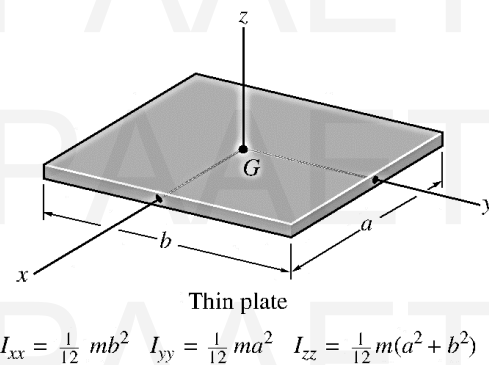
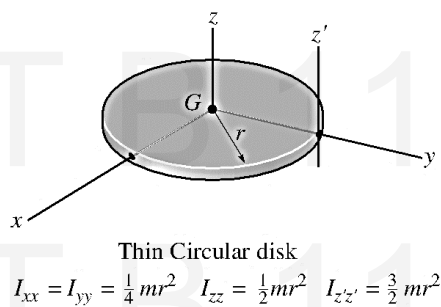
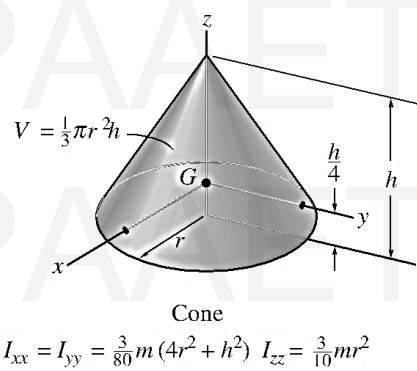
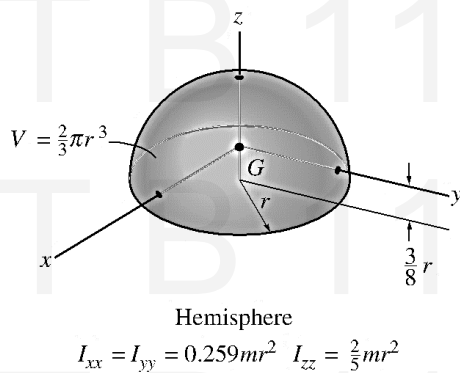
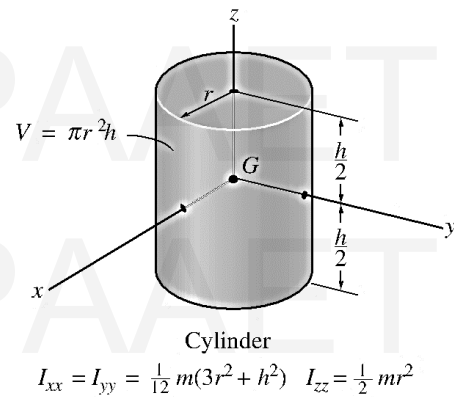
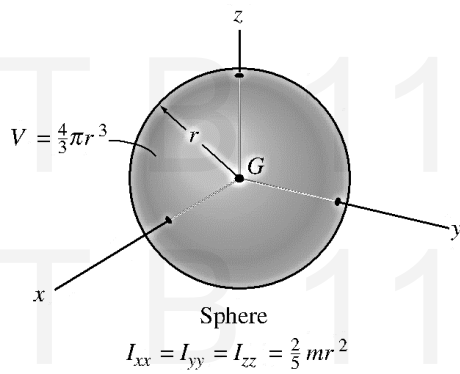
$$A = \pi \left[ Q(R - r) + (R^2 - r^2) + RS \right]$$

$$Q = \sqrt{r^2 + \left( \frac{Hr}{R - r} \right)^2}$$

$$S = \sqrt{(R - r)^2 + H^2}$$

Volume:  $V = \frac{\pi h}{3}(r^2 + rR + R^2)$

8.2 Center of Gravity and Mass Moment of Inertia of Homogenous Solids:



### 8.3 Fundamental Equations of Statics:

#### Cartesian Vector

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

#### Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

#### Directions

$$\begin{aligned} \mathbf{u}_A &= \frac{\mathbf{A}}{A} = \frac{A_x}{A} \mathbf{i} + \frac{A_y}{A} \mathbf{j} + \frac{A_z}{A} \mathbf{k} \\ &= \cos \alpha \mathbf{i} + \cos \beta \mathbf{j} + \cos \gamma \mathbf{k} \\ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= 1 \end{aligned}$$

#### Dot Product

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= AB \cos \theta \\ &= A_x B_x + A_y B_y + A_z B_z \end{aligned}$$

#### Cross Product

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

#### Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1) \mathbf{i} + (y_2 - y_1) \mathbf{j} + (z_2 - z_1) \mathbf{k}$$

#### Cartesian Force Vector

$$\mathbf{F} = F \mathbf{u} = F \left( \frac{\mathbf{r}}{r} \right)$$

#### Moment of a Force

$$\begin{aligned} M_o &= Fd \\ \mathbf{M}_o &= \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \end{aligned}$$

#### Moment of a Force About a Specified Axis

$$M_a = \mathbf{u} \cdot \mathbf{r} \times \mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

#### Simplification of a Force and Couple System

$$\begin{aligned} \mathbf{F}_R &= \Sigma \mathbf{F} \\ (\mathbf{M}_R)_O &= \Sigma \mathbf{M} + \Sigma \mathbf{M}_O \end{aligned}$$

#### Equilibrium

##### Particle

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$

##### Rigid Body-Two Dimensions

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0$$

##### Rigid Body-Three Dimensions

$$\begin{aligned} \Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0 \\ \Sigma M_{x'} = 0, \Sigma M_{y'} = 0, \Sigma M_{z'} = 0 \end{aligned}$$

#### Friction

Static (maximum)  $F_s = \mu_s N$

Kinetic  $F_k = \mu_k N$

#### Center of Gravity

##### Particles or Discrete Parts

$$\bar{r} = \frac{\Sigma \tilde{r} W}{\Sigma W}$$

##### Body

$$\bar{r} = \frac{\int \tilde{r} dW}{\int dW}$$

#### Area and Mass Moments of Inertia

$$I = \int r^2 dA \quad I = \int r^2 dm$$

##### Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \quad I = \bar{I} + md^2$$

##### Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \quad k = \sqrt{\frac{I}{m}}$$

#### Virtual Work

$$\delta U = 0$$

#### 8.4 SI Prefixes:

<i>Multiple</i>	<i>Exponential Form</i>	<i>Prefix</i>	<i>SI Symbol</i>
1 000 000 000	$10^9$	giga	G
1 000 000	$10^6$	mega	M
1 000	$10^3$	kilo	k
<i>Submultiple</i>			
0.001	$10^{-3}$	milli	m
0.000 001	$10^{-6}$	micro	$\mu$
0.000 000 001	$10^{-9}$	nano	n

#### 8.5 Conversion Factors (FPS) to (SI)

<i>Quantity</i>	<i>Unit of Measurement (FPS)</i>	<i>Equals</i>	<i>Unit of Measurement (SI)</i>
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

#### 8.6 Conversion Factors (FPS):

$$\begin{aligned}
 1 \text{ ft} &= 12 \text{ in. (inches)} \\
 1 \text{ mi. (mile)} &= 5280 \text{ ft} \\
 1 \text{ kip (kilopound)} &= 1000 \text{ lb} \\
 1 \text{ ton} &= 2000 \text{ lb}
 \end{aligned}$$



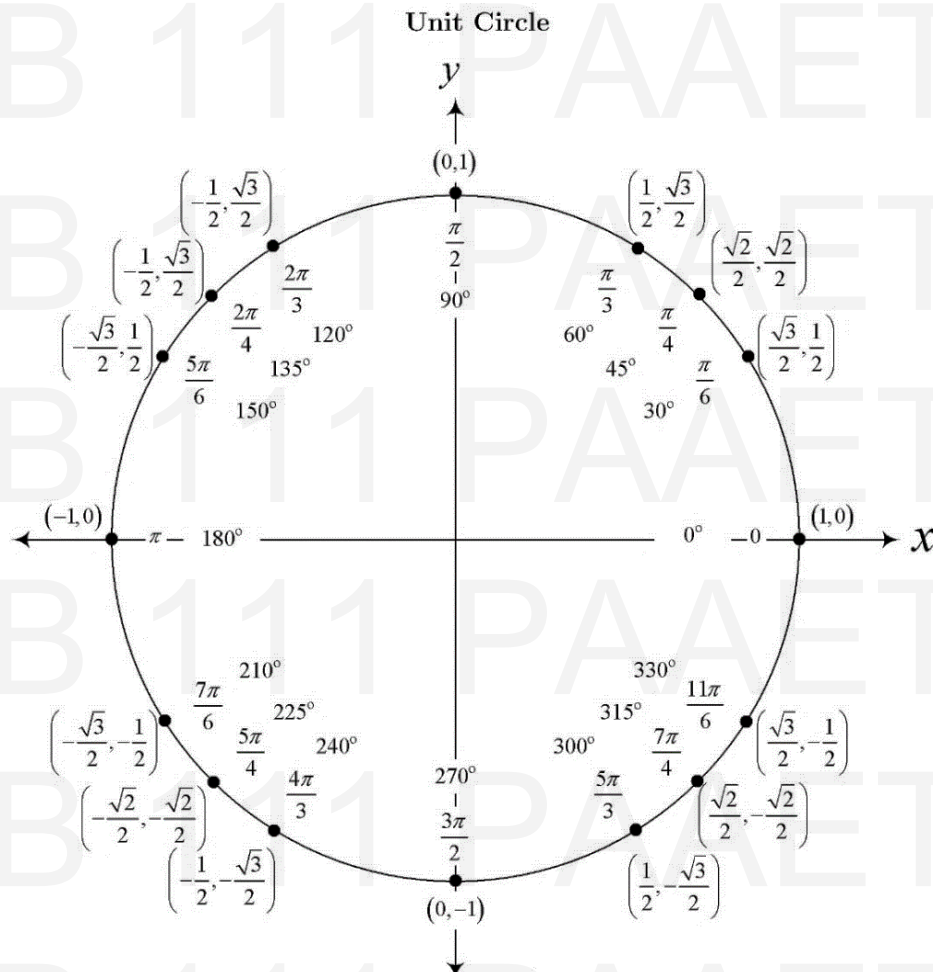
8.7 Conversion Factors Table:

Conversion Factors					
Multiply	By	To Obtain	Multiply	By	To Obtain
acre	43560	square feet (ft <sup>2</sup> )	joule (J)	9.478×10 <sup>-4</sup>	Btu
ampere-hr (A-hr)	3600	coulomb (C)	J	0.7376	ft-lbf
ångström (Å)	1×10 <sup>-10</sup>	meter (m)	J	1	newton-m (N·m)
atmosphere (atm)	76	cm, mercury (Hg)	J/s	1	watt (W)
atm, std	29.92 in	mercury (Hg)			
atm, std	14.7	lbf/in <sup>2</sup> abs (psia)	kilogram (kg)	2.205	pound (lbm)
atm, std	33.9	ft, water	kgf	9.8066	newton (N)
atm, std	1.013×10 <sup>5</sup>	pascal (Pa)	kilometer (km)	3281	feet (ft)
			km/hr	0.621	mph
bar	1×10 <sup>5</sup>	Pa	kilopascal (kPa)	0.145	lbf/in <sup>2</sup> (psi)
barrels-oil	42	gallons-oil	kilowatt (kW)	1.341	horsepower (hp)
Btu	1055	joule(J)	kW	3413	Btu/hr
Btu	2.928×10 <sup>-4</sup>	kilowatt-hr (kWh)	kW	737.6	(ft-lbf)/sec
Btu	778	ft-lbf	kW-hour (kWh)	3413	Btu
Btu/hr	3.930×10 <sup>-4</sup>	horsepower (hp)	kWh	1.341	hp-hr
Btu/hr	0.293	watt (W)	kWh	3.6×10 <sup>6</sup>	joule (J)
Btu/hr	0.216	ft-lbf/sec	kip (K)	1000	lbf
			K	4448	newton (N)
calorie (g-cal)	3.968×10 <sup>-3</sup>	Btu	liter (L)	61.02	in <sup>3</sup>
cal	1.560×10 <sup>-6</sup>	hp-hr	L	0.264	gal (US Liq)
cal	4.186	joule (J)	L	10×10 <sup>-3</sup>	m <sup>3</sup>
cal/sec	4.186	watt (W)	L/second (L/s)	2.119	ft <sup>3</sup> /min (cfm)
centimeter (cm)	3.281×10 <sup>-2</sup>	foot (ft)	L/s	15.85	gal (US)/min (gpm)
cm	0.394	inch (in)			
centipoise (cP)	0.001	pascal-sec (Pa·s)	meter (m)	3.281	feet (ft)
centistokes (cSt)	1×10 <sup>-6</sup>	m <sup>2</sup> /sec (m <sup>2</sup> /s)	m	1.094	yard
cubic feet/second (cfs)	0.646317	million gallons/day (mgd)	metric ton	1000	kilogram (kg)
cubic foot (ft <sup>3</sup> )	7.481	gallon	m/second (m/s)	196.8	feet/min (ft/min)
cubic meters (m <sup>3</sup> )	1000	Liters	mile (statute)	5280	feet (ft)
electronvolt (eV)	1.602×10 <sup>-19</sup>	joule (J)	mile (statute)	1.609	kilometer (km)
			mile/hour (mph)	88	ft/min (fpm)
foot (ft)	30.48	cm	mph	1.609	km/h
ft	0.3048	meter (m)	mm of Hg	1.316×10 <sup>-3</sup>	atm
ft-pound (ft-lbf)	1.285×10 <sup>-3</sup>	Btu	mm of H <sub>2</sub> O	9.678×10 <sup>-5</sup>	atm
ft-lbf	3.766×10 <sup>-7</sup>	kilowatt-hr (kWh)			
ft-lbf	0.324	calorie (g-cal)	newton (N)	0.225	lbf
ft-lbf	1.356	joule (J)	N·m	0.7376	ft-lbf
ft-lbf/sec	1.818×10 <sup>-3</sup>	horsepower (hp)	N·m	1	joule (J)
gallon (US Liq)	3.785	liter (L)	pascal (Pa)	9.869×10 <sup>-6</sup>	atmosphere (atm)
gallon (US Liq)	0.134	ft <sup>3</sup>	Pa	1	newton/m <sup>2</sup> (N/m <sup>2</sup> )
gallons of water	8.3453	pounds of water	Pa-sec (Pa·s)	10	poise (P)
gamma (γ, Γ)	1×10 <sup>-9</sup>	tesla (T)	pound (lbm,avdp)	0.454	kilogram (kg)
gauss	1×10 <sup>-4</sup>	T	lbf	4.448	N
gram (g)	2.205×10 <sup>-3</sup>	pound (lbm)	lbf-ft	1.356	N·m
			lbf/in <sup>2</sup> (psi)	0.068	atm
hectare	1×10 <sup>4</sup>	square meters (m <sup>2</sup> )	psi	2.307	ft of H <sub>2</sub> O
hectare	2.47104	acres	psi	2.036	in of Hg
horsepower (hp)	42.4	Btu/min	psi	6895	Pa
hp	745.7	watt(W)			
hp	33000	(ft-lbf)/min	radian	$\frac{180}{\pi}$	degree
hp	550	(ft-lbf)/sec			
hp-hr	2544	Btu	stokes	1×10 <sup>-4</sup>	m <sup>2</sup> /s
hp-hr	1.98×10 <sup>6</sup>	ft-lbf			
hp-hr	2.68×10 <sup>6</sup>	joule (J)	therm	1×10 <sup>5</sup>	Btu
hp-hr	0.746	kWh			
inch (in)	2.54	centimeter (cm)	watt (W)	3.413	Btu/hr
in of Hg	0.0334	atm	W	1.341×10 <sup>-3</sup>	horsepower (hp)
in of Hg	13.6	in of H <sub>2</sub> O	W	1	joule/sec (J/s)
in of H <sub>2</sub> O	0.0361	lbf/in <sup>2</sup> (psi)	weber/m <sup>2</sup> (Wb/m <sup>2</sup> )	10000	gauss
in of H <sub>2</sub> O	0.002458	atm			

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8.8 Cheat Sheet:

**Trigonometry:**



For any ordered pair on the unit circle  $(x,y)$  :  $\cos \theta = x$  and  $\sin \theta = y$

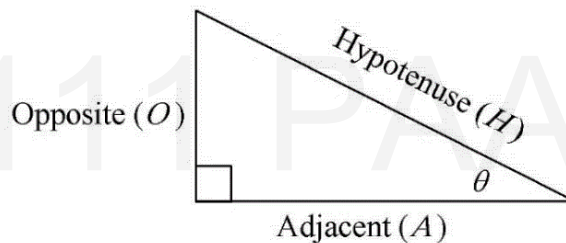
**Degrees to Radians Formulas**

If  $x$  is an angle in degrees and  $t$  is an angle in radians then

$$t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

**Right Triangle**

For this definition we assume that  $0 < \theta < \frac{\pi}{2}$  or  $0^\circ < \theta < 90^\circ$



$$\begin{aligned} \sin(\theta) &= \frac{O}{H} & \cos(\theta) &= \frac{A}{H} & \tan(\theta) &= \frac{O}{A} \\ \csc(\theta) &= \frac{H}{O} & \sec(\theta) &= \frac{H}{A} & \cot(\theta) &= \frac{A}{O} \end{aligned}$$

**Reciprocal Identities**

$$\begin{aligned} \sin(\theta) &= \frac{1}{\csc(\theta)} & \cos(\theta) &= \frac{1}{\sec(\theta)} & \tan(\theta) &= \frac{1}{\cot(\theta)} \\ \csc(\theta) &= \frac{1}{\sin(\theta)} & \sec(\theta) &= \frac{1}{\cos(\theta)} & \cot(\theta) &= \frac{1}{\tan(\theta)} \end{aligned}$$

**Pythagorean Identities**

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad \cot^2(\theta) + 1 = \csc^2(\theta)$$

**Even/Odd Formulas**

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) & \csc(-\theta) &= -\csc(\theta) \\ \cos(-\theta) &= \cos(\theta) & \sec(-\theta) &= \sec(\theta) \\ \tan(-\theta) &= -\tan(\theta) & \cot(-\theta) &= -\cot(\theta) \end{aligned}$$

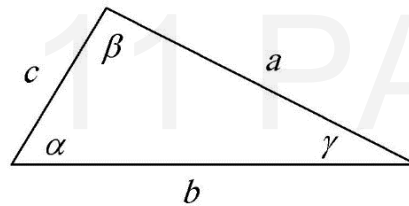
**Inverse Trig Functions**

$$y = \sin^{-1}(x) \text{ is equivalent to } x = \sin(y)$$

$$y = \cos^{-1}(x) \text{ is equivalent to } x = \cos(y)$$

$$y = \tan^{-1}(x) \text{ is equivalent to } x = \tan(y)$$

**Law of Sines, Cosines and Tangents**



**Law of Sines**

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

**Law of Cosines**

$$a^2 = b^2 + c^2 - 2ac \cos(\alpha) \quad b^2 = a^2 + c^2 - 2bc \cos(\beta) \quad c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

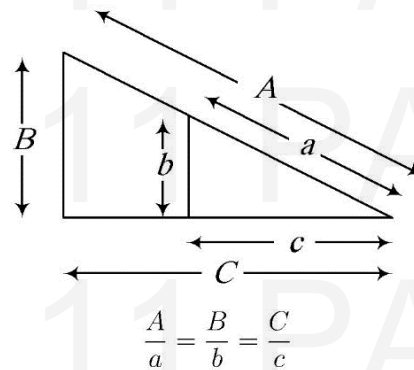
**Law of Tangents**

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha-\beta)}{\tan \frac{1}{2}(\alpha+\beta)} \quad \frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta-\gamma)}{\tan \frac{1}{2}(\beta+\gamma)} \quad \frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha-\gamma)}{\tan \frac{1}{2}(\alpha+\gamma)}$$

**Mollweide's Formula**

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

**Similar Triangles**



**Algebra:**

**Arithmetic Operations**

$$ab + ac = a(b + c) \quad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\left(\frac{a}{b}\right) = \frac{a}{bc} \quad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, \quad a \neq 0 \quad \left(\frac{\frac{a}{b}}{\frac{c}{d}}\right) = \frac{ad}{bc}$$

**Exponent Properties**

$$a^n a^m = a^{n+m} \quad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$(a^n)^m = a^{nm} \quad a^0 = 1, \quad a \neq 0$$

$$(ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{a^n}{b^n} \quad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$$

**Properties of Radicals**

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{\sqrt[n]{a}} = a, \quad \text{if } n \text{ is odd} \quad \sqrt[n]{\sqrt[n]{a}} = |a|, \quad \text{if } n \text{ is even}$$

**Distance Formula**

If  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Logarithms and Log Properties****Definition**

$y = \log_b x$  is equivalent to  $x = b^y$

**Special Logarithms**

$\ln x = \log_e x$  natural log

$\log x = \log_{10} x$  common log where  $e = 2.718281828 \dots$

**Logarithm Properties**

$$\begin{array}{llll} \log_b b = 1 & \log_b 1 = 0 & \log_b b^x = x & b^{\log_b x} = x \\ \log_b (x^r) = r \log_b x & \log_b (xy) = \log_b x + \log_b y & \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y & \end{array}$$

The domain of  $\log_b x$  is  $x > 0$

## Answers to Problems:

### Chapter (2)

- 2.1  $R = 3.30 \text{ kN}$ ,  $\alpha = 66.6^\circ$   
 2.2  $P = 101.4 \text{ N}$ ,  $R = 196.6 \text{ N}$   
 2.3  $F_x = 61.3 \text{ N}$ ,  $F_y = 51.4 \text{ N}$   
 $F_x = 41.0 \text{ N}$ ,  $F_y = 112.8 \text{ N}$   
 $F_x = -122.9 \text{ N}$ ,  $F_y = 86.0 \text{ N}$   
 2.4  $F_x = 640 \text{ N}$ ,  $F_y = 480 \text{ N}$   
 $F_x = -224 \text{ N}$ ,  $F_y = -360 \text{ N}$   
 $F_x = 192 \text{ N}$ ,  $F_y = -360 \text{ N}$   
 2.5  $R = 54.9 \text{ lb}$ ,  $\alpha = 48.9^\circ$   
 2.6  $R = 202 \text{ lb}$ ,  $\alpha = 33.2^\circ$

### Chapter (3)

- 3.1  $T_{AC} = 6.37 \text{ kN}$ ,  $T_{BC} = 12.47 \text{ kN}$   
 3.2  $T_{AC} = 1244 \text{ lb}$ ,  $T_{BC} = 115.4 \text{ lb}$   
 3.3 (a)  $x_{AC} = 0.739 \text{ m}$ ,  $x_{AB} = 0.467 \text{ m}$   
 (b)  $m = 8.56 \text{ kg}$   
 3.4  $F_1 = 1.83 \text{ kN}$ ,  $F_2 = 9.60 \text{ kN}$   
 3.5  $T_{AC} = 312 \text{ N}$ ,  $T_{BC} = 144 \text{ N}$   
 3.6  $P = 1249 \text{ N}$ ,  $\alpha = 62.5^\circ$

### Chapter (4)

- 4.1 (a)  $M_B = -115.7 \text{ lb-in}$   
 (b)  $\alpha = 23.2^\circ$   
 4.2  $M_B = -361.77 \text{ lb-in}$   
 4.3  $M_S = -611 \text{ lb-in}$   
 4.4 (a)  $M_1 = 336 \text{ lb-in}$   
 (b)  $d_1 = 28 \text{ in}$   
 (c)  $\alpha = 54.05^\circ$   
 4.5  $M_O = -5600 \text{ lb-ft}$   
 4.6  $F = 167 \text{ lb}$

### Chapter (5)

- 5.1  $N_A = 0 \text{ N}$ ,  $V_A = 100 \text{ N}$ ,  $M_A = 40 \text{ N-m}$   
 5.2  $N_A = 0 \text{ N}$ ,  $V_A = 400 \text{ lb}$ ,  $M_A = -1900 \text{ lb-ft}$   
 5.3  $N_A = 1.88 \text{ kN}$ ,  $V_A = -2.68 \text{ kN}$ ,  $M_A = 0.56 \text{ kN-m}$   
 5.4  $N_A = 0 \text{ N}$ ,  $V_A = -125 \text{ N}$ ,  $M_A = 53.1 \text{ N-m}$   
 5.5  $N_A = 0 \text{ N}$ ,  $V_A = 84 \text{ lb}$ ,  $M_A = 522 \text{ lb-ft}$   
 5.6  $N_A = 0 \text{ N}$ ,  $V_A = 24 \text{ lb}$ ,  $M_A = 522 \text{ lb-ft}$   
 5.7  $N_A = 0 \text{ N}$ ,  $V_A = -592 \text{ lb}$ ,  $M_A = 950 \text{ lb-ft}$   
 5.8  $V = 0$ ,  $M = \frac{wl^2}{8}$

### Chapter (6)

- 6.1  $F_{AB} = 720 \text{ lb}(T)$ ,  $F_{BC} = -780 \text{ lb}(C)$ ,  
 $F_{AC} = -1200 \text{ lb}(C)$   
 6.2  $F_{AB} = F_{AE} = 671 \text{ lb}(T)$ ,  $F_{BC} = F_{DE} = -600 \text{ lb}(C)$ ,  
 $F_{AC} = F_{AD} = -1000 \text{ lb}(C)$ ,  $F_{CD} = 200 \text{ lb}(T)$   
 6.3  $F_{BD} = 10 \text{ kN}(T)$ ,  $F_{BE} = 8 \text{ kN}(T)$ ,  $F_{BG} = -5 \text{ kN}(C)$   
 6.4  $F_{CD} = -9 \text{ kN}(C)$ ,  $F_{DF} = 12 \text{ kN}(T)$   
 6.5  $F_{CE} = 8000 \text{ lb}(T)$ ,  $F_{DE} = 2600 \text{ lb}(T)$ ,  
 $F_{DF} = -9000 \text{ lb}(C)$   
 6.6  $F_{CD} = -20 \text{ kN}(C)$ ,  $F_{DF} = -52 \text{ kN}(C)$

### Chapter (7)

- 7.1  $\bar{X} = 9.60 \text{ in}$   
 7.2  $\bar{X} = 116 \text{ mm}$   
 7.3  $\bar{X} = 6.97 \text{ in}$ ,  $\bar{Y} = 3.79 \text{ in}$   
 7.4  $\bar{X} = 9.9 \text{ in}$ ,  $\bar{Y} = 0 \text{ in}$   
 7.5  $h = 18.2 \text{ mm}$ ,  $b = 39.7 \text{ mm}$   
 7.6  $\bar{X} = 0 \text{ in}$ ,  $\bar{Y} = 7.48 \text{ in}$   
 7.7  $\bar{X} = 9.64 \text{ in}$ ,  $\bar{Y} = 4.60 \text{ in}$

## Glossary

Here is a simple glossary of some of the most used terminology in statics and structural analysis courses.

### A

Abrupt	مفاجئ
Absolute	مطلق
Absolute Value	القيمة المطلقة
Absolute system of units	نظام الوحدات المطلقة
Acceleration	تسارع
Accuracy	دقة
Accurate	دقيق
Action	عمل / فعل
Active force	القوة الفعالة / القوة النشطة
Actual	فعلي
Addition	إضافة / جمع
Addition of forces	جمع القوى
Addition of vectors	جمع المتجهات
Adjacent vectors	المتجهات المجاورة
Advantage	أفضلية
Aerostatics	الإيروساتيكس / علم توازن الهواء و الغازات
Algebra	علم الجبر
Algebraic	جبري
Algebraic expression	تعبير جبري
Algebraic sum	جمع جبري
Allow	يسمح
Analysis	تحليل
Analytical	تحليلية
Analyze	تحليل
Anchor bolts	مرساة البراغي
Anemometers	أنيموميتر / جهاز قياس شدة الريح

Angle	زاوية
Angular	زاوي / ذو علاقة بالزاوية
Answer	إجابة
Apex	ذروة
Application	تطبيق
Applied force	القوة المطبقة
Approximate	تقريبي
Arbitrary shapes	الأشكال العشوائية
Arches	أقواس
Area	مساحة
Area moments of inertia	عزم المساحة (عزم القصور الذاتي)
Area of cross-section	مساحة المقطع العرضي
Arm	ذراع
Arrow	سهم
Associative	ترابطي
Associative addition	الجمع الترابطي
Associative property	الخاصية الترابطية
Assume	افتراض
Assumption	افتراض
Atmospheric pressure	الضغط الجوي
Available	متاح
Average	معدل
Axes	محاور
Axial	محوري
<b>B</b>	
Balanced	متوازن
Bar	قضيب (معدني)
Barrel arches	أقواس ذات مقطع علوي شبه اسطواني
Base	قاعدة



Beam	كمره
Beam cross section	المقطع العرضي للكمرة
Cantilever beam	كمرة معلقة (كابولي)
Deep beam	كمرة عميقة
Overhanging beam	كمرة المتدلية
Simply supported beam	كمرة بسيطة
Bearing	تحمل / ضغط
Bearing friction	تحمل / ضغط الاحتكاك
Bearing stress	إجهاد التحمل / الضغط
Behavior	سلوك
Belt	حزام
Belt friction	حزام الاحتكاك
Belts and pulleys	أحزمة و بكرات
Bending	تقوس
Bending moment	عزم الانحناء
Bending moment diagram	الرسم البياني لعزم الانحناء
Bending rigidity	صلابة الانحناء
Bending stress	إجهاد الانحناء
Bernoulli's principle of virtual displacements	مبدأ برنولي للإزاحة الافتراضية
Body	الجسم
Body force	قوة الجسم
Body rotation	دوران الجسم
Bond	رابطة
Boundary	حدود
Boundary conditions	شروط / حالات الحدود
Braced frame	إطار غير قابل للتمايل (مثبت)
Bracing	تثبيت
Bridge	جسر
British system of units	النظام البريطاني للوحدات
Brittle	هش

Buckling	التواء
Buckling load	حمل الالتواء
Buckling moment	عزم الالتواء
Building	بناء
Building code	قانون البناء
Building materials	مواد بناء
Buoyancy	الطفو

## C

Cables	الكابلات
Calculus	حساب التفاضل والتكامل
Cambered beam	الكمرة المقوسة تصميمياً
Cantilever	ناتئ / بارز / (كابولي)
Capstan	رحوية
Cartesian	ديكارتي
Cartesian components	المكونات الديكارتية
Cartesian coordinates	الإحداثيات الديكارتية
Catenary	سلسال
Center	مركز
Center line	خط الوسط
Center of mass	مركز الكتلة
Center of pressure	مركز الضغط
Center of gravity	مركز الجاذبية
Centroid	مركز المساحة / الجسم
Centroidal axes	محور مركز المساحة / الجسم
Chord	وتر
Circle of friction	دائرة الاحتكاك
Circular	دائري
Circular area	مساحة دائرية
Circular sector	قطاع دائري

Circumference	محيط
Civil engineers	مهندس مدني
Clamps	مشابك
Classification	تصنيف
Clockwise	عقارب الساعة
Coefficient	مُعامل
Coefficient of friction	معامل الاحتكاك
Coincide	يتزامن
Collapse	انهدام
Collinear	على خط واحد
Column	عامود
Common	مشترك
Commutative property	خاصية التبديل
Compatible	متوافق
Complementary	مكمل
Component	مكون
Composite	مركب
Compound	مركب
Compound beam	كمر مركبة
Compound truss	جمالون مركب
Compression	ضغط
Computation	حساب
Computer analysis	تحليل باستخدام الحاسوب
Concave	مقعر
Concentrated	مركز
Concentrated force	قوة مركزة
Concentrated load	حمل مركز
Conceptual design	التصميم النظري
Concrete	الخرسانة
Concrete bridges	جسور خرسانية

Reinforced concrete	خرسانة مسلحة
Concurrent	بنفس الوقت
Concurrent force system	نظام القوة المتزامنة
Condition	شرط / حالة
Cone of friction	مخروط الاحتكاك
Conservative	متحفّظ
Conservation of energy states	حفظ حالات الطاقة
Conservative system	أنظمة متحفّظة
Constant	ثابت
Constant of gravitation	ثابت الجاذبية
Constrained	مقيدة
Constraint	قيود
Construction	أعمال بناء
Contact	اتصل / اتصال
Continuity	استمرارية
Continuous	مستمر
Convention	عُرف
Conversion	تحويلات
Convex	محدب
Coordinates	إحداثيات
Coordinate systems	نظم الإحداثيات
Coordinate transformation	تحول الإحداثيات
Coplanar	في نفس المسطح
Copper	نحاس
Corner	ركن
Corresponding	المقابلة
Corrosion	تآكل
Cosines	جيب التمام (cos)
Coulomb theory of friction	نظرية كولومب للاحتكاك
Counterclockwise	عكس عقارب الساعة

Couple	زوجان / مزدوج
Cover	غطاء
Crack	شرخ
Create	يُحدِث
Creep	زحف
Critical	حرج
Cross	عكس / ضرب
Cross bracing	تثبيت متعاكس
Cross or vector product	حاصل الضرب المتجهي
Crush	سحق
Curvature	انحناء
Curve	منحنى
Customary units (U.S.)	الوحدات الأمريكية المتعارف عليها
Cutout	تم استقطاعه / جزء مقطوع من كل
Cylinder	اسطوانة
<b>D</b>	
Dam	سد
Dampers	مخمّدات / لامتصاص الطاقة
Dead load	الحمل الميت
Debris impact load	حمولة تأثير الحطام
Deck truss	جمالون لحمل الأسطح
Deep	عميق
Definition	تعريف
Deflection	انحراف / هبوط
Deform	تشوه / تغير بالشكل
Deformable body	جسم مشوه
Deformation	تشويه
Degree	درجة
Degree of freedom (DOF)	درجة الحرية

Degree of redundancy	درجة التكرار
Degree of Statical indeterminacy	درجة عدم الثبات الاستاتيكي
Density	كثافة
Dependent	يعتمد على
Depth	عمق
Derivative	المشتقة
Derived units	الوحدات المشتقة
Design	تصميم
Determinacy	الاحتمية
Determinate	مُحدد / (استاتيكيًا)
Deviation	الانحراف
Diagonal	قطري
Diagram	رسم بياني / رسم
Diameter	قطر الدائرة
Deferential	تفاضلي
Differential element	عنصر التفاضلية
Differential equation	المعادلة التفاضلية
Dimension	بُعد
Dimensionless	عديم أبعاد
Direct	مباشرة
Direction	اتجاه
Disk friction	احتكاك القرص
Displacement	الإزاح
Distorted sketch	رسم مشوهة
Distribute	يوزع
Distributed loads	الأحمال الموزعة
Distribution	توزيع
Distribution factor (DF)	عامل التوزيع
Distributive laws	قوانين التوزيع
Distributive property	خاصية التوزيع

Divide	يقسم
Dot products	ضرب المتجهات عددياً
Double	مزدوج
Double integration	التكامل المزدوج
Draw	رسم
Dry friction	الاحتكاك الجاف
Ductile	قابل للسحب
Dummy load	حمل وهمي
Durable	متين
Dynamic	ديناميكي

## E

Earthquake	زلزال
Eccentric	غير محوري
Edge	حافة
Effect	تأثير
Effective	فعال
Efficiency	كفاءة
Elastic	مرن
Electromagnetic forces	القوى الكهرومغناطيسية
Element	جزء
Elevations	الارتفاعات
Elongation	استطالة
Empirical formula	الصيغة التجريبية
Energy	طاقة
Engineering	هندسة
Engineering mechanics	الميكانيكا الهندسية
Equation	معادلة
Equilibrium	اتزان
Equilibrium equations	معادلات الاتزان

Equilibrium position	موضع الاتزان
Equivalent	مكافئ
Equivalent systems of forces	أنظمة مكافئة للقوى
Errors in computation	أخطاء في الحساب
Evaluation of design	تقييم التصميم
Exceed	يتجاوز
Expansion	توسيع
External	خارجي
<b>F</b>	
Fabrication errors	أخطاء التصنيع
Factor	عامل
Factor of safety	عامل السلامة
Gust factor	عامل العاصفة
Impact factor	عامل التأثير
Reduction factor	عامل التخفيض
Failure	فشل
Feet (ft)	قدم (وحدة قياس)
Fibers	ألياف
Finite	محدود
Fink trusses	جمالون (نوع Fink)
First moment of area	العزم الأول للمساحة
First-order analysis	تحليل من الدرجة الأولى
Fixed	ثابت
Flat roofs	أسطح مستوية
Flexibility	المرونة
Flexible cables	الكابلات المرنة
Flexural stiffness	صلابة الانحناء
Flood loads	أحمال الفيضانات
Floor systems	أنظمة الأرضيات



Fluids	الموائع
Footing	قاعدة / أساس
Force	قوة القصور الذاتي
Formula	معادلة
Formulation of problems	صياغة المشاكل
Foundation	أساس
Frame	الإطار
Free	حر
Free-body diagrams (FBD)	مخططات الجسم الحر
Friction	احتكاك
Frictionless	عديم الاحتكاك
Function	دالة / تطبيق رياضي
<b>G</b>	
Gage pressure	سعة الضغط
Gaps	ثغرات
Gas	غاز
General	عام / (عكس خاص)
General loading	تحميل عام
Geometrically unstable structure	هيكل غير مستقر هندسيا
Girder	عارضة / كمره رئيسية عرضية
Global coordinate system	نظام الإحداثيات العالمية
Graphical	بياني / رسومي
Graphical representation	تمثيل رسومي
Graphical solutions	حلول رسومية
Gravitation	الجاذبية الأرضية
Gravitational potential energy	طاقة الجاذبية الكامنة
Gravity	الجاذبية
Gyration	دوران / التفاف

**H**

Hard	صعب / صلب
Height	ارتفاع
High	مرتفع
High-strength steel wires	أسلاك الفولاذ عالية القوة
Highway bridges	جسور الطريق السريع
Hinge	مفصل
Hollow	أجوف
Homogenous	متجانس
Hooke's law	قانون هوك
Horizontal	أفقي
Horsepower (hp)	حصان (وحدة قياس)
Hour (h)	ساعة (وحدة قياس)
Howe truss	جمالون (نوع Howe)
Hydraulics	علم السوائل المتحركة / هيدروليكا
Hydrostatics	علم الهيدروستاتيكا
Hydrostatic loads	الأحمال الهيدروستاتيكية
Hydrostatic pressure	الضغط الهيدروليكي

## I

I-beams	كمرة بمقطع حرف ال I
Idealizing structures	هياكل مثالية
Identical	مطابق
Imaginary	خيالي
Impact factor	عامل التأثير
Impeding	وشيك
Impending motion	حركة وشيكة
Impending slip	انزلاق وشيك
Improper	غير سليم / غير لائق
Improper constraints	القيود غير السليمة
Improper supports	الدعامات غير السليمة

Inclined	مائل
Indeterminate	غير محدد
Inelastic behavior	سلوك غير مرن
Inertia force	قوة القصور الذاتي
Infinity	ملا نهاية
Inflection	التواء / انثناء / تغيير مسار
Influence area	منطقة التأثير
In-plane	في نفس المسطح الهندسي
Integration	تكامل رقمي
Intermediate	متوسط
Internal	داخلي
International	دولي
International Code Council	مجلس القانون الدولي
International System of units (SI units)	النظام الدولي للوحدات
Isolate	يعزل
<b>J</b>	
Joint	فصالة / مفصل
Joule	جول (وحدة طاقة)
<b>K</b>	
K truss	جمالون (نوع K)
Kilo-	كيلو (1000 وحدة)
Kilogram (kg)	كيلوغرام (وحدة قياس)
Kilometer (km)	الكيلومتر (وحدة قياس)
Kilonewtons (kN)	كيلونيوتونات (وحدة قياس)
Kilopound (kip)	كيلوباوند (وحدة قياس)
Kinetic energy	الطاقة الحركية
<b>L</b>	
Lateral bracing	ربط / تدعيم جانبي

Law	قانون
Law of cosines	قانون (cosines)
Law of sines	قانون (sines)
Laws of motion	قوانين الحركة
Length	الطول
Level	مستوى
Limit	حد
Line	خط
Line of action	خط العمل / خط تأثير القوى
Linear	خطي
Link	حلقة وصل
Liquids	السوائل
Loads	أحمال
Dead loads	الأحمال الميتة
Earthquake loads	أحمال الزلازل
Flood loads	أحمال الفيضانات
Live loads	الأحمال الحية
Rain loads	أحمال الأمطار
Roof loads	أحمال الأسطح
Snow loads	أحمال الثلوج
Wind loads	أحمال الرياح
Load intensity	كثافة / شدة الحمل
Loading conditions	حالات التحميل
Loading curve	منحنى التحميل
Local coordinate system	نظام الإحداثيات المحلية
Longitudinal fibers	الألياف الطولية
Low-rise buildings	مباني منخفضة الارتفاع
<b>M</b>	
Machines	آلات

Magnitude	قيمة
Mass	كتلة
Material	مادة
Mathematics	الرياضيات
Mathematical model	نموذج رياضي
Matrix	مصفوفة
Maximum	أقصى
Mechanical efficiency	الكفاءة الميكانيكية
Mechanics	علم الميكانيكا
Mechanism	آلية
Mega gram (Mg)	ميغرام (وحدة قياس)
Member	عضو / عنصر
Member coordinate system	نظام الإحداثيات للعناصر / للأعضاء
Member stiffness	صلابة الأعضاء
Meter (m)	متر (وحدة قياس)
Method	طريقة
Metric	مترى
Middle	وسط
Mild steel	الفولاذ الطري
Mile (mi)	ميل (وحدة قياس)
Minimum	الحد الأدنى
Minute (min)	دقيقة (وحدة قياس)
Modulus	معامل
Modulus of elasticity	معامل المرونة
Young's modulus	معامل يونج
Mohr s circle	دائرة (Mohr)
Moment	عزم
Motion	حركة وشيكة
Multi-force members	عناصر متعددة القوى

## N

Negative	سلبى
Neutral	محايد
Neutral axis	المحور المحايد
Neutral plane	المسطح المحايد
Newton (N)	نيوتن (وحدة قياس)
Newton's law of gravitation	قانون نيوتن للجاذبية
Newton's three fundamental laws	قوانين نيوتن الثلاثة الأساسية
Nonlinear	غير الخطية
Normal force	قوى طبيعية
Notations	الترميزات / الرموز
Numerical	عددي / رقمي
Numerical Analysis	تحليل رقمي
Numerical integration	تكامل رقمي

## O

Object	جسم
One-story building	مبنى من طابق واحد
Opposite	مقابل / عكس
Ordinate	الإحداثي الصادي (ص) لنقطة
Origin	الأصل
Original	أصلي

## P

Parallel	موازي
Parallelogram	متوازي الاضلاع
Partial constraints	قيود جزئية
Particle	جسيم
Pascals (Pa)	باسكالز (وحدة قياس)
Passing a section	يمر خلال مقطع
Perimeter	محيط

Permanent	دائم
Perpendicular	عمودي
Pin-support	دعامة (Pin)
Plane	مستوى / مسطح
Planar	ذو مستوى / ذو مسطح
Point	نقطة
Point of application	نقطة التطبيق
Point of inflection	نقطة التواء / تغير المسار / تغير التقوس
Polygon	المضلع
Position	موضع
Possible	ممکن
Potential energy	الطاقة الكامنة
Pound (lb)	جنيه (وحدة قياس)
Practical	عملي
Pratt truss	جمالون نوع (Pratt)
Prefixes	البادئات
Pressure	الضغط
Pressure distribution	توزيع الضغط
Pressure intensity	شدة الضغط
Primary moment	عزم أساسي
Principal axes	المحاور الرئيسية
Principle	مبدأ
Product	حاصل الضرب
Projection	إسقاط
Properties of areas	خصائص المساحات
Proportion	نسبة
Proportional limit	الحد النسبي
Pulleys	البكرات
Pure bending	العزم النقي
Pythagorean theorem	نظرية فيثاغورث

## Q

Quality	جودة
Quantity	كمية

## R

Radian	راديان (وحدة قياس)
Radius	نصف القطر
Range	نطاق
Ratio	نسبة
Reaction	رد فعل
Real work	العمل الحقيقي / الفعلي
Rectangular components	المكونات المستطيلة للمتجه
Reduction factor	عامل التخفيض
Redundant	زائد عن الحاجة / متوفر
Redundancy	وفرة
Redundant supports	دعم زائدة عن الحاجة
Reinforced concrete	خرسانة مسلحة
Relationship	صلة
Relative	نسبيا
Resistance	مقاومة
Resolution	تحليل
Result	نتيجة
Resultant	محصلة
Revolution	دوران
Right	عامودي / قائم
Right triangle	مثلث قائم
Right-hand rule	قاعدة اليد اليمنى
Right-handed coordinate system	نظام إحداثيات اليد اليمنى
Rigid	جامد
Rivet	برشام



Roof	سقف
Rotate	يدور
Rotated axes	محاور تم استدارتها
Rotation	دوران
Rough surfaces	الأسطح الخشنة
Rounding off	التقريب
Rule	قاعدة / قانون

## S

Safe	آمن
Scalar	عددي / رقمي
Scale	مقياس
Screw	برغي
Second (s)	الثانية (وحدة قياس)
Section	جزء
Semicircular area	منطقة نصف دائرية
Sense	إحساس
Series	سلسلة / متتالية
Service loads	أحمال خدمية
Shear	قص
Shear force	قوة القص
Shear force diagram	مخطط قوة القص
Shear stress	إجهاد القص
Sidesway	التمايل الجانبي
Sign conventions	توقيع الاتفاقيات
Similar	مماثل
Simple support	دعامة بسيطة
Slender	نحيل
Slip	انزلاق
Slope	ميل

Smooth surfaces	الأسطح الملساء
Solution	حل
Space	الفراغ / فضاء
Span	امتداد
Specific weight	الوزن المحدد
Spherical domes	قبة كروية
Spring	زنبرك
Spring constant	ثابت الزنبرك
Stable	مستقر
Static	ثابت / ساكن
Static equilibrium equations	معادلات التوازن الساكنة
Static friction	الاحتكاك الساكن
Statically determinate	محدد استاتيكيًا
Statically equivalent set	مجموعة مكافئة الستاتيكيًا
Statically indeterminate	غير محدد استاتيكيًا
Static-friction force	قوة الاحتكاك الثابت
Statics	علم الثوابت / السكون / الأجسام الساكنة
Stationary	ثابت
Stiffness	صلادة
Strategies	استراتيجيات
Strength	قوة
Stress	إجهاد
Bearing stress	إجهاد التحمل / الضغط
Normal stress	الإجهاد العمودي
Shear stress	إجهاد القص
Stretch	تمتد
Structural analysis	تحليل إنشائي
Structure	هيكل / منشأ
Subtraction	طرح
Sufficient conditions	ظروف كافية

Summary	ملخص
Superposition	تراكب
Super-positioned forces	قوى متراكبة
Super-positioned loads	أحمال متراكبة
Superimposing displacements	إزاحات متراكبة
Support	الدعم / دعامة
Fixed	ثابت / مرتكز
Hinged	فصالة
Roller	منزلق / قابل للانزلاق أفقياً
Surface force	قوة السطح
Suspended cables	الكابلات المعلقة
Symbol	رمز
Symmetry	تناظر
System	النظام

## T

Table	جدول / طاولة
Tangential	تماسي
Taylor series	متتالية تايلور
Temperature variation	تباين درجة الحرارة
Tension	شد
Test	اختبار
Theory	نظرية
Thickness	سماعة
Thin plates	لوحات رقيقة
Time	زمن / وقت
Ton (t)	طن (وحدة قياس)
Torque	عزم الدوران
Torsion	التواء
Translation	حركة / انتقال

Trapezoid	شبه منحرف
Triangle	مثلث
Triangle law	قانون المثلث
Tributary areas	المناطق الرافدة
Trigonometry	علم المثلثات
Truss	جمالون
Deck truss	جمالون لحمل الأسطح
Fink truss	جمالون (نوع Fink)
Howe truss	جمالون (نوع Howe)
K truss	جمالون (نوع K)
Pratt truss	جمالون (نوع Pratt)
Vierendeel truss	جمالون (نوع Vierendeel)
Warren truss	جمالون (نوع Warren)
Tsunami	تسونامي
Tube	إنبوب

## U

Ultimate	أقصى
Unbalanced moment (UM)	عزم غير متوازن
Underestimate	يقلل من شأن
Uniformly distributed load	الحمل الموزع بشكل موحد
Unit	وحدة
Universal gravitational constant	ثابت الجاذبية العالمي
Unknowns	المجاهيل
Unstable	غير مستقر

## V

Value	قيمة
Variable	متغير
Vector	متجه
Velocity	السرعة

Vertical	عمودي
Vierendeel truss	جمالون (نوع Vierendeel)
Virtual	افتراضي / غير واقعي
Volume	حجم

## W

Warren truss	جمالون (نوع Warren)
Watt (W)	واط (وحدة قياس)
Wave	موجة
Wedge	وند
Weight	وزن
Welding	لحام
Wheel friction	احتكاك العجلة
Width	عرض
Winches	الروافع
Wind	رياح
Wires	الأسلاك
Work	عمل
Wrench	مفتاح الربط

## Y

Yield	خضوع
Yield strain	انفعال الخضوع
Yield stress	إجهاد الخضوع
Young's modulus	معامل يونج

## Z

Zero-force members	العناصر ذات القوى الصفرية
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